THE R-FUNCTIONS METHOD IN MATHEMATICAL MODELING OF CONVECTIVE HEAT TRANSFER IN FUEL CARTRIDGE WITH FUEL RODS

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Conjugate boundary value problems of heat transfer in cases, when viscous incompressible fluid flows via channels of non-canonical sections, bypassing the bundle of rods, is considered. The influence of the packaging pattern on the velocity and temperature distribution is researched. The theory of R-functions in combination with the Ritz variational method were used for the solution. Different packaging of fuel rods are examined. Each package contains 91 rods, and the corresponding equations are constructed with the new design tools of the theory of R-functions.

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1. INTRODUCTION

At the core of modern nuclear power plants the nuclear fuel is concentrated in the fuel elements (fuel rods). One of the main problems in the reactor calculation process is a definition of the temperature field, since the requirements to the reliability of the fuel rods are highest. The failure of several fuel rods, while the reactor has thousands of them, can lead to an emergency situation. In this case, there is a need to meet the relevant transport problems in the dual formulation, as these solutions allow to research the real heat transfer processes, which essentially show the mutual influence of the moving fluid, the channel walls, fuel rods, etc.

Consider the conjugate heat transfer boundary value problems for cases, when viscous incompressible fluid flows through the channel of the non-canonical section, bypassing the rods. It is assumed that the physical properties of the fluid are constant, laminar flow is hydro-dynamically stable, and heat exchange process is stationary. Conditions of the first, second and third kind can be specified on the outer surface of the channel. It is assumed that the change of the heat flux along the channel due to the axial thermal conductivity is negligible compared to the change of the heat flux due to convection. It is also assumed that the pipe walls and the inner rods are made of an isotropic material, and the thermal conductivity of the latter can be considered as constant in this temperature range [1-3].

The presence of internal heat sources in the reactor components complicates both the heat equation, and the methods for its solution. A heat transfer in the bundle of infinite cylinders bypassed lengthwise is considered in [1,3]. A symmetry of the temperature field due to the symmetry of the system is assumed, and only the region is considered (Fig. 1).

The aim of this work is the improvement of design tools and algorithms of the R-functions method for mathematical and computer modeling of the conjugate problem of convective heat transfer in arrays of fuel rods and the study of the packaging influence on the velocity and temperature distributions.

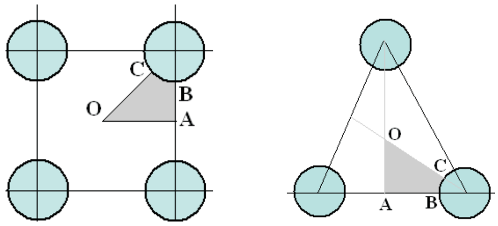

Fig.1. *Location of operating channels in the reactor core by the pattern: hallway-on the left; checkerboard-on the right*

2. MAIN PART

The basic system of equations describing the process of heat transfer in a flow of viscous fluid with constant physical properties of the liquid and the temperature has the form

$$\begin{cases} \frac{DT}{D\tau} = a\triangle T + \frac{q_V}{\rho c_p} + \frac{\mu\Phi}{\rho c_p}\,, \\ \frac{D\vec{V}}{D\tau} = -\frac{1}{\rho}\vec{\nabla}p + \nu\triangle\vec{V}\,, \\ div\,\vec{V} = 0\,, \end{cases}$$

where $\frac{D}{D\tau} = \frac{\partial}{\partial\tau} + (\vec{V}\cdot\vec{\nabla})$ is a substantial (or total) derivative,

$\mu\Phi$ is a dissipation function;

$a = \frac{\lambda}{\rho c_p}$ is a thermal diffusivity;

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c_p is a heat capacity of the medium;
q_V is a power of internal heat sources.

For stationary processes, the temperature of an object is independent of time, and the heat conduction equation, while bypassing the fuel rods lengthwise, takes the form

$$V_z \frac{\partial T}{\partial z} = a\triangle T + \frac{q_V}{\rho c_p},$$

and a mathematical model of the velocity field for laminar flow is given by

$$\triangle V_z = -\frac{\nabla P}{\mu l} = -C,$$

where ∇P is a constant pressure drop along the pipe on the randomly selected portion of length l.

