

# QUANTUM ELECTRODYNAMICS IN THE STRONG PULSED LASER FIELDS

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The some results on the resonant processes of quantum electrodynamics (QED) proceeding in the strong pulsed light fields, realized in modern powerful pulsed lasers is presented. The appearance of resonances in a laser field is one of the fundamental problems of QED in electromagnetic fields. Following QED processes of the second order in the fine structure constant in the pulsed laser field are considered: resonant spontaneous bremsstrahlung by an electron scattered by a nucleus, resonant photocreation of electron–positron pairs on a nucleus, and resonant scattering of a lepton by a lepton. The resonant peak’s altitude and width are defined by the external pulsed wave properties. It is demonstrated that the resonant cross sections may be several orders of magnitude greater than the corresponding cross sections in the absence of an external field. Results obtained may be experimentally verified, for example, by the scientific facilities at the SLAC National Accelerator Laboratory and FAIR (Facility for Antiproton and Ion Research, Darmstadt, Germany).

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## 1. INTRODUCTION

Use of a powerful coherent light source in modern applied and fundamental research has stimulated study of the external strong field influence on quantum electrodynamic (QED) processes [1]. A characteristic feature of electrodynamic processes of second order in the fine-structure constant in a laser field is associated with the fact that such processes may occur under both nonresonant and resonant conditions [1]-[5]. The resonant character relates to the fact that lower-order processes, such as spontaneous emission and one-photon creation and annihilation of electron-positron pairs, may be allowed in the field of a light wave. Therefore, within a certain range of energy and momentum, a particle in an intermediate state may fall within the mass shell. Then the considered higher-order process is effectively reduced to two sequential lower-order processes [1]-[5]. The appearance of resonances in a laser field is one of the fundamental problems of QED in strong fields.

As a result of laser technology development different types of coherent light sources have become available, with intensities that have increased up to  $10^{22} \text{W} \cdot \text{cm}^{-2}$  in recent years. The new experimental conditions have required constant improvements in calculations and model development. The amplitude of the field intensity of powerful ultrashort pulsed lasers changes greatly in space and time. In the description of QED processes in the presence of a pulsed laser the external field is usually modeled as a plane nonmonochromatic wave, when a characteristic pulse width  $\tau$  obeys the condition [2]-[5]

$$\omega\tau \gg 1. \quad (1)$$

The four-potential of the pulsed plane wave propagating along the z-axis has form [2]-[5]:

$$A(\varphi) = A_0 g\left(\frac{\varphi}{\omega\tau}\right) (e_x \cos \varphi + \delta e_y \sin \varphi), \quad (2)$$

$$\varphi = (kx) = \omega(t - z),$$

where  $A_0 = F_0/\omega$ ,  $k = (\omega, \mathbf{k})$  is the wave vector,  $F_0$ ,  $\omega$  and  $\delta$  are the strength, the frequency and the ellipticity parameter of the wave,  $e_x = (0, \mathbf{e}_x)$ ,  $e_y = (0, \mathbf{e}_y)$  are the four-vectors of wave polarization. The function in expression (2)  $g(\varphi/\omega\tau)$  is the envelope function of the four-potential, which must be equal the unit in center of a pulse,  $g(0) = 1$ , and to decrease exponentially ( $g \rightarrow 0$ ) when  $\varphi \gg \omega\tau$ .

Following QED processes of the second order in the fine structure constant in the pulsed laser field are considered: resonant spontaneous bremsstrahlung by an electron scattered by a nucleus [3], resonant photocreation of electron–positron pairs on a nucleus [4], and resonant scattering of a lepton by a lepton [5]. There are two characteristic parameters in these processes of QED in the field of a pulsed electromagnetic wave. The first is the classical relativistic-invariant parameter [1]-[5],

$$\eta_0 = \frac{eF_0\lambda}{mc^2}, \quad (3)$$

which in the pulse peak equals numerically the ratio of work done by the field within the distance equal to a wavelength to the electron rest energy ( $e$  and  $m$

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are the charge and the mass of an electron,  $F_0$  and  $\lambda = c/\omega$  are the strength and the wave-length of an electric field in the pulse peak). The Bunkin-Fedorov quantum parameter is specified [1]-[5]:

$$\gamma_i = \frac{mv_i c}{\hbar\omega}. \quad (4)$$

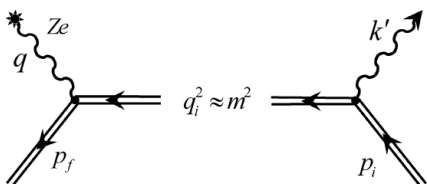
( $v_i$  is the electron speed). We treat these problems of QED within the range of moderate-strong-field intensities, when

$$\eta_0 \ll 1, \quad \gamma_i \gtrsim 1. \quad (5)$$

Consequently, the quantum Bunkin-Fedorov parameter is the main parameter which determines multi-photon processes. Hereafter, we use the relativistic system of units  $\hbar = c = 1$ .

