MATHEMATICAL OPTIMIZATION MODEL FOR ALTERNATING-PHASE FOCUSING (APF) LINAC

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Mathematical model of beam dynamic optimization in an equivalent traveling wave is suggested. The problem of the APF linac parameters optimization is being discussed. Beam dynamics in the 14 MeV deuteron accelerator are considered.

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INTRODUCTION

At present time the design of accelerators with accelerating field focusing has become important. For example, feasibility of this focusing is possible in RFQ and APF accelerators. Problems of design and optimization of these structures were developed in [1 - 17]. Among these, we mention [15], which was first obtained synchronous phase sequence for focusing period in APF structure. RFQ structures are usually used as the initial part of the high-energy accelerators. Also, they are used independently for different application purposes. However, their applying is limited to low energies (2…5 MeV). Therefore a combination of RFQ and APF is considered to be a good solution for a high-energy accelerator. A high acceleration rate and an absence of magnetic field focusing in APF structure makes it more efficient and less expensive in comparison with Alvarez linac. Therefore, the development problems of these accelerators and improving the quality of beams are still important and vital.

Various mathematical optimization models in RFQ structures were proposed in [1, 9-12, 14, 16]. In particular, some models allow us to analyze the longitudinal and transversal motions separately. Beam dynamic optimization in an equivalent traveling wave is carried out and the limitation of the defocusing factor is taken into account. It is possible to obtain a guaranteed good transversal dynamics. These ideas are considered in the case of acceleration in the APF structures too.

In this paper we examine an approach of beam dynamics simulations in an equivalent traveling wave. To estimate the beam quality we introduce the integral functional. The numerical gradient method is suggested to minimize this functional.

1. THE APF PRINCIPLE

The main parameter that determines the beam dynamics in an APF accelerator is the synchronous phase sequence φ_s. The concept of APF is to obtain this sequence providing longitudinal and transversal motions stability alternately so that in general motion is stable. For example, we can obtain this sequence using the analysis of stability diagrams for Hill equation. Next we consider the possibility of optimizing the phase of synchronous particle in order to improve the beam dynamics.

2. MATHEMATICAL MODEL OF BEAM DYNAMICS

Denote by L_i coordinate of the i -th acceletaring cell and by D_i coordinate of this center. Then accelerating field approximation may be assumed as

$$E_i(z,t) = (-1)^i E_{max} \cos\left(\frac{\pi(z-D_i)}{L_i - L_{i-1}}\right)\cos(\omega t).$$

This standing wave approximation allows to accept the following mathematical model of beam dynamics in the equivalent traveling wave [18, 19].

$$\frac{d\beta_s}{d\tau} = \alpha\sqrt{1-\beta_s^2}\cos(\varphi_s(\tau)), \quad (1)$$

$$\frac{d\psi}{d\tau} = \frac{2\pi(1-\beta_s^2)^{3/2}}{\beta_s^2} p, \quad (2)$$

$$\frac{dp}{d\tau} = \alpha\beta_s(\cos(\varphi_s(\tau)) - \cos(\varphi_s(\tau)+\psi)), \quad (3)$$

$$\frac{dS_{11}}{d\tau} = 2S_{21}, \quad (4)$$

$$\frac{dS_{21}}{d\tau} = -\frac{\alpha\pi(1-\beta_s^2)^{3/2}}{\beta_s}\sin(\varphi_s(\tau)+\psi)S_{11} + S_{22}, \quad (5)$$

$$\frac{dS_{22}}{d\tau} = -2\frac{\alpha\pi(1-\beta_s^2)^{3/2}}{\beta_s}\sin(\varphi_s(\tau)+\psi)S_{21}. \quad (6)$$

Where $\psi = \varphi - \varphi_s$, $p = \gamma_s - \gamma$, $\varphi_s(\tau)$, $\tau = ct/\lambda$, $\alpha = q\lambda E_{max}/(2m_0c^2)$ is the accelerating amplitude parameter, S_{11}, S_{21}, S_{22} is the element of matrix $G = \begin{pmatrix} S_{11} & S_{21} \\ S_{21} & S_{22} \end{pmatrix}$, that describe dynamics of initial ellipse G_0 in the radial plane (ρ,η), where $\rho = r/\lambda$ is the radial position of particle, $\eta = d\rho/d\tau$. Step-wise function $\varphi_s(\tau)$ defined as

$$\varphi(t) = \varphi_i, \quad \text{if} \quad \tau \in [\tau_{i-1}, \tau_i],$$
$$\varphi_i \in [-2\pi, 2\pi]$$
$$\tau_i - \tau_{i-1} = (\varphi_i - \varphi_{i-1} + \pi)/2\pi.$$

Thus, $\varphi_s(\tau)$ is parameterized only by parameters $\varphi_s^{(i)}$.