In the thermal stability area, when $\frac{\partial T}{\partial z} = const$, we will take

$$-div(\lambda \nabla T) = q_V - V_z C_l .$$

Thus, the mathematical model of heat transfer in laminar flow of the fluid in the cartridge with fuel rods is reduced to the system of equations

$$\begin{cases} \triangle V_z = -C, & in\ \Omega_b \bigcap \overline{\Omega_{t\nu}}; \\ -div(\lambda_i \nabla T_i) = F_i, & in\ \Omega_b, \end{cases}$$

where

$$\begin{cases} F_1 = -V_z, & in\ \Omega_b \bigcap \overline{\Omega_{t\nu}}; \\ F_2 = q_V, & in\ \Omega_{t\nu}, \end{cases}$$

with boundary conditions

$$V_z \left|_{\partial\Omega_b \bigcap \overline{\partial\Omega_{t\nu}}} = 0, \quad \frac{\partial T}{\partial n} + hT \right|_{\partial\Omega_b} = 0,$$
$$T_1 |_{\partial\Omega_{t\nu}} = T_2 |_{\partial\Omega_{t\nu}} ,$$
$$\lambda_1 \frac{\partial T_1}{\partial n_1} |_{\partial\Omega_{t\nu}} = \lambda_2 \frac{\partial T_2}{\partial n_2} |_{\partial\Omega_{t\nu}} . \qquad (1)$$

Consider a typical constructive pattern of the reactor core, which is collected of a large number of fuel cartridges [2, 3]. Cartridges are hexagonal casings, possessing the fuel rods. Construct an equation of fuel cartridge with 91 fuel rods and the triangular packaging moved apart (Fig. 2,a), which is called checkerboard sometimes. Note that ordinary technique used in the theory of R-functions allows to obtain 93 R-operations in the equation as a result. Cumbersome formula will lead not only to increase of the computation time, but also, perhaps, to some symmetry breaking due to the non-associativity of R-operations. Therefore, technique developed in [4, 5, 6] will be used for the construction of the equation of hexagonal casing.

Consider the equation of a line

$$\sigma \equiv R_\nu - x \geq 0$$

and a periodic function

$$\mu_\nu = \frac{4}{3\pi} \sum_k (-1)^{k+1} \frac{\sin[(2k-1)3\theta]}{(2k-1)^2} .$$

The result is

$$\omega_b \equiv R_\nu - r \cos \mu_\nu = 0 ,$$

where $r = \sqrt{x^2 + y^2}$, $\theta = arctg \frac{y}{x}$.

To construct a triangle packing of fuel rods, define $f_1 = R^2 - \mu_x^2 - \mu_y^2 \geq 0$,
where

$$\mu_x = \frac{4h_x}{\pi^2} \sum_k (-1)^{k+1} \frac{\sin\left[(2k-1)\frac{\pi x}{h_x}\right]}{(2k-1)^2},$$

$$\mu_y = \frac{4h_y}{\pi^2} \sum_k (-1)^{k+1} \frac{\sin\left[(2k-1)\frac{\pi x}{h_y}\right]}{(2k-1)^2},$$

$f_2 = R^2 - \mu_{x1}^2 - \mu_{y1}^2 \geq 0$,
where

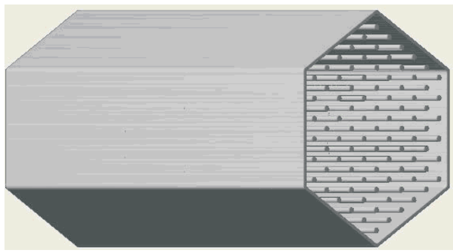
$$\mu_{x1} = \frac{4h_x}{\pi^2} \sum_k (-1)^{k+1} \frac{\sin\left[\frac{(2k-1)\pi(x-h_x/2)}{h_x}\right]}{(2k-1)^2},$$

$$\mu_{y1} = \frac{4h_y}{\pi^2} \sum_k (-1)^{k+1} \frac{\sin\left[\frac{(2k-1)\pi(y-h_y/2)}{h_x}\right]}{(2k-1)^2},$$

Then the equation of fuel cartridge is
$\omega \equiv \omega_b \wedge_0 \overline{\omega_{t\nu}} \geq 0, \quad \omega_{t\nu} \equiv (f_1 \vee_0 f_2)$.

