QUANTUM COSMOLOGY

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We discuss quantum-cosmological approach to the problem of initial singularity and origin of our universe, starting with the pioneer idea of quantum creation of the universe put forward by P.I. Fomin.

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1. INTRODUCTION

Quantum cosmology is an audacious attempt to understand the laws of origin and primordial evolution of the universe.

In the modern context, the problem arose as soon as it was firmly established in the 1920s that the universe is expanding, hence, almost certainly should have had a beginning. The backwards extrapolation of the general-relativistic Friedmann law [1] of the universe expansion

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho + \frac{\Lambda}{3}, \tag{1}$$

where a is the scale factor, ρ is the average matter-energy density, and Λ is the cosmological constant, inevitably runs into a singularity $a = 0$ in a finite time of the order of the Hubble time $1/H_0 \approx 1.36 \times 10^{10}$ years. Here, $H_0 = \dot{a}_0/a_0$ is the current value of the Hubble parameter. For some time, this cosmological singularity was deemed to be an artefact of the idealized assumption of perfect homogeneity and isotropy of the Friedmann models of the universe. In the 1960s, however, powerful theorems were proved by Penrose and Hawking (see [2]) which demonstrated the generic character of such a singularity.

There have been several theoretical attempts to avoid the cosmological singularity on the classical level. One of the first proposals in this direction was the Tolman's model of the cyclic universe [3], in which an epoch of universe contraction is more or less promptly changed into expansion by virtue of a bounce. Modern investigations in loop quantum cosmology [4] and multidimensional cosmology [5] can provide theoretical background for such a behaviour. A different type of singularity-free model was proposed by Starobinsky [6] and, in fact, represented one of the first models of eternal inflation. The subsequent development of the theory of eternal inflation has shown, however, that the issue of cosmological singularity is still present in this theory [7], suggesting that the true resolution of the singularity problem may be of quantum nature.

The first proposals of quantum origin of the universe were made by Fomin [8, 9] and by Tryon [10] in 1973. Fomin used the well-known observation that a universe which is (almost) spatially closed has its total energy close to zero. He argued that such a universe could, therefore, be spontaneously created in a quantum-mechanical process as a baby universe from the space of a big mother universe (Fig. 1). This process, therefore, replaces the classical singularity.

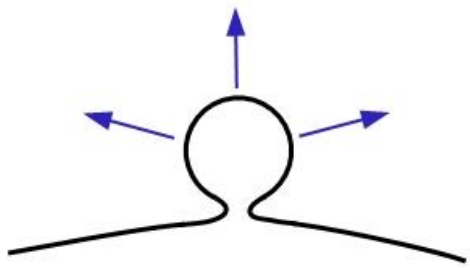

Fig.1. *Quantum birth of a baby universe from the vacuum space of a mother universe*

Mathematically, one can estimate the energy *E(M)* of a distribution of matter with total mass M and spatial radius a as [8, 9]

$$E(M) = Mc^2 - \eta\frac{GM^2}{a} = 0, \tag{2}$$

where η is a numerical coefficient of order one that depends on the matter distribution. The gravitational energy enters with negative sign, and one can see that, apart from the trivial solution for the mass, $M = 0$, the above equation admits a non-trivial solution $M = c^2 a/\eta G$. From the general-relativistic viewpoint, the corresponding space is curved and spatially closed, having zero total energy (and electric charge).

During the revival of quantum cosmology in the early 1980s, a theory of quantum birth of a universe literally "from nothing" was advanced by Vilenkin [11], Hartle and Hawking [12], Linde [13] and other researches. Its theoretical basis was quantum geometrodynamics proposed in the 1960s by Wheeler [14,

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15] and De Witt [16] and the related path-integral formalism developed by Hartle and Hawking [12]. The central object in this approach is the wave function of the universe $\Psi[^{(3)}g, \Phi]$, where $^{(3)}g$ is the geometry of three-dimensional space, and Φ is a collection of matter fields defined in this space. The wave function obeys the functional Wheeler–De Witt equation of the form $H\Psi = 0$, and formally can be represented as a path integral over Euclidean four-geometries which have the field configuration $^{(3)}g, \Phi$ as its boundary (Fig. 2):

