

# NON-RELATIVISTIC ELECTRONICS

## TRANSITION RADIATION OF AN ELECTRON CROSSING AN INTERFACE BETWEEN A DIELECTRIC AND A LAYERED SUPERCONDUCTOR

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We investigate the transition radiation of an electron crossing an interface between a dielectric and a layered superconductor. The electron's direction of motion and the orientation of the superconductor's layers are perpendicular to the interface. The analysis of the radiation patterns reveals the strong anisotropy of the radiation intensity distribution with respect to the azimuth angle in the interface plane.

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### INTRODUCTION

As is known, the transition radiation effect, i.e. the effect of radiation of a uniformly moving electron due to its transition from one medium into another, was discovered by V.L. Ginzburg and I.M. Frank in 1945 [1]. Since then the numerous papers have been written on the subject (see, e.g., [2 - 6]). Nowadays a good deal of attention to the transition radiation effect is caused by a large number of its applications. For instance, the transition radiation is used in high-energy physics to the detection of charged particles [2]. The electron bunch bombardment of the solids [6] and transit of the bunches through diaphragms [8, 9] allow to generate short high power pulses which are widely used for radiolocation. The transition radiation also finds application in investigation of two-dimensional electron systems which are the basis for many up-to-date electronic devices [10, 11]. Besides, the radiation of modulated electron beams crossing a boundary of a plasma-like medium is a very effective method for generation of surface electromagnetic waves [12 - 13].

The characteristic properties of the transition radiation of both the electrons and electron bunches crossing anisotropically conducting interfaces such as wire shields have been considered in [14 - 16]. Specifically, the possibility of obtaining an elliptical polarization of electromagnetic waves has been shown. V.E. Pafomov in 1959 first proved the possibility of excitation of the electromagnetic waves with negative group velocity with the aid of the Vavilov-Cherenkov radiation in the medium which possesses negative permittivity and negative permeability simultaneously (so-called left-handed medium) [17]. Namely, he first demonstrated that if an electron moves from a vacuum into the medium, the maximum of the intensity of the Vavilov-Cherenkov radiation is in a vacuum. A similar effect occurs when an electron crosses an interface between a vacuum and a uniaxial anisotropic conducting medium (e.g., layered superconductor) in the case where the conducting layers are parallel to the interface [18].

In the present paper we theoretically investigate the transition radiation of an electron crossing a layered superconductor in the case where its layers are perpendicular to the interface. Specifically, it turned out that

unlike the results of work [18], in our system the excitation of the electromagnetic waves with negative group velocity in the layered superconductor is principally impossible, i.e. the energy flux density vector in the superconductor does not form an obtuse angle with the direction of the electron motion. In addition, we have found that energy flux distribution of transition radiation exhibits a strong anisotropy on azimuth angle in the interface plane.

### 1. STATEMENT OF THE PROBLEM AND BASIC EQUATIONS

Let the structure under study be comprised of the dielectric (the region with index 1) with permittivity  $\epsilon_1$  and the layered superconductor (the region with index 2) with dielectric permittivity given by the diagonal permittivity tensor with components  $\epsilon_{xx} = \epsilon_{yy} = \epsilon_{ab}$ ,  $\epsilon_{zz} = \epsilon_c$  (Fig. 1), where

$$\epsilon_{ab} = \epsilon \left( 1 - \gamma^2 / \bar{\omega}^2 + i \gamma^2 v_{ab} / \bar{\omega} \right), \quad (1)$$

$$\epsilon_c = \epsilon \left( 1 - 1 / \bar{\omega}^2 + i v_c / \bar{\omega} \right). \quad (2)$$

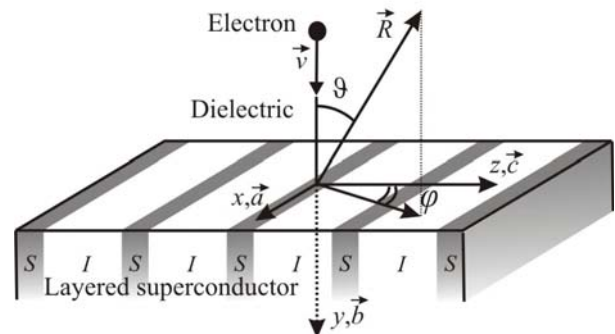


Fig. 1. Geometry of the problem

Here, we introduce the dimensionless parameters  $\bar{\omega} = \omega / \omega_J$ ,  $v_{ab} = 4\pi\sigma_{ab} / (\epsilon\omega_J\gamma^2)$  and  $v_c = 4\pi\sigma_c / (\epsilon\omega_J)$ ,  $\gamma = \lambda_c / \lambda_{ab} \gg 1$  is the current-anisotropy parameter,  $\lambda_c$  and  $\lambda_{ab}$  are the magnetic-field penetration depths along and across the layers, respectively. The relaxation frequencies  $v_{ab}$  and  $v_c$  are proportional to the averaged quasi-particle conductivities  $\sigma_{ab}$  (along the layers) and  $\sigma_c$  (across the layers), respectively,  $\omega_J = (8\pi e D J_c / \hbar \epsilon)^{1/2}$