Construction of the $\omega(x, y)$ is performed for the following values of parameters: $R = 0.2$; $h_x = 2.32$; $h_y = 1.35$; $n_0 = 6$; $r_k = 6.7$.

It should be noted that R-operations are used only twice in new method of the cartridge equation construction.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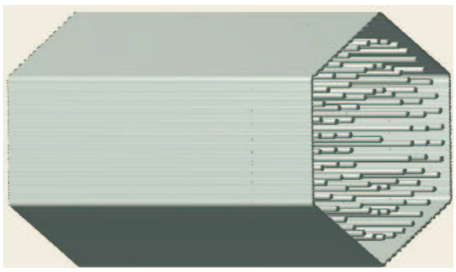
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Fig.2. *Fuel cartridge with 91 fuel rods located by: a - checkerboard pattern; b - distribution with cyclic symmetry and central fuel rod*

Let us also consider the construction of the function $\omega(x, y) \equiv \omega_b \wedge_0 \overline{\omega_{t\nu}}$, when fuel rod is distributed with cyclic symmetry n_d times along a circle of radius R, n_b times along a circle of radius R_1, and n_c times along a circle of radius R_2.

To construct the equations of the boundary of fuel rods distributed with cyclic symmetry n_d times along a circle of radius R, the function

$$\omega_0 \equiv \frac{1}{2R_{t\nu}}\left(R_{t\nu}^2-(x-R)^2-y^2\right)$$

and the formula

$$\mu_d=\frac{8}{n_d\pi}\sum_k(-1)^{k+1}\frac{\sin\left[(2k-1)\frac{n_d\theta}{2}\right]}{(2k-1)^2},$$

will be used. The result is

$$\omega_{t\nu 1}\equiv\frac{R_{t\nu}^2-(r\cos\mu_d-R)^2-(r\sin\mu_d)^2}{2R_{t\nu}}\geq 0\,.$$

To construct the equations of the boundary of fuel rods distributed with cyclic symmetry n_b times along a circle of radius R_1, the function

$$\omega_{01} \equiv \frac{1}{2R_{t\nu}}\left(R_{t\nu}^2-(x-R_1)^2-y^2\right)$$

and the formula

$$\mu_b=\frac{8}{n_b\pi}\sum_k(-1)^{k+1}\frac{\sin\left[(2k-1)\frac{n_b\theta}{2}\right]}{(2k-1)^2},$$

will be used. The result is

$$\omega_{t\nu 2}\equiv\frac{R_{t\nu}^2-(r\cos\mu_b-R_1)^2-(r\sin\mu_b)^2}{2R_{t\nu}}\geq 0\,.$$

To construct the equations of the boundary of fuel rods distributed with cyclic symmetry n_c times along a circle of radius R_2, the function

$$\omega_{02} \equiv \frac{1}{2R_{t\nu}}\left(R_{t\nu}^2-(x-R_2)^2-y^2\right)$$

and the formula

$$\mu_c=\frac{8}{n_c\pi}\sum_k(-1)^{k+1}\frac{\sin\left[(2k-1)\frac{n_c\theta}{2}\right]}{(2k-1)^2},$$

will be used. The result is

$$\omega_{t\nu 3}\equiv\frac{R_{t\nu}^2-(r\cos\mu_c-R_2)^2-(r\sin\mu_c)^2}{2R_{t\nu}}\geq 0\,.$$