## 2. RESONANT BREMSSTRAHLUNG OF AN ELECTRON SCATTERED BY AN ION IN A PULSED LIGHT FIELD

Here we describe a theory of resonant spontaneous bremsstrahlung (SB) produced by the scattering of an electron by a Coulomb center in the presence of pulsed external electromagnetic field (Fig.1) [3]. The problem under consideration is of vast scientific interest with respect to the concept substantiation of electromagnetic interaction realization as the virtual particle exchange. In the study of resonant processes in the field of plane monochromatic wave the resonance infinities were phenomenologically eliminated by Breit and Wigner procedure. The main point of this procedure is that radiative corrections in the Green function of an intermediate electron were included under consideration. It is important to emphasize that the pulsed character of external field was taken into account in the study of electron-nucleus SB. It enables to eliminate the resonance infinity in the process amplitude by the sequential method in the frame of examined approaches [3].



**Fig.1.** Resonant SB related to the scattering of an electron by a nucleus in the field of a pulsed light wave. Here  $p_i$ ,  $p_f$  are the four-momenta of initial and final electrons;  $k'$  is the four-momentum of a spontaneous photon;  $q_i$  is the four-momentum of an intermediate electron;  $q$  is the transferred momentum

The electron interaction with a nucleus is considered in the frame of the Born approximation, i.e. the case of rather fast electrons is studied ( $v_i \gg Z/137$ ,  $Z$  is the nucleus charge number). The process of electron-nucleus SB in the presence of pulsed light wave may occur under resonant conditions when the

four-momentum of an intermediate electron lies near the mass surface

$$q_i^2 - m^2 \lesssim \frac{(kq_i)}{\omega\tau} \sim \frac{m}{\tau} \ll \omega m. \quad (6)$$

Four characteristic domains of the resonant frequency can be separated: in the nonrelativistic case,  $\omega_{res} \approx \omega$ ; for an ultrarelativistic electron moving within a narrow cone with the photon from the external field,  $\omega_{res} \ll \omega$ ; for an ultrarelativistic electron moving within a narrow cone with the spontaneous photon,  $\omega_{res} \gg \omega$ ; otherwise,  $\omega_{res} \sim \omega$ . The process of resonant electron-nucleus SB in the field of a pulsed light wave can be effectively reduced to two sequential processes of the first order in the fine-structure constant: emission of a photon with a four-momentum  $k'$  by an electron  $p_i$  in a pulsed light wave and scattering of an electron  $q_i$  by a nucleus in the field of the pulsed wave (see Fig. 1). The resonant differential cross section of electron-nucleus SB in the field of pulsed light wave for moderately strong intensities when the electron is scattered by a large angle is:

$$\frac{d\sigma_{res}}{d\Omega'} = \frac{1}{\pi^2} \frac{E_i \omega'^2 |\mathbf{q}_i| (kp_i)}{(kk')^2 |\mathbf{p}_i|} P_{res} d\sigma_s dW^{(1)}. \quad (7)$$

Here  $d\sigma_s$  is the differential cross section of scattering of an intermediate electron with a four-momentum  $q_i$  by a nucleus in the field of the wave,  $dW^{(1)}$  is the probability that an electron with a four-momentum  $p_i = (E_i, \mathbf{p}_i)$  absorbs one photon from the external field and spontaneously emits a photon with a four-momentum  $k' = (\omega', \mathbf{k}')$ ,

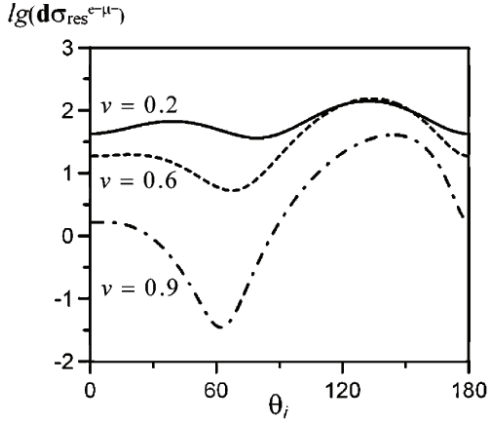
$$P_{res} = \pi(\omega\tau)^2 \frac{\exp\{-\beta^2/2\}}{64(kq_i)} \frac{1}{2\rho} \int_{-\rho}^{\rho} d\phi |\operatorname{erf}(\phi + \frac{i\beta}{2}) + 1|^2, \quad (8)$$