$$\Psi[^{(3)}g, \Phi] = \sum_{geometries} exp(-S_{Euclid}). \qquad (3)$$

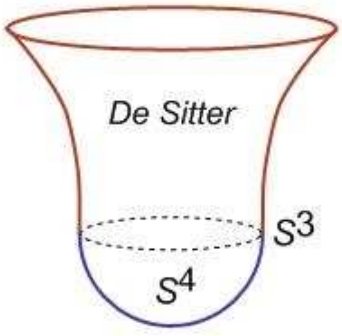

Fig.2. *Creation of a closed universe with spherical geometry S^3 from nothing is described as a sum over four geometries with the corresponding boundary. The saddle-point contribution is given by the half of the four-sphere S^4. The created universe then evolves as De Sitter space*

In the so-called minisuperspace approach, the universe is approximated by a closed homogeneous and isotropic space, whose three-geometry is described by a single quantity — the scale factor a. For such a universe filled by a scalar field ϕ with potential $V(\phi)$, employing their no-boundary proposal in calculating the path functional (3), Hartle and Hawking [12] obtained the wave function

$$\Psi_{NB}(a, \phi) \propto exp\left[\frac{1}{3V(\phi)}\right] \times cos\left[\frac{(a^2V(\phi) - 1)^{3/2}}{3V(\phi)} - \frac{\pi}{4}\right]. \qquad (4)$$

The tunnelling proposal by Vilenkin [17] leads to a somewhat different expression for the wave function in this model:

$$\Psi_T(a, \phi) \propto exp\left[-\frac{1}{3V(\phi)}\right] \times exp\left[-\frac{i(a^2V(\phi) - 1)^{3/2}}{3V(\phi)}\right]. \qquad (5)$$

For sufficiently large values of the scale factor a, the wave function (5) corresponds to a classical inflationary expanding universe with the effective cosmological constant given by $V(\phi)$, while the wave function (4) represents a superposition of a contracting and expanding universe.

Looking closer at expressions (4) and (5), one can see two immediate problems with this approach. Firstly, they both, as well as the general wave function of the universe (3), do not depend explicitly on any time variable. Hence, it is not clear how such a function can correspond to a real universe which depends on time. In an attempt to answer this problem it was argued that any one of the suitable physical parameters can be treated as a time parameter; in the case of wave functions (4) and (5), it is usually taken to be the scale factor a. However, this resolution of the time problem looks rather artificial and arbitrary and does not shed any light on the universally observed phenomenon of time flow.

Secondly, solutions of the Wheeler–De Witt equation usually are not normalizable in any sense. This is reflected in expressions (4) and (5) which, being squared, are non-normalizable with respect to the variable ϕ either at small or at large values of ϕ. Hence, it is not clear what kind of quantum statistics is described by such solutions. Furthermore, the general scheme of quantum mechanics, with its measurement problem and wave-function collapse, cannot be straightforwardly applied to the wave function of the universe in the absence of any external observer.

2. LOOP QUANTUM COSMOLOGY

In the last decades, quantum cosmology was substantially updated due to the development of loop quantum gravity [18]. By adopting the Ashtekar variables [19] as the proncipal canonical variables in general relativity, the proponents of loop quantum gravity succeeded in constructing a mathematically rigorous quantum representation of the algebra of the diffeomorphism group. The basic variables in this approach turned out to be the holonomies $h_l(A) = Pexp\left(\int_l A^i_a \dot{e}^a \tau_i dt\right)$ of the Ashtekar connection A^i_a along spatial curves l and fluxes $F^{(f)}_S(E) = \int_S E^a_i n_a f^i d^2y$ of the densitized triad E^a_i through spatial surfaces S, with SU(2) generators τ_i proportional to the Pauli matrices and surface-supported smearing functions f^i. In the homogeneous and isotropic context, the connection and the densitized triad reduce to $A^i_a = c\delta^i_a$ and $E^a_i = p\delta^a_i$, respectively. The analogue of the Wheeler–De Witt equation in the thus constructed loop quantum cosmology becomes a difference equation with respect to the discrete spectrum of the operator $\hat{p}$. This resolves the classical singularity issue in the present theory [20]. Moreover, the effective Friedmann equation in this theory is modified by quantum corrections at high energy density [4]:

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_0}\right), \qquad (6)$$

where ρ_0 is the density parameter of the order of the Planck value. Equation (6) obviously leads to the bounce at high energy densities. It should be noted that a very similar form (6) is obtained in the cosmological evolution in the braneworld theory with time-like extra dimension [5], where the role of ρ_0 is played by the vacuum energy density of the brane.