Thus, the equation of the boundary of the cartridge with 91 fuel rods has the form

$$\omega\equiv(\omega_b\wedge_0\overline{\omega_{t\nu 1}\vee_0\omega_{t\nu 2}\vee_0\omega_{t\nu 3}})\geq 0\,,$$

when $n_d = 38$, $R = 6$, $n_b = 32$, $R_1 = 4.5$, $n_c = 21$, $R_2 = 3$ and it is a seven-parametric $(n_b, n_d, n_c, R, R_1, R_2, R_{t\nu})$ family of curves (Fig. 2,b). It should be noted that the R-operations were used only three times. If the central fuel rod is present, the result is

$$\omega\equiv\omega_b\wedge_0\overline{\omega_{t\nu 1}\vee_0\omega_{t\nu 2}\vee_0\omega_{t\nu 3}\vee_0\frac{{R_{t\nu}}^2-x^2-y^2}{2R_{t\nu}}}\geq 0\,,$$

when $n_c = 20$.

When the equation of the cartridge and fuel rods are known, we can rewrite the problem (1) in the form

$$\begin{cases}\triangle V_z=-C,\\ -div(\lambda\nabla T)=F,\end{cases}$$

with boundary conditions

$$V_z\Big|_{\partial\Omega_b\bigcap\overline{\partial\Omega_{t\nu}}}=0,\quad \frac{\partial T}{\partial n}+hT\,|_{\Omega_b}=0,$$
$$T_1\,|_{\partial\Omega_{t\nu}}=T_2\,|_{\partial\Omega_{t\nu}}\,,$$
$$\lambda_1\left.\frac{\partial T_1}{\partial n_1}\right|_{\partial\Omega_{t\nu}}=\lambda_2\left.\frac{\partial T_2}{\partial n_2}\right|_{\partial\Omega_{t\nu}},$$

where

$$\lambda=\lambda_1\frac{1-\operatorname{sgn}(\omega_{t\nu})}{2}+\lambda_2\frac{1+\operatorname{sgn}(\omega_{t\nu})}{2},$$

$$F=-V_z\frac{1-\operatorname{sgn}(\omega_{t\nu})}{2}+q_V\frac{1+\operatorname{sgn}(\omega_{t\nu})}{2}\,.$$

The R-functions method in conjunction with the Ritz variational method were used for the solving. The structure of solution for the problem of laminar longitudinal flow of fuel rods by fluid has the form

$$V_z=\omega p_1\,,$$

where $\omega(x,y)\equiv\omega_b\wedge_0\overline{\omega_{t\nu}}\geq 0$ is equation of the boundary of cartridge's cross-section, $p_1=\sum_{i=1}^N c_{ik}\varphi_{ik}(x,y)$ is the undefined component, which will be found, minimizing the functional

$$I=\int_\Omega\left[(\nabla V_z)^2-2CV_z\right]d\Omega\,.$$

Note that the solution V_z is obtained analytically and used without any further treatment (approximation, interpolation). Therefore, the resulting distribution of the velocity is substituted in the right side of the heat conductivity equation. The structure of solution for the problem of determining the temperature field was used both as exactly satisfying the boundary conditions on $\partial\Omega_b$

$$u=p_2+\omega_b(-D_1p_2+hp_2)\,,$$

and as $T = p_2$ where, as before,

$$p_2=\sum_{i=1}^N d_{ik}\varphi_{ik}(x,y)\,.$$

It should be noted that the boundary conditions

$$\frac{\partial T}{\partial n}+hT\,|_{\Omega_b}=0\,,$$
$$\lambda_1\left.\frac{\partial T_1}{\partial n_1}\right|_{\partial\Omega_{t\nu}}=\lambda_2\left.\frac{\partial T_2}{\partial n_2}\right|_{\partial\Omega_{t\nu}}$$

are natural and result from the Ritz functional

$$I=\int_\Omega\left[\lambda(\nabla T)^2-2FT\right]d\Omega+\int_{\partial\Omega_b}hT^2d\partial\Omega_b\,.$$

As an approximation of $\varphi_{ik}(x,y)$ cubic splines of Schoenberg with $N = 6400$, 10000 were used. Computational experiments were carried out in POLYERL system developed in the department of applied mathematics and computational methods of IPMash

NASU. The results of researches for different packages of fuel rods are shown below (Figs. 3-6). Each package contains 91 fuel rods with same other conditions $\lambda_1 = 1$, $\lambda_2 = 10$, $h = 1$, $q_V = 4$.