is the Josephson plasma frequency,  $e$  is the electron charge,  $D$  is the period of the layered structure,  $J_c$  is the critical Josephson current density,  $\varepsilon$  is the inter-layer dielectric constant. All media are supposed to be nonmagnetic. We define our coordinate system so that the dielectric occupies the half-space  $y < 0$ , the layered superconductor occupies the half-space  $y > 0$ , and the  $z$ -axis coincides with the crystallographic  $\bar{c}$ -axis of superconductor. An electron moves uniformly in the dielectric along the  $y$ -axis at a velocity  $v \ll c$  (where  $c$  is the speed of light in a vacuum) and crosses the interface. The electron-charge density  $n(\vec{r}, t)$  is determined by the formula

$$n(\vec{r}, t) = e\delta(x)\delta(v - yt)\delta(z), \quad (3)$$

where  $\delta(x)$  is the Dirac delta function. The electromagnetic fields of the electron are expressed in terms of Fourier integrals

$$\vec{E}_\ell^e(\vec{r}, t) = \int \vec{E}_\ell^e(\vec{k}, \omega) \exp[i(\vec{k}\vec{r} - \omega t)] d\vec{k} d\omega, \quad (4)$$

where  $\ell = 1, 2$  is the number of the medium,  $\vec{k} = (k_x, k_y, k_z)$ . The Fourier components for the electric and magnetic fields of the electron in the dielectric are

$$\vec{E}_1^e(\vec{k}, \omega) = \frac{ie}{2\pi^2 \varepsilon_1} \frac{\omega \varepsilon_1 \vec{v} / c^2 - \vec{k}}{k^2 - (\omega/c)^2 \varepsilon_1} \delta(\omega - k_y v), \quad (5)$$

$$\vec{H}_1^e(\vec{k}, \omega) = \frac{\varepsilon_1}{c} \vec{v} \times \vec{E}_1^e(\vec{k}, \omega). \quad (6)$$

The corresponding field components in the superconductor are

$$E_{2x}^e(\vec{k}, \omega) = -\frac{ievk_x k_y \Lambda_1}{2\pi^2 \omega \Delta^{(o)} \Delta^{(e)}} \delta(\omega - k_y v), \quad (7)$$

$$E_{2y}^e(\vec{k}, \omega) = -\frac{iev\Lambda_2}{2\pi^2 \omega \Delta^{(o)} \Delta^{(e)}} \delta(\omega - k_y v), \quad (8)$$

$$E_{2z}^e(\vec{k}, \omega) = -\frac{ievk_z k_y}{2\pi^2 \omega \Delta^{(e)}} \delta(\omega - k_y v), \quad (9)$$

$$\vec{H}_2^e(\vec{k}, \omega) = \frac{1}{c} \vec{v} \times \vec{D}_2^e(\vec{k}, \omega), \quad (10)$$

where  $D_{2j}^e(\vec{k}, \omega) = \varepsilon_{jk}(\omega) E_{2k}^e(\vec{k}, \omega)$ ,

$$\Lambda_1 = k^2 - \frac{\omega^2}{c^2} \varepsilon_c, \quad (11)$$

$$\Lambda_2 = \left( k_y^2 - \frac{\omega^2}{c^2} \varepsilon_{ab} \right) \left( k^2 - \frac{\omega^2}{c^2} \varepsilon_c \right) - k_z^2 \frac{\omega^2}{c^2} (\varepsilon_c - \varepsilon_{ab}), \quad (12)$$

$$\Delta^{(o)} = k^2 - \frac{\omega^2}{c^2} \varepsilon_{ab}, \quad (13)$$

$$\Delta^{(e)} = k_z^2 \varepsilon_c + (k_x^2 + k_y^2) \varepsilon_{ab} - \frac{\omega^2}{c^2} \varepsilon_{ab} \varepsilon_c. \quad (14)$$