$$\beta = \frac{q_i^2 - m^2}{4(kq_i)} \omega\tau. \quad (9)$$

The parameter  $\beta$  (9) specifies how close the four-momentum of an intermediate electron coincides with the value on the mass surface in the resonant conditions. The dependence of the function  $P_{res}$  on the parameter  $\beta$  defines a magnitude and a shape of the resonant peak in the cross section of electron-nucleus SB process in the pulsed light field. The parameter  $\rho$  is the relation between observation time and pulse-width. The function  $P_{res}$  can be easily written in the form

$$P_{res} = \frac{a_1}{(q_i^2 - m^2)^2 + (2m\Gamma_\tau)^2}, \quad \Gamma_\tau = \frac{2}{\sqrt{a_2}} \frac{(kq_i)}{m(\omega\tau)}. \quad (10)$$

Here coefficients  $a_1$  and  $a_2$  weakly depend on the parameter  $\rho$ . A transit resonant width  $\Gamma_\tau$  arose from the finite time of particle-field interaction.



**Fig.2.** The ratio  $R_{res}$  as a function of the electron velocity for preset orientations of the electron momentum in the initial and final states and fixed orientation of the spontaneous photon ( $\theta' = 120^\circ$  and  $\varphi' = 10^\circ$  solid line;  $\theta' = 120^\circ$  and  $\varphi' = 60^\circ$  dashed line)

Let's consider the relation between resonant differential cross section of electron-nucleus SB and the cross-section of electron-nucleus SB in an absence of external field. The following parameters: the laser wave frequency  $\omega = 2.35$  eV; the laser pulse-width  $\tau = 1.5$  ps; the field strength in pulse peak  $F_0 = 6 \cdot 10^9$  V · cm<sup>-1</sup> were chosen for the calculation. Fig.2 displays ratio  $R_{res}$  as a function of the initial velocity of the electron. As can be seen from Fig.2, within the range of relativistic electron energies, the resonant differential cross section of electron-nucleus SB may be five orders of magnitude higher than the corresponding cross section in the absence of the external field. Within the range of ultrarelativistic electron energies, this ratio drastically decreases.

### 3. RESONANT PHOTOCREATION OF ELECTRON-POSITRON PAIRS ON A NUCLEUS IN A PULSED LIGHT FIELD

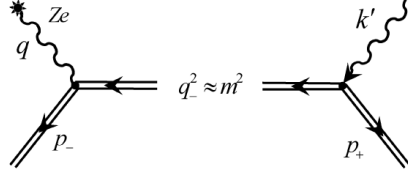
Here we describe a theory of resonant photocreation of electron-positron pairs on a nucleus in a pulsed light field (Fig.3) [4]. The photocreation process of electron-positron pairs on a nucleus in the presence of pulsed light wave may occur under resonant conditions when the four-momentum of an intermediate electron lies near the mass surface

$$q_-^2 - m^2 \lesssim \frac{(kq_-)}{\omega\tau} \ll \omega m. \quad (11)$$

In this case resonances are possible only for an ultrarelativistic positron moving within a narrow cone with the initial photon and the resonant frequency of an initial photon is

$$\omega_{res} = \frac{E_+}{1 - W_{th}/E_+}, \quad W_{th} = \frac{(1 + \delta_+^2)}{4\sin^2(\theta_i/2)} \frac{m^2}{\omega}, \quad (12)$$

$$\delta_+ = \theta_+(E_+/m), \quad \theta_+ = \angle(\mathbf{k}_i, \mathbf{p}_+) \ll 1. \quad (13)$$



**Fig.3.** Resonant photocreation of electron-positron pairs on a nucleus in a pulsed light field. Here  $p_-$ ,  $p_+$  are the four-momenta of electron and positron;  $k_i$  is the four-momentum of an initial photon;  $q_i$  is the four-momentum of an intermediate electron;  $q$  is the transferred momentum

The resonant differential cross section of photocreation of electron-positron pairs on a nucleus in the field of pulsed light wave for moderately strong intensities when the electron is scattered by a large angle is:

$$d\sigma_{res}^{(\pm)} = \sqrt{\frac{\pi}{2}} \frac{\tau}{8\sin^2(\theta_i/2)} d\sigma_s(q_-) dW_{pair}^{(1)}. \quad (14)$$