Loop quantum cosmology, just as any of quantum theories with intrinsic diffeomorphism invariance, still has the problem of time evolution. As a consequence, it also lacks a clear quantum-mechanical probabilistic interpretation. It is interesting that the original proposal, due to Fomin [8, 9], of creation of a baby universe from the vacuum space of a mother universe is potentially capable of solving (at least partly) both problems. We are going to discuss this issue in the next section.

3. CREATION OF A UNIVERSE FROM THE VACUUM

In the proposal by Fomin [8, 9], a baby universe can be regarded as a quantum fluctuation of spatial geometry, as depicted in Fig. 1. It represents a semi-closed world attached to the mother universe by a narrow spatial bridge. The quantum evolution of such a baby universe can be approximated by quantum geometrodynamics; however, there is an important fact that one can take into account here. Since the baby universe is attached to the mother universe, hence, moves in the external four-geometry, there arises an external time with respect to which one can determine the evolution of its quantum state. As a consequence, the problem of wave function normalization and interpretation are also resolved [21, 22], as we are about to show.

In the minisuperspace approach, the geometry of a baby universe depicted in Fig. 1 is described by the internal metric of the form

$$ds^2 = N^2(t)dt^2 - a^2(t)d\Omega, \tag{7}$$

where $N(t)$ is the so-called lapse function, $a(t)$ is the scale factor, and $d\Omega$ is the metric of the a unit three-sphere. Since the baby universe is attached to the mother universe, its intrinsic time should be correlated with the time of its motion along the mother universe. This leads to the condition $\int_0^1 N(t)dt = T$, where T is the proper time along the world line of the baby universe. The amplitude of transition from the state with scale factor a_1 to the state with scale factor a_2 in time T is then given by a restricted path integral over geometries (7) of the form:

$$\begin{aligned} U(T, a_2, a_1) = \int DNDa exp\left(\frac{iS[N,a]}{\hbar}\right) \\ \times \delta\left(\int_0^1 N(t)dt - 1\right), \end{aligned} \tag{8}$$

where $S[N, a]$ is the Hilbert–Einstein action reduced to the minisuperspace geometry under consideration:

$$S = -\frac{3\pi}{4G}\int_0^1 \left(\frac{a\dot{a}^2}{N^2} - a + \frac{\Lambda}{3}a^3\right) N dt \ . \tag{9}$$

The classical problem described by action (9) corresponds to particle motion in the potential $U(a) = a^2 - \Lambda a^4/3$, depicted in Fig.3.

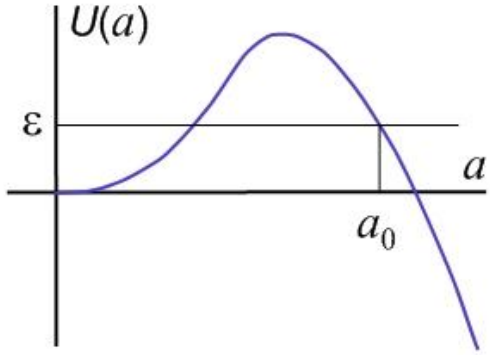

Fig.3. *The classical potential corresponding to action (9). The quantum particle is tunnelling from the quasistationary state with energy ε localized around $a = 0$ to the classical escape point a_0*

In the dimensionless Planckian variables

$$q = \left(\frac{\sqrt{\pi}a}{l_P}\right)^{3/2}, \ \tau = \frac{3\sqrt{\pi}T}{4t_P}, \ \lambda = \frac{l_P^2\Lambda}{3\pi}, \tag{10}$$

where l_P and t_P are, respectively, the Planck length and time, the amplitude (8) leads to a non-stationary Schrödinger equation of the type:

$$i\frac{\partial\psi(\tau, q)}{\partial\tau} = \left[\frac{\partial^2}{\partial q^2} - q^{2/3} + \lambda q^2\right]. \tag{11}$$

In these variables, the integration measure in the q-space that defines the probability density is equal simply to dq.