The radiation fields we express in terms of the following Fourier integrals

$$\vec{E}_\ell(\vec{r}, t) = \int \vec{E}_\ell(\vec{k}, \omega) \exp[i(\vec{k}\vec{\rho} + k_{ly} y - \omega t)] d\vec{k} d\omega, \quad (15)$$

where  $\vec{k} = (k_x, k_z)$  is the wave vector in the  $xz$  plane,  $\vec{\rho} = (x, y)$  is the radius vector in the  $xz$  plane. We assume that the radiation field in the dielectric is the

superposition of the ordinary and extraordinary electromagnetic waves with components  $(E_{1x}^{(o)}, E_{1y}^{(o)}, 0)$ ,  $(H_{1x}^{(o)}, H_{1y}^{(o)}, H_{1z}^{(o)})$  and  $(E_{1x}^{(e)}, E_{1y}^{(e)}, E_{1z}^{(e)})$ ,  $(H_{1x}^{(e)}, H_{1y}^{(e)}, 0)$ , respectively. From the Maxwell equations we attain the following relationships between the Fourier components of electric and magnetic fields of  $o$  and  $e$  types:

$$E_{1y}^{(o)}(\vec{k}, \omega) = -\frac{k_x}{k_{1y}} E_{1x}^{(o)}(\vec{k}, \omega), \quad (16)$$

$$H_{1x}^{(o)}(\vec{k}, \omega) = \frac{ck_x k_z}{\omega k_{1y}} E_{1x}^{(o)}(\vec{k}, \omega), \quad (17)$$

$$H_{1y}^{(o)}(\vec{k}, \omega) = \frac{ck_z}{\omega} E_{1x}^{(o)}(\vec{k}, \omega), \quad (18)$$

$$H_{1z}^{(o)}(\vec{k}, \omega) = -\frac{c}{\omega} \frac{k_x^2 + k_{1y}^2}{k_{1y}} E_{1x}^{(o)}(\vec{k}, \omega), \quad (19)$$

$$E_{1y}^{(e)}(\vec{k}, \omega) = \frac{k_{1y}}{k_x} E_{1x}^{(e)}(\vec{k}, \omega), \quad (20)$$

$$E_{1z}^{(e)}(\vec{k}, \omega) = -\frac{k_x^2 + k_{1y}^2}{k_x k_z} E_{1x}^{(e)}(\vec{k}, \omega), \quad (21)$$

$$H_{1x}^{(e)}(\vec{k}, \omega) = -\frac{\omega}{c} \frac{k_{1y}}{k_x k_z} \varepsilon_1 E_{1x}^{(e)}(\vec{k}, \omega), \quad (22)$$

$$H_{1y}^{(e)}(\vec{k}, \omega) = \frac{\omega}{ck_z} \varepsilon_1 E_{1x}^{(e)}(\vec{k}, \omega), \quad (23)$$

where  $k_{1y} = -\sqrt{\omega^2 \varepsilon_1 / c^2 - \kappa^2}$ . In the layered superconductor the radiation field we represent as the superposition of electromagnetic waves of  $o$  and  $e$  types with following Fourier components:

$$E_{2y}^{(o)}(\vec{k}, \omega) = -\frac{k_x}{k_{2y}^{(o)}} E_{2x}^{(o)}(\vec{k}, \omega), \quad (24)$$

$$H_{2x}^{(o)}(\vec{k}, \omega) = \frac{ck_x k_z}{\omega k_{2y}^{(o)}} E_{2x}^{(o)}(\vec{k}, \omega), \quad (25)$$

$$H_{2y}^{(o)}(\vec{k}, \omega) = \frac{ck_z}{\omega} E_{2x}^{(o)}(\vec{k}, \omega), \quad (26)$$

$$H_{2z}^{(o)}(\vec{k}, \omega) = -\frac{c}{\omega} \frac{k_x^2 + k_{2y}^{(o)2}}{k_{2y}^{(o)}} E_{2x}^{(o)}(\vec{k}, \omega), \quad (27)$$

$$E_{2y}^{(e)}(\vec{k}, \omega) = \frac{k_{2y}^{(e)}}{k_x} E_{2x}^{(e)}(\vec{k}, \omega), \quad (28)$$

$$E_{2z}^{(e)}(\vec{k}, \omega) = -\frac{\varepsilon_{ab}}{\varepsilon_c} \frac{k_x^2 + k_{2y}^{(e)2}}{k_x k_z} E_{2x}^{(e)}(\vec{k}, \omega), \quad (29)$$

$$H_{2x}^{(e)}(\vec{k}, \omega) = -\frac{\omega}{c} \frac{k_{2y}^{(e)}}{k_x k_z} \varepsilon_{ab} E_{2x}^{(e)}(\vec{k}, \omega), \quad (30)$$

$$H_{2y}^{(e)}(\vec{k}, \omega) = \frac{\omega}{ck_z} \varepsilon_{ab} E_{2x}^{(e)}(\vec{k}, \omega), \quad (31)$$

where

$$k_{2y}^{(o)} = \sqrt{\omega^2 \varepsilon_{ab} / c^2 - \kappa^2}, \quad (32)$$

$$k_{2y}^{(e)} = \sqrt{\omega^2 \varepsilon_c / c^2 - k_x^2 - \varepsilon_c k_z^2 / \varepsilon_{ab}}. \quad (33)$$