Different distributions of the researched fields were taken by changing the parameter values.

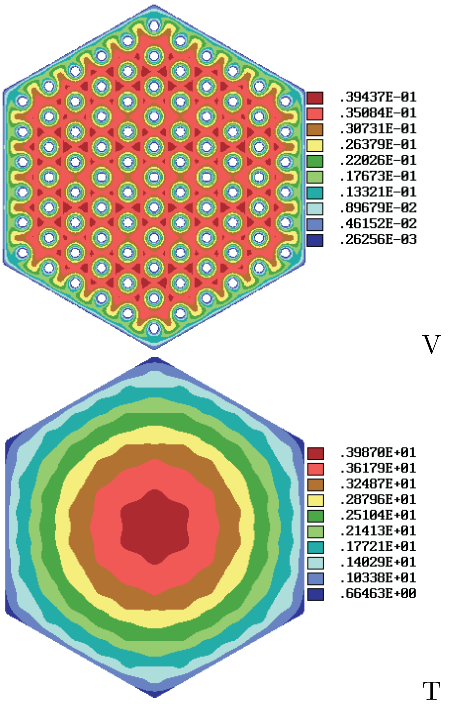

Fig.3. *Picture of the velocity and temperature distribution in the cartridge with fuel rods arranged in checkerboard pattern*

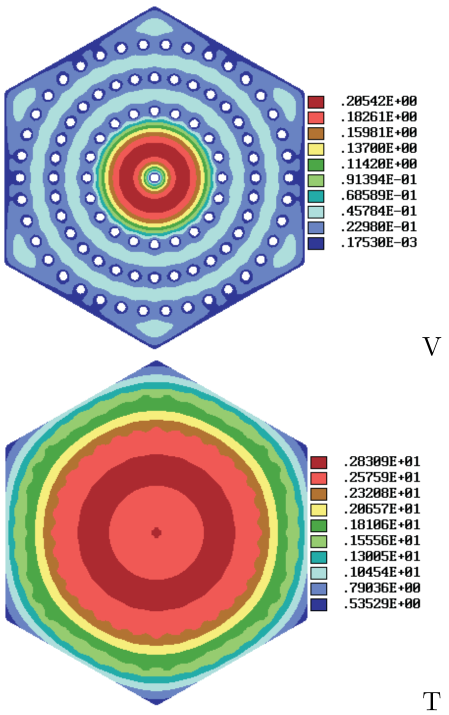

Fig.4. *Picture of the velocity and temperature distribution in the cartridge with cyclically spaced fuel rods and central fuel rod*

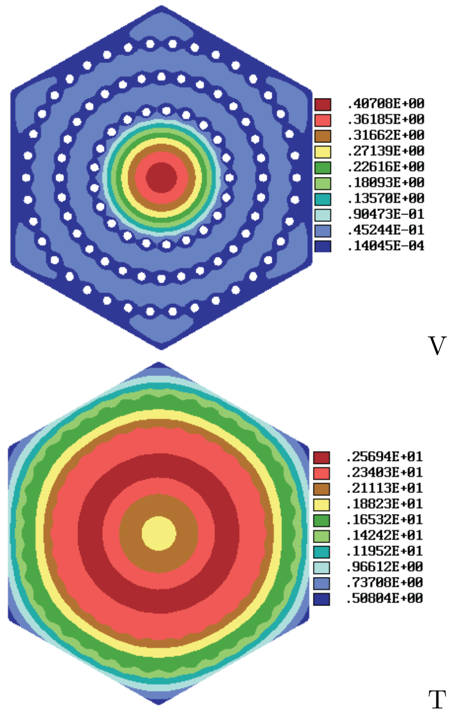

Fig.5. *Picture of the velocity and temperature distribution in the cartridge with cyclically spaced fuel rods without central fuel rod*