Equation (11) allows one to develop a theory of a quasi-stationary quantum state of the universe initially localized in the neighbourhood of $q = 0$, slowly evolving in time τ (or T) and tunnelling through its barrier. It is very similar to the Gamow's theory of alpha-decay of a radioactive nucleus. The energy of the quasi-discrete level in Planckian units turns out to be of the order unity, $\varepsilon \sim 1$, and the classical escape size of the universe is determined by the effective cosmological constant as $a_0 \approx \sqrt{3/\Lambda}$.

The tunnelling probability is given by the usual exponent

$$\Gamma \sim exp\left(-2\int_{q_1}^{q_2}\sqrt{q^{2/3} - \lambda q^2 - \varepsilon dq}\right), \tag{12}$$

and in the case of a small parameter λ, is approximated by

$$\Gamma \sim exp\left(-\frac{1}{\lambda} + \frac{3\pi\varepsilon}{4\sqrt{\lambda}}\right). \tag{13}$$

The tunnelling time of the baby universe in the Planckian units is given by the inverse of this quantity:

$$T_{tun} \sim t_P exp\left(\frac{1}{\lambda} - \frac{3\pi\varepsilon}{4\sqrt{\lambda}}\right), \tag{14}$$

This expression can be used also in the case where the vacuum energy density is represented not by a cosmological constant but by a potential of slowly varying scalar field, as in the theory of inflationary universe. As regards the current value of the time (14), since today we have $\lambda \sim 10^{-122}$, the tunnelling time is exponentially large, which makes our present space exponentially stable with respect to formation of baby universes. However, in the past, because of a larger value of λ, this time could be relatively smaller. In the Planckian epoch, where we have $\lambda \sim 1$, the tunnelling time approaches the Hubble (and Planckian) time.

Once the baby universe becomes classical and starts its inflationary evolution, the notion of the universal external time of the mother universe ceases to become legitimate. In fact, such a grown-up baby universe for all practical purposes should decouple from the mother universe, and perhaps even physically do so as the narrow bridge initially connecting the two spaces collapses and disappears (see Fig. 1). Speaking of the quantum evolution of such a baby universe, we should effectively return to something like the Wheeler–De Witt equation with zero total energy, and with its problem of temporal evolution discussed above. One way to address this problem is offered by the pilot-wave formulation of the quantum theory in general and of quantum gravity in particular.

4. PILOT-WAVE QUANTUM COSMOLOGY

The basic idea of the pilot-wave theory can be summarised as follows (see [23]). Any physical system is described by a deterministic evolution of its configuration variables. These are the same as in classical physics and are just the spatial coordinates of the elementary particles and the field configurations. The only difference between classical and quantum theory is in the dynamics of these configuration variables. In classical physics, their dynamics is determined by the principle of extremal action, or by any of the equivalent principles. In quantum physics the evolution of the configuration variables is guided (piloted, in de Broglie's terminology) by a quantum wave which obeys the Schrödinger equation. Quantum probabilities in this theory have purely statistical origin and arise because of our ignorance of and inability to control the actual (initial) values of particle and field configuration variables in every system of an ensemble.

In applying this general scheme to the theory of gravity, we start with the action in the 3+1 canonical form

$$I = \int d^3\mathbf{x}\; dt \left(\pi^{ab}\dot{g_{ab}} + \pi_\Phi \dot{\Phi} - N^\mu H_\mu\right), \qquad (15)$$

where H_0 and H_a are, respectively, the Hamiltonian and momentum constraints, and N_μ are the Lagrange multipliers—the lapse (N_0) and shift (N_a) functions. The symbol Φ symbolically denotes the collection of bosonic fields, and g_{ab} is the positive-definite three-metric—the main dynamical object in canonical gravity. Their conjugate momenta are denoted by π_Φ and π^{ab}, respectively. The general-relativistic constraints have the form

$$H_0 \equiv \frac{1}{2\kappa} G_{abcd}\pi^{ab}\pi^{cd} + \kappa\sqrt{g}\left(2\Lambda -^{(3)} R\right) + H_\Phi = 0, \qquad (16)$$

$$H_a \equiv -2\nabla_b \pi_a^b + H_a^\Phi = 0. \qquad (17)$$

Here, $\kappa = 1/16\pi G$, and the Wheeler's "supermetric" is

$$G_{abcd} = \frac{1}{\sqrt{g}}\left(g_{ac}g_{bd} + g_{ad}g_{bc} - g_{ab}g_{cd}\right). \qquad (18)$$