From the continuity conditions for the tangential components of the electric and magnetic fields at the interface we derive the following expressions for the total radiation fields in the dielectric:

$$E_{1x}(\vec{k}, \omega) = -\frac{i\omega}{c} \frac{\alpha_1 Q_2 + \alpha_3 Q_1}{\Delta_{OSW}}, \quad (34)$$

$$E_{1y}(\vec{k}, \omega) = \frac{i\omega}{ck_{1y}\Delta_{OSW}} \times \quad (35)$$

$$\times [(\alpha_3 k_x + \alpha_2 k_z) Q_1 + (\alpha_1 k_x - \alpha_4 k_z) Q_2],$$

$$E_{1z}(\vec{k}, \omega) = \frac{i\omega}{c} \frac{\alpha_4 Q_2 - \alpha_2 Q_1}{\Delta_{OSW}}, \quad (36)$$

$$H_{1x}(\vec{k}, \omega) = \frac{c}{\alpha_1 k_{1y} \omega} E_{1x}(\vec{k}, \omega) \times \quad (37)$$

$$\times \left[ \alpha_1 k_x k_z - \alpha_4 (k_z^2 + k_{1y}^2) \right] - \frac{i(k_z^2 + k_{1y}^2)}{\alpha_1 k_{1y}} Q_1,$$

$$H_{1y}(\vec{k}, \omega) = \frac{c}{\omega} \left( k_z + \frac{\alpha_4}{\alpha_1} k_x \right) E_{1x}(\vec{k}, \omega) + \frac{ik_x}{\alpha_1} Q_1, \quad (38)$$

$$H_{1z}(\vec{k}, \omega) = -\frac{c}{\alpha_1 k_{1y} \omega} E_{1x}(\vec{k}, \omega) \times \quad (39)$$

$$\times \left[ \alpha_1 (k_x^2 + k_{1y}^2) - \alpha_4 k_x k_z \right] + \frac{ik_x k_z}{\alpha_1 k_{1y}} Q_1,$$

where

$$\alpha_1 = -k_x k_z \frac{k_{2y}^{(o)} - k_{1y}}{k_{1y} k_{2y}^{(o)}}, \quad \alpha_2 = -k_x k_z (\varepsilon_{ab} - \varepsilon_1), \quad (40)$$

$$\alpha_3 = k_z^2 (\varepsilon_{ab} - \varepsilon_1) + \varepsilon_{ab} k_{1y} (k_{1y} - k_{2y}^{(e)}), \quad (41)$$

$$\alpha_4 = k_{2y}^{(o)-1} \left( \frac{\omega^2}{c^2} \varepsilon_{ab} - k_z^2 \right) - k_{1y}^{-1} \left( \frac{\omega^2}{c^2} \varepsilon_1 - k_z^2 \right), \quad (42)$$

$$\Delta_{OSW} = \alpha_1 \alpha_2 + \alpha_3 \alpha_4, \quad (43)$$

$$Q_1 = -\frac{eck_x}{2\pi^2 \nu \omega \varepsilon_1 k_{2y}^{(o)} \Delta_1 \Delta^{(o)} \Delta^{(e)}} \times$$

$$\times \left\{ \left[ \Delta^{(o)} \Delta^{(e)} - \varepsilon_1 \Delta_1 \left( k^2 - \frac{\omega^2}{c^2} \varepsilon_c \right) \right] \left( \frac{\omega^2}{c^2} \varepsilon_{ab} - k_z^2 \right) - \quad (44)$$

$$-\frac{\omega \nu}{c^2} k_{2y}^{(o)} \varepsilon_1 \left[ \Delta^{(o)} \Delta^{(e)} - \varepsilon_{ab} \Delta_1 \left( k^2 - \frac{\omega^2}{c^2} \varepsilon_c \right) \right] +$$

$$+ k_z^2 \Delta^{(o)} \left( \Delta^{(e)} - \varepsilon_1 \Delta_1 \right) \left. \right\},$$

$$Q_2 = \frac{eck_z k_{1y}}{2\pi^2 \nu \omega^2 \Delta_1 \Delta^{(o)} \Delta^{(e)}} \times$$

$$\times \left\{ \nu \Delta^{(o)} \left( \Delta^{(e)} - \varepsilon_c \Delta_1 \right) \left( \frac{\omega^2}{c^2} \varepsilon_{ab} - k_z^2 \right) - \quad (45)$$

$$-\nu k_x^2 \left[ \Delta^{(o)} \Delta^{(e)} - \varepsilon_{ab} \Delta_1 \left( k^2 - \frac{\omega^2}{c^2} \varepsilon_c \right) \right] -$$

$$-\frac{\omega \varepsilon_{ab}}{\varepsilon_1} k_{2y}^{(e)} \Delta^{(o)} \left( \Delta^{(e)} - \varepsilon_1 \Delta_1 \right) \left. \right\},$$

$$\Delta_1 = k^2 - \frac{\omega^2}{c^2} \varepsilon_1, \quad k^2 = k_x^2 + k_z^2 + \frac{\omega^2}{v^2}. \quad (46)$$