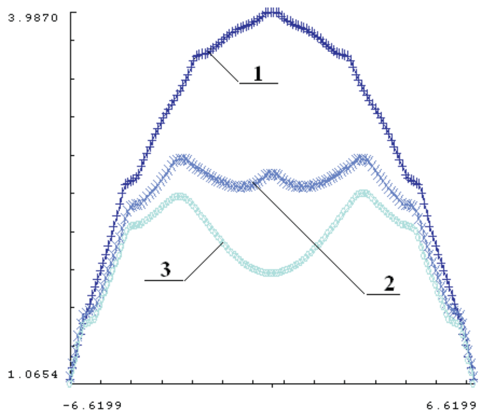

Fig.6. *Graphs of the temperature field for different packages of fuel rods in the cross section : 1 is checkerboard pattern; 2 is cyclical symmetry with central fuel rod; 3 is cyclical symmetry without central fuel rod*

Analyzing the results, we can conclude that the presence of fuel rods in the central area leads to a higher temperature. Therefore, changing the packaging pattern and types of symmetry, we can adjust the nature of the flow and temperature distribution in the cartridge, achieving the value stated by technical requirements. Analysis of the velocity and temperature distribution allows to conclude that the consideration of the velocity field of the cell (see Fig. 1), if it is sufficiently far from the border, is appropriate. However, the temperature field at the same time is far from reality, as evidenced by the results obtained for the whole cartridge.

3. CONCLUSIONS

It is shown that the R-functions method is effective for solving the problems of the physical field calculation in complex shape structural elements of nuclear power plants. The developed design tools for constructing the equations of domain boundaries with translational and cyclic symmetry types allowed to significantly reduce the number of operations with subsequent automation of the process, and, hence, to reduce the time for solving the problems. These experiments allow designers to choose certain types of packaging according to the specifications. During this, the essential point is to calculate the temperature field for the whole cartridge. Mathematical modeling and associated computer experiment are indispensable in cases, when the natural experiment is impossible or difficult, for various reasons. In addition, working with a mathematical model of the process and computer experiment allows to investigate the properties and behavior of the process in different situations painlessly, relatively quickly, and without significant cost. At the same time, the computational experiments with object models allow, based on modern numerical methods, to study them deeply in details. The reliability of analytical identification of geometric objects is approved by their visualization, while the reliability of calculation methods, results and conclusions is confirmed by comparison with the information known from the literature and the analysis of the numerical convergence of solutions and the calculation of the residual.

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МЕТОД R-ФУНКЦИЙ В МАТЕМАТИЧЕСКОМ МОДЕЛИРОВАНИИ КОНВЕКТИВНОГО ТЕПЛООБМЕНА В ТОПЛИВНОЙ КАССЕТЕ ТВЭЛОВ

К.В. Максименко-Шейко, Р.А. Уваров, Т.И. Шейко

Рассмотрены сопряженные краевые задачи теплообмена для случаев, когда вязкая несжимаемая жидкость движется по каналам неканонического сечения, обтекая пучок стержней. Исследовано влияние вида упаковки на распределение скорости и температуры. Для решения использовалась теория R-функций в сочетании с вариационным методом Ритца. Рассмотрены различные упаковки ТВЭлов. Каждая упаковка содержит 91 стержень, а соответствующие уравнения построены с использованием новых конструктивных средств теории R-функций.

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К.В. Максименко-Шейко, Р.О. Уваров, Т.І. Шейко

Розглянуто пов'язані крайові задачі теплообміну для випадків, коли в'язка нестисла рідина рухається по каналах неканонічного перерізу, обтікаючи пучок стрижнів. Досліджено вплив виду пакування на розподіл швидкості і температури. Для розв'язування використовувалася теорія R-функцій у поєднанні з варіаційним методом Рітца. Розглянуто різні пакування ТВЕлів. Кожне пакування містить 91 стрижень, а відповідні рівняння побудовані з використанням нових конструктивних засобів теорії R-функцій.