OPTIMIZATION OF APF ACCELERATORS

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Optimization in an equivalent traveling wave of APF accelerating structures is considered. New approach to obtain synchronous phase sequence in APF structure is suggested. Space charged was taken into account in considering mathematical optimization model. The result of numerical optimization is discussed.

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INTRODUCTION

Linear accelerators with the accelerating field focusing have long been a part of any modern accelerator complex. Particularly, the combination of the radio frequency quadrupole (RFQ) accelerator and the accelerator with alternating-phase focusing (APF) is a good solution for the initial part of a high-energy accelerator. In this case, the bunched in RFQ ions beam can be injected in a cavity with APF.

At present there is a significant progress in the development of APF accelerators [1 - 5]. But the problem of intensity beams optimization is still actual, because increasing of the beam current leads to decreasing of the beam quality. Minimizing the negative effects of this phenomenon is the purpose of this work. A similar problem was solved earlier (for RFQ accelerators [6 - 15]) by using the approach of modeling the beam dynamics in an equivalent traveling wave.

The same principle is applied in this work for APF structures. A numerical optimization method for intense beam dynamics is proposed.

1. MATHEMATICAL MODEL OF BEAM DYNAMICS

Let us consider the beam dynamics in following variables: β_s is the velocity of synchronous particle, $\psi = \varphi - \varphi_s$ is the phase deviation, $p_\psi = \gamma_s - \gamma$ is the energy deviation S_{11}, S_{21}, S_{22} are the elements of the matrix $G = \begin{pmatrix} S_{11} & S_{21} \\ S_{21} & S_{22} \end{pmatrix}$, that describe dynamics of the initial ellipse G_0 in the radial plane (η, κ) where $\eta = r/\lambda$ is the radial position of particle and $\kappa = d\eta/d\tau$ (where $\tau = ct/\lambda$ is an independent variable). Then, to describe the dynamics of the beam in an equivalent traveling wave we can use the following mathematical model [1, 16, 17]:

$$\frac{d\beta_s}{d\tau} = \alpha\sqrt{1-\beta_s^2}\cos(\varphi_s(\tau)), \qquad (1)$$

$$\frac{d\psi}{d\tau} = \frac{2\pi(1-\beta_s{}^2)^{3/2}}{\beta_s{}^2} p_\psi, \qquad (2)$$

$$\frac{dp_\psi}{d\tau} = \alpha\beta_s(\cos(\varphi_s(\tau)) - \cos(\varphi_s(\tau)+\psi)) + F_{\text{int}}, \qquad (3)$$

$$\frac{dG}{d\tau} = -A^T G - GA, \qquad (4)$$

$$A = \begin{pmatrix} 0 & 1 \\ Q & 0 \end{pmatrix},$$

$$Q = \frac{\alpha\pi(1-\beta_s{}^2)^{3/2}}{\beta_s}\sin(\phi_s(\tau)+\psi) + \frac{q\lambda I}{2\pi m_0 c^3 \varepsilon_0 \beta S_{11}},$$

where $\alpha = q\lambda E_{max}/(2m_0c^2)$ is the accelerating wave amplitude parameter, $\varphi_s(\tau)$ is the synchronous phase function.

Also, we are taking into account the interection of the charged particles by using the «large particles» method.

Let us consider the dynamics of N large particles ("thick disks"). Then the intensity function F_{int} can be written as follows

$$F_{\text{int}}(\tau,\beta_s,\psi_j,\psi_i) = \frac{a^2 qI\lambda^2}{2\pi d^2 R^2 \varepsilon_0 m_0 c^3}\sum_{i=1}^{N}\sum_{m=1}^{\infty}\frac{J_1^2(\nu_m R/a)}{\nu_m^4 J_1^2(\nu_m)}\times$$

$$\times\Big[2g_m\big((h(\psi_j)-h(\psi_i))\big) - g_m\big((h(\psi_j)-h(\psi_i)+2d)\big)$$

$$-g_m\big((h(\psi_j)-h(\psi_i))-2d)\big)\Big],$$

where $2d$ is the disk thickness, a is the aperture radius, ν_m are the roots of $J_0(t)$ function,

$$h(t) = \begin{cases} \dfrac{-\lambda x(t+\varphi_S)}{2\pi}, \text{если } t+\varphi_S > 0, \\ \dfrac{-\lambda x(t+\varphi_S+2\pi)}{2\pi}, \text{если } t+\varphi_S < 0, \end{cases}$$

$$g_m(t) = \text{sign}(t)\big(1-\exp(-\nu_m|t|/a)\big).$$

2. OPTIMIZATION METHOD

Let us introduce the following designations:

$x = \beta_s$,

$y_1 = \{\psi, p_\psi\}^T$,

$y_2 = \{S_{11}, S_{21}, S_{22}\}^T$.