In order to derive the spatial and time dependencies of the total electromagnetic fields in the explicit forms, we need to integrate expressions (34) - (39) with respect to  $k_x$  and  $k_z$  by means of the stationary phase method for two-dimensional integrals [19]. According to this method, we obtain following stationary points:

$$k_{x0} = \frac{\omega}{c} \sqrt{\varepsilon_1} \sin \vartheta \sin \varphi, \quad (47)$$

$$k_{z0} = \frac{\omega}{c} \sqrt{\varepsilon_1} \sin \vartheta \cos \varphi, \quad (48)$$

where  $\vartheta$  is the angle relative to the  $y$ -axis,  $\varphi$  is the azimuth angle in the interface plane (see Fig. 1).

To calculate the energy losses by the electron to radiation in the dielectric we find the energy flux of the total electromagnetic wave in the dielectric across the remote hemisphere using the following time-integrated Poynting vector:

$$\langle \vec{S} \rangle = \frac{c}{4\pi} \operatorname{Re} \int_{-\infty}^{\infty} dt \left[ \vec{E}(\vec{r}, t) \vec{H}^*(\vec{r}, t) \right]. \quad (49)$$

Eventually, we will get the following expression for the spectral density  $\Pi(\bar{\omega}, \vartheta, \varphi)$  of the radiation energy (in units of  $\Pi_0 = e^2/c$ ) per unit solid angle  $d\Omega = \sin \vartheta d\vartheta d\varphi$  averaged over all transit time of the electron:

$$\begin{aligned} \Pi(\bar{\omega}, \vartheta, \varphi) &= 2\pi^2 \varepsilon_1 \bar{\omega}^2 \cos^2 \vartheta \times \\ &\times \operatorname{Re} \left\{ \left[ \vec{E}_{1y}(\bar{\omega}, \vartheta, \varphi) \vec{H}_{1z}^*(\bar{\omega}, \vartheta, \varphi) - \right. \right. \\ &- \vec{E}_{1z}(\bar{\omega}, \vartheta, \varphi) \vec{H}_{1y}^*(\bar{\omega}, \vartheta, \varphi) \left. \right] \sin \vartheta \sin \varphi + \\ &+ \left[ \vec{E}_{1x}(\bar{\omega}, \vartheta, \varphi) \vec{H}_{1y}^*(\bar{\omega}, \vartheta, \varphi) - \right. \\ &- \vec{E}_{1y}(\bar{\omega}, \vartheta, \varphi) \vec{H}_{1x}^*(\bar{\omega}, \vartheta, \varphi) \left. \right] \sin \vartheta \cos \varphi - \\ &- \left[ \vec{E}_{1z}(\bar{\omega}, \vartheta, \varphi) \vec{H}_{1x}^*(\bar{\omega}, \vartheta, \varphi) - \right. \\ &- \left. \left. \vec{E}_{1x}(\bar{\omega}, \vartheta, \varphi) \vec{H}_{1z}^*(\bar{\omega}, \vartheta, \varphi) \right] \cos \vartheta \right\}, \end{aligned} \quad (50)$$

where  $\vec{E}_{1j}(\bar{\omega}, \vartheta, \varphi)$ ,  $\vec{H}_{1j}(\bar{\omega}, \vartheta, \varphi)$  are dimensionless Fourier components for the radiation fields (in units of  $e/\omega_j$ ) given by equations (34) - (39) and expressed in terms of dimensionless frequency  $\bar{\omega}$  with due account of equations (47), (48).

## 2. ANALYSIS OF THE SPECTRUM

Let us analyze the dependences of the spectral density  $\Pi(\bar{\omega}, \vartheta, \varphi)$  on the tilt angle  $\vartheta$  and the azimuth angle  $\varphi$ . Hereafter, we will make use of the following parameters of the adjacent media:

$$\varepsilon_1 = 1, \quad \varepsilon = 16, \quad \gamma = 200, \quad \nu_{ab} = 0, \quad \nu_c = 10^{-5}. \quad (51)$$

Fig. 2 shows the  $\vartheta$  dependence of  $\Pi$  for a number of  $\varphi$  values at  $\bar{\omega} = 0.7$ ,  $\beta = \nu/c = 0.3$  (the electron enters into the superconductor). In Fig. 2 curve 1 corresponds to  $\varphi = 90^\circ$ , curve 2 is for  $\varphi = 60^\circ$ , curve 3 is for  $\varphi = 30^\circ$ , and curve 4 is for  $\varphi = 0^\circ$ . As seen from

Fig. 2, at  $\varphi = 90^\circ$  the maximum of the spectral energy density is located at  $\vartheta \approx 90^\circ$ .