3. OPTIMIZATION PROBLEM

Let us consider the formalization of the beam dynamics optimization problem. We may consider (1)-(6) as equations that described program and disturbed motions of charged particles [9].

$$\frac{dx}{dt} = f(t, x(t), u(t)),$$

$$\frac{dy}{dt} = F(t, x(t), y(t), u(t)),$$

$$x(0) = x_0,\ y(0) = y_0 \in \overline{M}_0.$$

Where x is a scalar and y is a n - dimensional vector of phase coordinates, f is a scalar function and F is a n - dimensional vector function. Control $u(t)$ is correspond to function $\varphi_s(\tau)$. Accordingly, the control function will be parameterized as

$$u(t) = u_i, \quad \text{if} \quad t \in [t_{i-1}, t_i],$$
$$u_i \in [-2\pi, 2\pi] \tag{7}$$
$$t_i - t_{i-1} = (u_i - u_{i-1} + \pi)/2\pi.$$

Such control is admissible. Let us consider the following functional

$$I(u) = \int_{M_{T,u}} g_1(y_T)dy_T + g_2(x_T). \tag{8}$$

This functional is an integral estimate the beam quality at the moment $t = T$. Here the set $M_{T,u}$ is a cross-section of the beam of trajectories at the moment $t = T$, i.e. $y_T = y(T) \in M_{T,u}$, non-negative functions g_1 and g_2 is characterization the dynamics of the disturbed and program motions. We consider the problem of functional (8) minimizing for the admissible controls.

To solve the problem of reducing the loss of particles and decrease the beam radius it is possible to take

$$g_1 = c_1 F_1(\psi_T) + c_2 F_2(S_{11}),$$

where

$$F_1 = \begin{cases} (\psi_T + \psi_1)^2, & \text{if} \quad \psi_T < -\psi_1, \\ 0, & \text{if} \quad \varphi_T \in [\psi_1, \psi_2], \\ (\psi_T - \psi_2)^2, & \text{if} \quad \psi_T > \psi_2, \end{cases}$$

$$F_2 = \begin{cases} 0, & \text{if} \quad S_{11} < S, \\ (S_{11} - S)^2, & \text{if} \quad S_{11} > S. \end{cases}$$

Where $c_1, c_2, \psi_1, \psi_2, S$ are the non-negative constants.

4. SOLUTION METHOD

The variation of functional (8) can be written as [9]

$$\delta I = -\int_0^T \left[\int_{M_{t,u}} \left(\mu^T \Delta_u F + \nu^T \Delta_u f \right) dy_t + \psi^T \Delta_u f \right] dt,$$

where ψ, μ, ν are satisfying the following equations

$$\frac{d\mu}{dt} = -\left(\frac{\partial F}{\partial y} \right)^T \mu,$$

$$\frac{d\nu}{dt} = -\left(\frac{\partial f}{\partial x} \right)^T \nu - \left(\frac{\partial F}{\partial x} \right)^T \mu,$$

$$\frac{d\psi}{dt} = -\left(\frac{\partial f}{\partial x} \right)^T \psi.$$

$$\mu(T, x(T)) = -\left(\frac{\partial g_1(y_T)}{\partial y} \right)^T,$$

$$\nu(T, x(T)) = 0,$$

$$\psi(T, x(T)) = -\left(\frac{\partial g_2(x_T)}{\partial x} \right)^T.$$

Having considered the parameterization (7) we can obtain the functional gradient

$$\begin{aligned} \frac{\partial I}{\partial u_i} = \frac{1}{2\pi} \int_{M_{t_i,u}} & [\mu^T(t_i)(F(t_i, x(t_i), y(t_i), u_i) - \\ & - F(t_i, x(t_i), y(t_i), u_{i-1})) + \nu^T(t_i) \times \\ & \times \left(f(t_i, x(t_i), u_i) - f(t_i, x(t_i), u_{i-1}) \right)] dy_t - \\ & - \int_{t_i}^{t_{i+1}} \int_{M_{t,u}} [\mu^T(t) \frac{\partial F(t, x(t), y(t), u_i)}{\partial u} + \\ & + \nu^T(t) \frac{\partial f(t, x(t), u_i)}{\partial u}] dy_t dt. \end{aligned} \tag{9}$$

By using the analytic representation (9) of the functional gradient we can build gradient methods for optimizing the accelerator parameters.

5. NUMERICAL SIMULATIONS

5.1. ACCELERATOR PARAMETERS

To illustrate the approach described earlier, we consider the problem of design parameters for 433 MHz, 3.5 m long deuteron accelerator for 14 MeV output energy. Input energy is 3.5 MeV, amplitude of accelerating field is 110 kV/cm, and 6 focusing periods consist of 115 accelerating cells. Provided NIIEFA input emittances are present at Figs. 1, 2. Main parameters of structure are present at Figs. 3, 4.