The classical equations of motion for the three-metric read

$$\dot{g}_{ab} = \frac{N^0}{\kappa} G_{abcd}\pi^{cd} + \nabla_a N_b + \nabla_b N_a. \qquad (19)$$

In quantum theory, the system is described by a wave functional $\Psi[g_{ab}(\mathbf{x}), \Phi(\mathbf{x})]$ which obeys the quantum constraint equations

$$\hat{H}_\mu \Psi = 0, \qquad (20)$$

which are quantum counterparts of the classical constraint equations (16) and (17) obtained by replacing the generalized momenta π_Φ and π^{ab} by the corresponding functional derivatives.

In the pilot-wave formulation of this dynamics [24], the wave functional is presented in the polar form $\Psi = R\; exp(iS/\hbar)$ with real amplitude R and phase S, and the pilot-wave quantum evolution of the metric has the classical form (19) in which instead of momenta one substitutes the derivatives of the phase S:

$$\pi^{ab}(\mathbf{x}) \to \frac{\delta S}{\delta g_{ab}(\mathbf{x})}. \qquad (21)$$

The evolution of the the field Φ is defined in a similar way. Thus one recovers the time evolution of the universe, which in the quasi-classical approximation approaches the classical general-relativistic dynamics since the phase functional $S[g_{ab}(\mathbf{x}), \Phi(\mathbf{x})]$ in this approximation obeys the Hamilton–Jacobi equation.

5. QUANTUM ASPECT OF COSMOLOGICAL INFLATION

According to the theory of inflationary universe (see [25]), the observable large-scale structure came out as a result of gravitational instability and clustering from the initial perturbations which were produced during inflation as quantum (zero-point) fluctuations in the density of a special scalar field (the inflaton) and space-time geometry. It is fare to say that the theoretical prediction of the shape of the primordial spectrum of such fluctuations and its adiabatic character agree very well with the current observations of the large-scale structure and of the temperature anisotropy of the cosmic microwave background. However, in the theory of the origin of primordial fluctuations, there remains an issue connected with the problem of quantum-state reduction. The problem is that the quantum state of the universe during inflation is spatially homogeneous and isotropic to a very high degree. Meanwhile, primordial fluctuations clearly break this invariance, and the subsequent evolution enhances this by producing structure (galaxies and clusters), thereby distinguishing between the spatial points. The problem is, how can such a symmetry be broken in a causal process of quantum evolution.

There are several approaches trying to cope with this fundamental issue inherent in the quantum theory as a whole. Among them, one can mention the sophisticated theories of "consistent histories" or "many worlds" (see their description in [23]). It should be noted that the pilot-wave formulation discussed in the previous section also naturally resolves this problem. In this formulation, even if the wave function of quantum fields can be assumed to be spatially homogeneous and isotropic (in any suitable sense), this need not be the case for a specific causal field configuration $\Phi(\mathbf{x})$. The inhomogeneity, so to speak, is of purely classical nature and origin in this theory. This, however, calls for a theory of origin of a specific inhomogeneity $g_{ab}(\mathbf{x})$ and $\Phi(\mathbf{x})$ in this formulation, which remains to be developed.

6. CONCLUSIONS

Quantum cosmology is an attempt to deal with the quantum processes that lead to the origin of the universe as a whole, or at least to its large-scale structure. It is primarily motivated by the problem of cosmological singularity. The task of testing theories of the origin of our universe is very difficult since the connection of its remote past with its observable state of today is usually very vague. However, the development of the inflationary theory has shown that links of such a kind can in principle be testable. This brings a hope that quantum cosmology some day will be subject to observational tests and, thereby, will become an established part of physics.

ACKNOWLEDGEMENTS

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КВАНТОВАЯ КОСМОЛОГИЯ

Ю.В. Штанов

Обсуждается квантово-космологический подход к проблеме начальной сингулярности и происхождения нашей Вселенной, начатый пионерской идеей квантового рождения Вселенной, предложенной П.И. Фоминым.

КВАНТОВА КОСМОЛОГІЯ

Ю.В. Штанов

Обговорюється квантово-космологічний підхід до проблеми початкової сингулярності і походження нашого Всесвіту, започаткований піонерською ідеєю квантового народження Всесвіту, запропонованою П.І. Фоміним.