Physically this implies that radiation energy flux at  $\vartheta \approx 90^\circ$  is directed at a grazing angle to the interface.

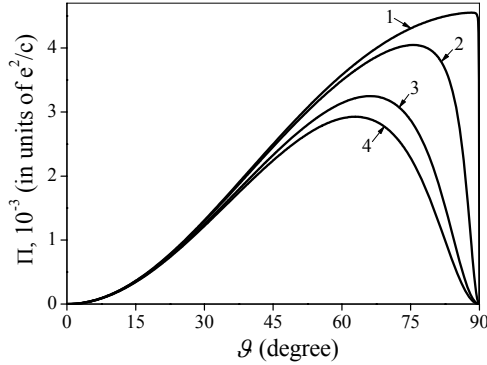


Fig. 2. The dependences of  $\Pi(\vartheta)$  for a number of  $\varphi$  values at  $\bar{\omega} = 0.7$ ,  $\beta = 0.3$

It easily seen from Fig. 2 that the maximum of the spectral energy density decreases and shifts towards lower values of the angle  $\vartheta$  with decreasing the angle  $\varphi$ . Note that at angles  $\varphi \approx 0^\circ$  and  $\varphi \approx 90^\circ$  the radiation field is TM polarized. At  $\varphi \approx 0^\circ$  the radiation field has components  $(0, E_{1y}, E_{1z})$ ,  $(H_{1x}, 0, 0)$ , while at  $\varphi \approx 90^\circ$  it has components  $(E_{1x}, E_{1y}, 0)$ ,  $(0, 0, H_{1z})$ . At angles  $0^\circ < \varphi < 90^\circ$  the radiation field, as mentioned earlier, is the superposition of  $o$  and  $e$  polarized waves and possesses all components of the electric and magnetic fields.

The dependences of  $\Pi(\vartheta)$  for a number of frequencies  $\bar{\omega}$  at  $\varphi = 0^\circ$ ,  $\beta = 0.3$  are shown in Fig. 3.

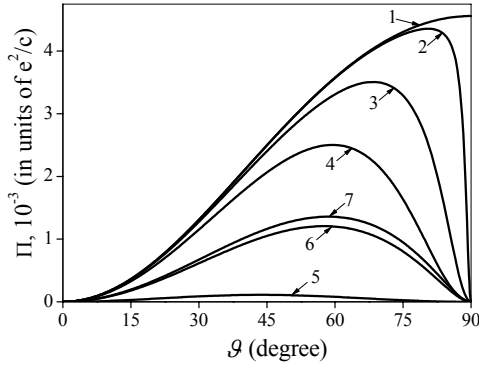


Fig. 3. The dependences of  $\Pi(\vartheta)$  for a number of  $\bar{\omega}$  values at  $\varphi = 0^\circ$ ,  $\beta = 0.3$

In Fig. 3 curve 1 corresponds to  $\bar{\omega} \rightarrow 0$ , curve 2 is for  $\bar{\omega} = 0.1$ , curve 3 is for  $\bar{\omega} = 0.5$ , curve 4 is for  $\bar{\omega} = 0.8$ , curve 5 is for  $\bar{\omega} = 1$ , curve 6 is for  $\bar{\omega} = 2$ , and curve 7 is for  $\bar{\omega} = 10$ . From Fig. 3 it follows that at  $\varphi = 0^\circ$  and the low frequencies ( $\bar{\omega} \rightarrow 0$ ) the maximum of the spectral energy density is located at  $\vartheta \approx 90^\circ$ . As the frequency further increases, the maximum decreases and tends to a certain minimum value at  $\bar{\omega} = 1$  (curve 5). At  $\bar{\omega} > 1$  the value of the maximum grows with the increase of the frequency and tends to the limit at  $\bar{\omega} \gg 1$  (curve 7).

Fig. 4 presents the analogous dependences at  $\varphi = 90^\circ$ ,  $\beta = 0.3$ .

In Fig. 4 curve 1 corresponds to  $\bar{\omega} \rightarrow 0$ , curve 2 is for  $\bar{\omega} = 2$ , and curve 3 is for  $\bar{\omega} = 10$ .