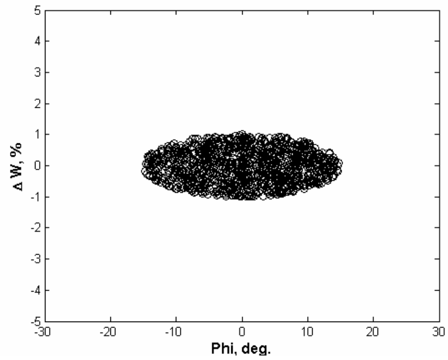

Fig. 1. Input longitudinal emittance

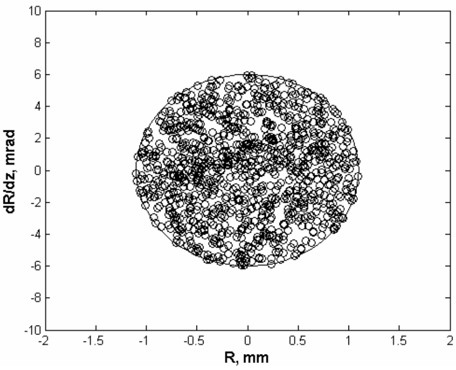

Fig. 2. Input transversal emittance

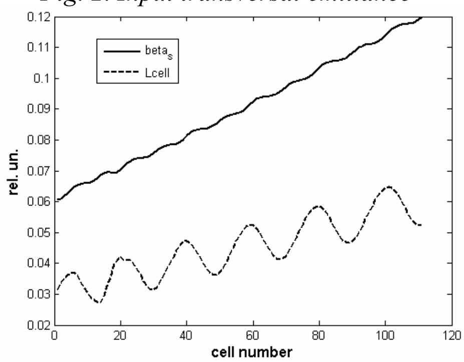

Fig. 3. Synchronous particle velocity and lengh of cells

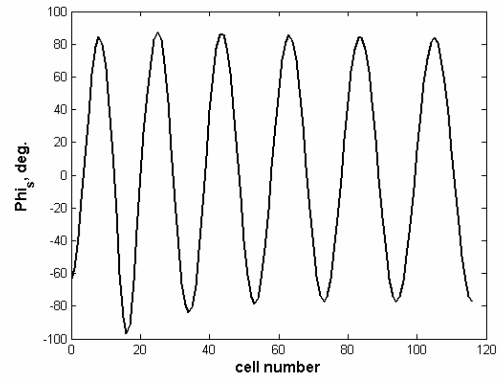

Fig. 4. Synchronous phase sequence

5.2. SIMULATIONS WITHOUT INTERACTION OF CHARGED PARTICLES

The results of beam dynamics simulations without interaction between charged particles are presented at Figs. 5 - 9. There are no particle losses at this model.