Here  $d\sigma_s$  is the differential cross section of scattering of an intermediate electron with a four-momentum  $q_-$  by a nucleus,  $dW_{pair}^{(1)}$  is the creation probability of the electron-positron pair ( $q_-$  and  $p_+$ ) by initial photon  $k_i$  as a result of absorption of one photon from the laser field. Let's consider the relation between resonant differential cross section of photocreation of electron-positron pairs on a nucleus in the field of pulsed light wave and the corresponding cross-section in an absence of laser field:

$$R_{res}^{(\pm)} = \frac{\pi}{8} \sqrt{\frac{\pi}{2}} \eta_0^2 \omega \tau \left[ \ln \frac{E_+}{m} \right]^{-1}. \quad (15)$$

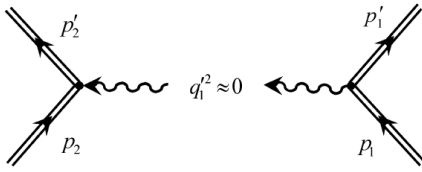
The following parameters: the laser wave frequency  $\omega = 1.17$  eV; the laser pulse-width  $\tau = 25$  ps; parameter  $\eta_0 \approx 0.1$ ;  $\omega_i = 5 \cdot 10^5$  m = 255 GeV were chosen for the calculation. We obtain from Eq. (15) the relation  $R_{res}^{(\pm)} \approx 40$ .

### 4. RESONANT SCATTERING OF A LEPTON BY A LEPTON IN A PULSED LIGHT FIELD

Here we describe a theory of resonant scattering of a lepton by a lepton in a pulsed light field (Fig.4) [5].

The scattering of a lepton by a lepton in the presence of pulsed light wave may occur under resonant conditions when the four-momentum of an intermediate photon lies near the mass surface

$$q_1'^2 \lesssim \frac{(kq_1')}{\omega\tau} \sim \frac{\omega^2}{\omega\tau} \ll \omega^2. \quad (16)$$

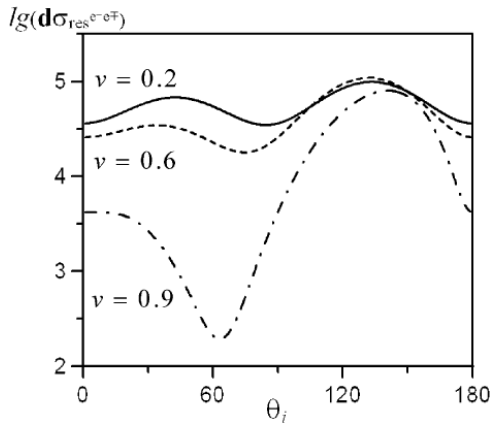


**Fig.4.** Resonant scattering of a lepton by a lepton in a pulsed light field. Here  $p_1, p_2$  and  $p'_1, p'_2$  are the four-momenta of leptons for initial and final particles states, respectively;  $q_1$  is the four-momentum of an intermediate photon

The resonance appears if leptons scatter by each other into the small angles in the frame of reference related to the center of inertia

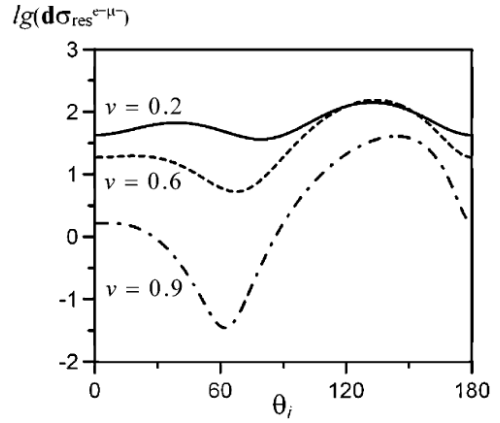
$$\theta_{res} = 2 \frac{\omega}{|\mathbf{p}|} \sin \theta_i \ll 1, \quad (17)$$

where  $\theta_i$  is the angle between directions of wave propagation and initial relative momentum  $\mathbf{p}$ .



**Fig.5.** The dependence of differential cross-section of scattering of an electron by an electron (an electron by a positron) in the pulsed light field (in units of respective cross-sections in the external field absence) on initial polar angle when azimuthal angle is fixed

Let's consider the ratio of derived resonant differential cross-section of scattering of leptons by each other into elementary azimuthal angle to the differential cross section of scattering of respective leptons in the external field absence for the most interesting processes of scattering of a lepton by a lepton: the scattering of an electron by an electron, the scattering of an electron by a positron, the scattering of an electron by a muon. The experimental investigation of processes of resonant scattering of a lepton by a lepton may be verified in the fields created by picosecond pulsed lasers which generate radiation within the frequency optical range.