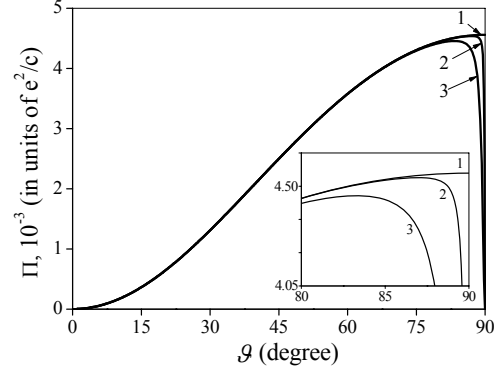


Fig. 4. The dependences of  $\Pi(\vartheta)$  for a number of  $\bar{\omega}$  values at  $\varphi = 90^\circ$ ,  $\beta = 0.3$

The dependences demonstrate that for all frequencies, which have a physical meaning, the maximums of the spectral energy density are directed at very large angles close to  $90^\circ$  with respect to the normal to the interface.

It is worthwhile to consider the Vavilov-Cherenkov radiation generated by the electron in the layered superconductor. As suggested earlier, the electron enters into the superconductor. In this case, the excited bulk electromagnetic wave in the superconductor has extraordinary polarization and we can write the following exact formula for  $k_{2y}^{(e)2}$ :

$$k_{2y}^{(e)2} = \text{Re} k_{2y}^{(e)2} + i \text{Im} k_{2y}^{(e)2}, \quad (52)$$

where

$$\text{Re} k_{2y}^{(e)2} = \varepsilon'_c \left( \frac{\omega^2}{c^2} - k_z^2 \frac{\varepsilon'_{ab}}{|\varepsilon_{ab}|^2} \right) - k_x^2 - k_z^2 \frac{\varepsilon''_{ab} \varepsilon''_c}{|\varepsilon_{ab}|^2}, \quad (53)$$

$$\text{Im} k_{2y}^{(e)2} = \varepsilon''_c \left( \frac{\omega^2}{c^2} - k_z^2 \frac{\varepsilon'_{ab}}{|\varepsilon_{ab}|^2} \right) + k_z^2 \frac{\varepsilon''_{ab} \varepsilon'_c}{|\varepsilon_{ab}|^2}, \quad (54)$$

$$\varepsilon'_{ab,c} = \text{Re} \varepsilon_{ab,c}, \quad \varepsilon''_{ab,c} = \text{Im} \varepsilon_{ab,c}. \quad (55)$$

From equations (52) - (54) it becomes evident that the excited extraordinary electromagnetic wave is a homogeneous one over the frequency range where  $\varepsilon'_{ab} < 0$ ,  $\varepsilon'_c > 0$  and the condition  $\text{Re} k_{2y}^{(e)2} > 0$ , in principle, can be satisfied. It is seen that over the same frequency range the condition  $\text{Im} k_{2y}^{(e)2} > 0$  is satisfied as well. Using the equations (52) - (54), we can easily demonstrate that over the frequency range where  $\varepsilon'_{ab} < 0$ ,  $\varepsilon'_c > 0$  the real and imaginary parts of  $k_{2y}^{(e)}$  are simultaneously positive. Physically this implies that the Vavilov-Cherenkov radiation generated by the electron entering the layered superconductor cannot be reversed, i.e. the energy flux density vector cannot form an obtuse angle with the direction of the electron motion. In other words, the superconductor does not behave like a left-handed medium. On the other hand, in case where the layers of the superconductor are parallel to the interface the reversed Vavilov-Cherenkov radia-

tion is possible [18]. Hence, the orientation of the superconductor layers with respect to the interface plane plays a critical role for the formation of the reversed Vavilov-Cherenkov radiation.

Let us consider the case where the electron escapes from the layered superconductor. In this case, the above-described characteristics of the transition radiation hold true. At the same time, the Vavilov-Cherenkov radiation can escape from the superconductor and its dependences of  $\Pi(\vartheta)$  are shown in Fig. 5.

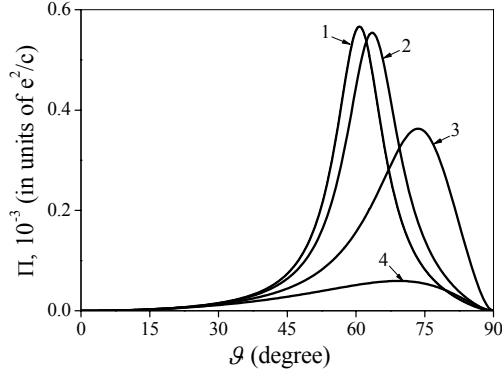


Fig. 5. The dependences of  $\Pi(\vartheta)$  for a number of  $\varphi$  values at  $\bar{\omega} \approx 1.81$ ,  $\beta = -0.3$

In Fig. 5 curve 1 is for  $\varphi = 0^\circ$ , curve 2 is for  $\varphi = 0.1^\circ$ , curve 3 is for  $\varphi = 0.2^\circ$ , and curve 4 is for  $\varphi = 0.3^\circ$ . It is seen that the Vavilov-Cherenkov radiation escapes from the semiconductor at very small angles  $\varphi$ . So, we can say that the almost all Vavilov-Cherenkov radiation is mainly concentrated in the  $yz$ -plane.