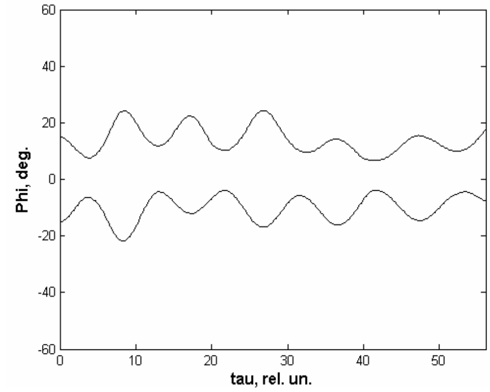

Fig. 5. Beam envelopes for phase motion

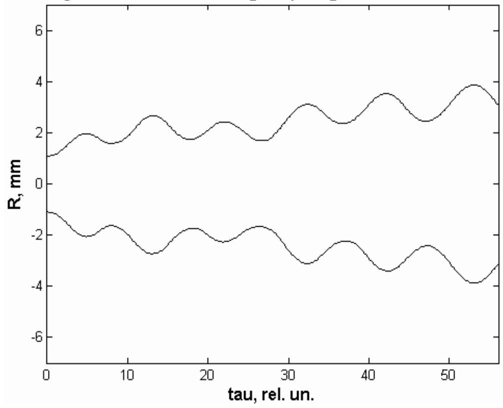

Fig. 6. Beam envelopes for radial motion

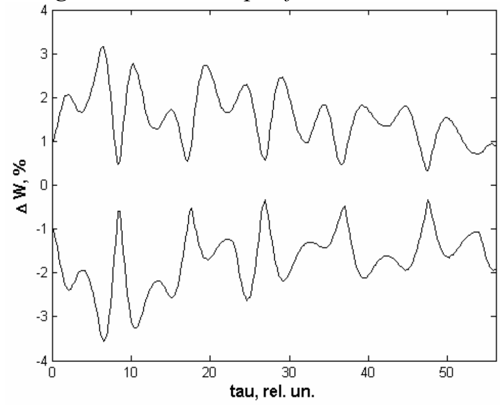

Fig. 7. Beam envelopes for energy motion

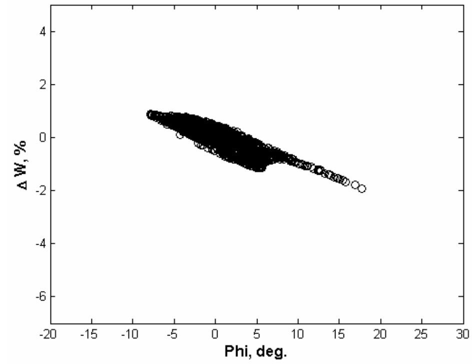

Fig. 8. Output longitudinal emittance

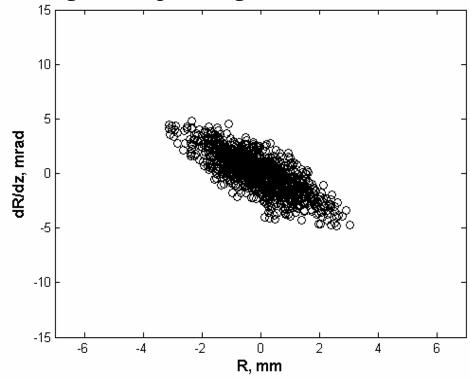

Fig. 9. Output transversal emittance

5.3. SIMULATIONS WITH 14 mA BEAM CURRENT

In this section the results of beam dynamics simulations with 14 mA beam current are presented. It should be noted that the optimization of the structure was carried out without taking into account the interaction of charged particles, but the results of this simulations show good beam quality. Output emittances are present at Figs. 10, 11. Particle losses are 8 %.

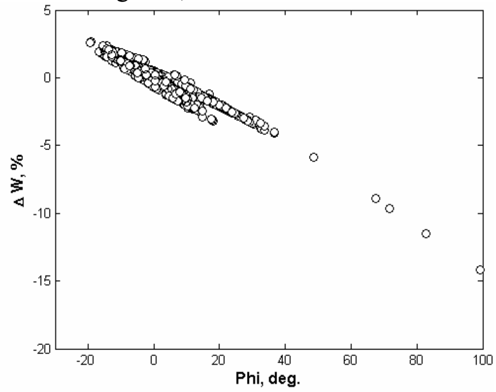

Fig. 10. Output longitudinal emittance

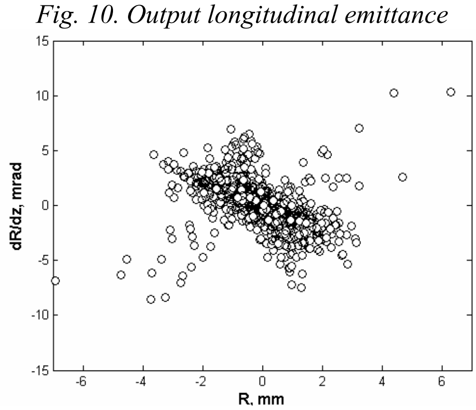

Fig. 11. Output transversal emittance

CONCLUSIONS

In this paper the problem of optimizing the parameters of longitudinal and transverse motion of the beam in an APF accelerator is examined. This problem is formalized and the solution based on analyzing the beam dynamics in an equivalent traveling wave is proposed. This approach does not impose rigid restrictions on the structure geometry, which later on allows to provide precision tuning of resonators to produce the desired field distribution. Analytic representation of the gradient of the quality functional allows us to provide the numerical optimization by other motion parameters. For instance it can be the width of the output energy and phase spectrum, the radial divergence of the beam, the effective emittance or the acceleration intensity.

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МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ОПТИМИЗАЦИИ ПАРАМЕТРОВ УСКОРИТЕЛЯ С ПЕРЕМЕННО-ФАЗОВОЙ ФОКУСИРОВКОЙ

Д.А. Овсянников, В.В. Алцыбеев

Предложена математическая модель оптимизации на основе уравнений динамики частиц в поле эквивалентной бегущей волны. Решена задача оптимизации параметров ускорителя с ПФФ. Рассмотрена динамика пучка в ускорителе дейтронов на 14 МэВ.

МАТЕМАТИЧНА МОДЕЛЬ ОПТИМІЗАЦІЇ ПАРАМЕТРІВ ПРИСКОРЮВАЧА З ПЕРЕМІННО-ФАЗОВИМ ФОКУСУВАННЯМ

Д.О. Овсянников, В.В. Алцибєєв

Запропоновано математичну модель оптимізації на основі рівнянь динаміки часток в полі еквівалентної біжучої хвилі. Вирішена задача оптимізації параметрів прискорювача з ПФФ. Розглянуто динаміку пучка в прискорювачі дейтронів на 14 МеВ.