**Fig.6.** The dependence of differential cross-section of scattering of an electron by muon in the pulsed light field (in units of respective cross-sections in the external field absence) on initial polar angle when azimuthal angle is fixed

The Fig. 5-6 show the dependences of the considered ratio on initial polar angle  $\theta_i$ . The external laser wave frequency amounts to value  $\omega = 2.35$  eV, the pulse-width is equal to  $\tau = 1.5$  ps, the field strength in pulse peak is  $F_0 = 6 \cdot 10^9$  V · cm<sup>-1</sup>. Hereby the most exceeding appears for the particles small relative velocities case, at that the exceeding reaches into five orders of magnitude in case of scattering of an electron by an electron (positron), and two orders in case of scattering of an electron by a muon.

## 5. CONCLUSIONS

- The QED processes of second order in the fine-structure constant in the presence of a pulsed light wave may occur under resonant conditions when the four-momentum of an intermediate particle lies near the mass surface.
- The resonant behavior of these processes is specified by characteristics of the laser pulse. The resonant singularity in the processes amplitude is eliminated by accounting for the pulsed character of the external field rather than by the phenomenological Breit-Wigner procedure.
- The resonant differential cross sections of the following processes proceeding in the strong pulsed light fields: resonant spontaneous bremsstrahlung by an electron scattered by a nucleus, resonant photocreation of electron-positron pairs on a nucleus, and resonant scattering of a lepton by a lepton may be several orders of magnitude higher than the corresponding cross sections in the absence of the laser field.
- The obtained results may be experimentally verified, for example, by the scientific facilities at the SLAC National Accelerator Laboratory and FAIR (Facility for Antiproton and Ion Research, Darmstadt, Germany).

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## КВАНТОВАЯ ЭЛЕКТРОДИНАМИКА В СИЛЬНЫХ ИМПУЛЬСНЫХ ЛАЗЕРНЫХ ПОЛЯХ

*С.П. Рошчупкин*

Рассматриваются некоторые резонансные процессы квантовой электродинамики (КЭД), протекающие в сильных импульсных световых полях, реализуемых в современных мощных импульсных лазерах. Появление резонансов в лазерном поле является одной из фундаментальных проблем КЭД в электромагнитных полях. Рассматриваются следующие процессы КЭД второго порядка по постоянной тонкой структуры в импульсном лазерном поле: резонансное спонтанное тормозное излучение электрона, рассеянного на ядре; резонансное фоторождение электрон-позитронных пар на ядре и резонансное рассеяние лептона на лептоне. Амплитуда и ширина резонансных пиков определяются параметрами внешней импульсной волны. Показано, что резонансные поперечные сечения могут быть на несколько порядков величины больше, чем соответствующие поперечные сечения в отсутствие внешнего поля. Полученные результаты могут быть экспериментально проверены в научных коллаборациях, таких как SLAC (National Accelerator Laboratory) и FAIR (Facility for Antiproton and Ion Research, Darmstadt, Germany).

## КВАНТОВА ЕЛЕКТРОДИНАМІКА В СИЛЬНИХ ІМПУЛЬСНИХ ЛАЗЕРНИХ ПОЛЯХ

*С.П. Рошчупкін*

Розглядаються деякі резонансні процеси квантової електродинаміки (КЕД), що протікають у сильних імпульсних світлових полях, реалізованих у сучасних потужних імпульсних лазерах. Поява резонансів у лазерному полі є однією з фундаментальних проблем КЕД в електромагнітних полях. Розглядаються наступні процеси КЕД другого порядку зі сталої тонкої структури в імпульсному лазерному полі: резонансне спонтанне гальмове випромінювання електрона, розсіянного на ядрі; резонансне фотонародження електрон-позитронних пар на ядрі й резонансне розсіювання лептона на лептоні. Амплітуда й ширина резонансних піків визначаються параметрами зовнішньої імпульсної хвилі. Показано, що резонансні поперечні перерізи можуть на декілька порядків величини бути більше, ніж відповідні поперечні перерізи у відсутності зовнішнього поля. Отримані результати можуть бути експериментально перевірені в наукових об'єднаннях, таких як SLAC (National Accelerator Laboratory) і FAIR (Facility for Antiproton and Ion Research, Darmstadt, Germany).