In order for the Vavilov-Cherenkov radiation to arise, the condition  $k_{2y}^{(e)} = \omega/v$  must be satisfied. This condition allows one to derive the following representative values of the azimuth angles  $\Delta\varphi$ , the radiation frequency  $\bar{\omega}_{\text{VCH}}$ , and the electron velocity  $\beta$ :

$$\tan^2 \varphi < \frac{\varepsilon_c}{|\varepsilon_{ab}|} \ll 1, \quad (56)$$

$$\bar{\omega}_{\text{VCH}} \approx \left[ 1 - (\varepsilon\beta^2)^{-1} \right]^{-1/2}, \quad (57)$$

where  $\beta^2 > 1/\varepsilon$ . The numerical estimates of the condition  $k_{2y}^{(e)} = \omega/v$  for the abovementioned parameters of the system show that the Vavilov-Cherenkov radiation exists over the narrow range of frequencies  $\Delta\bar{\omega} \propto 10^{-5}$  in the vicinity of  $\bar{\omega}_{\text{VCH}} \approx 1.81$  at  $\beta = -0.3$ . These circumstances are regarded to be new as compared to the known results obtained from the analysis of the transition radiation in the case where an electron crosses an interface of two isotropic media [2].

It is worthwhile to emphasize that the electron excites not only the above-considered bulk electromagnetic waves, but also the so-called oblique surface electromagnetic waves (OSWs) in the interface plane. The properties of the OSWs have recently been investigated in [20, 21]. Indeed, the value  $\Delta_{\text{OSW}}$  in the denomina-

tors of equations (34) - (36) corresponds to the dispersion relation of the OSWs  $\Delta_{\text{OSW}} = 0$ . In order to derive the spatial and time dependences of the OSW fields in the explicit form, we need to integrate the expressions (34) - (39) with respect to  $k_x$ ,  $k_z$ , and  $\omega$  taking into account the poles of the integrands in equations (34) - (39). We will consider the case of the OSWs excitation by means of the transition radiation effect in a subsequent paper.

## CONCLUSIONS

The problem of the transition radiation of an electron moving along the normal to the interface between an isotropic dielectric and a layered superconductor has been theoretically examined. It has been assumed that the layers of the superconductor are perpendicular to the interface. At azimuth angles  $0^\circ < \varphi < 90^\circ$  the radiation field of excited bulk electromagnetic waves is shown to be a superposition of the ordinary and extraordinary electromagnetic waves. At angles  $\varphi \approx 0^\circ$  and  $\varphi \approx 90^\circ$  the radiation field becomes TM polarized. At  $\varphi = 0^\circ$  the position of the maximum of the spectral energy density varies with frequency over a wide range of the tilt angle  $\vartheta$ . At  $\varphi = 90^\circ$  the position of the maximum of the spectral energy density is close to  $\vartheta \approx 90^\circ$  for all frequencies which have a physical meaning. It has been established that for the case under study the Vavilov-Cherenkov radiation generated by the electron entering the layered superconductor cannot be reversed. In the case where the electron escapes from the layered superconductor the Vavilov-Cherenkov radiation is mainly concentrated in the plane that comprises the normal to the interface and the crystallographic  $\vec{c}$ -axis. Besides, the so-called oblique surface electromagnetic waves can also be excited along with the bulk waves.

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#### **ПЕРЕХОДНОЕ ИЗЛУЧЕНИЕ ЭЛЕКТРОНА, ПЕРЕСЕКАЮЩЕГО ГРАНИЦУ РАЗДЕЛА ДИЭЛЕКТРИКА И СЛОИСТОГО СВЕРХПРОВОДНИКА**

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Исследовано переходное излучение электрона, пересекающего границу раздела диэлектрика и слоистого сверхпроводника. Направление движения электрона и слои сверхпроводника ориентированы перпендикулярно границе. Анализ диаграмм направленности излучения показал сильную анизотропию его интенсивности по азимутальному углу в плоскости границы раздела сред.

#### **ПЕРЕХІДНЕ ВИПРОМІНЮВАННЯ ЕЛЕКТРОНА, ЯКИЙ ПЕРЕТІНАЄ МЕЖУ ПОДІЛУ ДІЕЛЕКТРИКА І ШАРУВАТОГО НАДПРОВІДНИКА**

***Ю.О. Аверков, В.М. Яковенко, В.О. Ямпольський***

Досліджено перехідне випромінювання електрона, який перетинає межу поділу діелектрика і шаруватого надпровідника. Напрямок руху електрона і шари надпровідника орієнтовані перпендикулярно цієї межі. Аналіз діаграм спрямованості випромінювання показав сильну анізотропію його інтенсивності щодо азимутального кута у площині межі поділу середовищ.