Then the mathematical model (1) - (4) in the general form can be written as a system of integral-differential equations [16, 17]

$$\frac{dx}{dt} = f(t,x,u),$$

$$\frac{dy_1}{dt} = F_{11}(t,x,y_1,u) +$$

$$+\int_{M_{1_{t,u}}} F_{12}(t,x,y_1,z_t)\rho(t,z_t)dz_t = F_1(t,x,y_1,u),$$

$$\frac{\partial\rho(t,y_1)}{\partial t}+\frac{\partial\rho(t,y_1)}{\partial x}F_1(t,x,y_1,u)+$$
$$+\rho(t,y_1)\operatorname{div}_{y_1}F_1(t,x,y_1,u)=0,$$
$$\frac{dy_2}{dt}=F_2(t,x,y_1,y_2,u),$$

with initial conditions

$$x(0)=x_0,$$
$$y_1(0)=y_{10}\in M_{10},$$
$$y_2(0)=y_{20}\in M_{20},$$
$$\rho(0,y_1(0))=\rho_0,$$

where $\rho(t,y_1)$ is the distribution function of charged particles in the phase space. Control $u(t)$ is corresponding to function $\varphi_s(\tau)$. Let us consider the following functionals

$$I_1(u)=\int_0^T\varphi_1(t,x,u)dt+g_1(x(T)),$$
$$I_2(u)=\int_0^T\int_{M_{t,u}}\varphi_2(t,x,y,u,\rho(t,y))dy_t dt+$$
$$+\int_{M_{T,u}}g_2(y(T),\rho(T,y(T))dy_T,$$
$$I(u)=I_1(u)+I_2(u). \qquad (5)$$

We will consider the problem of minimization of the quality functional (5).

To solve the problem of reducing the loss of particles and decrease of beam radius it is possible to take

$$I(u)=\int_{M_{T,u}}c_1F_1(\psi_T)+c_2F_2(S_{11_T})d\psi_T dp_T dS_{11_T}dS_{21_T}dS_{21_T},$$

where

$$F_1=\begin{cases}(\psi_T+\psi_1)^2, & \text{при} \quad \psi_T<-\psi_1,\\ 0, & \text{при} \quad \varphi_T\in[\psi_1,\psi_2],\\ (\psi_T-\psi_2)^2, & \text{при} \quad \psi_T>\psi_2,\end{cases}$$
$$F_2=\begin{cases}0, & \text{при} \quad S_{11_T}<\bar{S},\\ (S_{11_T}-\bar{S})^2, & \text{при} \quad S_{11_T}>\bar{S}.\end{cases}$$

Where $c_1, c_2, \psi_1, \psi_2, \bar{S}$ are the non-negative constants.

By using the analytic representation [17] of the functional (5) variation and taking into account the space charge, we can develop various methods for optimization of the accelerator parameters.

3. BEAM DYNAMICS SIMULATION

By using the proposed mathematical model, an optimization of synchronous phase sequence (Fig. 1) was carried out for 433 MHz, 3.4 m long deuteron accelerator with 14 MeV output energy. Input energy is 3 MeV, amplitude of accelerating field is 110 kV/cm, and 6 focusing periods consist of 110 accelerating cells. Initial conditions are presented at Fig. 2, normalized transversal beam emittance is $\varepsilon_n=0.0373\,\pi\cdot\text{cm}\cdot\text{mrad}$.

Also we should mention that the first synchronous phase sequences for focusing period in APF structure were obtained in [18, 19]. In work [2] synchronous phases sequence dependency of a small number of parameters was presented. This representation allows to provide optimization by using a variety of methods that do not use the analytic representations of the functional gradient. In this paper, we use analytic representation of the functional variation that allows us to get the functional gradient with respect to the $\varphi_s(\tau)$ function.

The results of the beam dynamics simulations with 14 and 16 mA beam current are presented at Figs. 3-8. There are no particle losses for these currents.

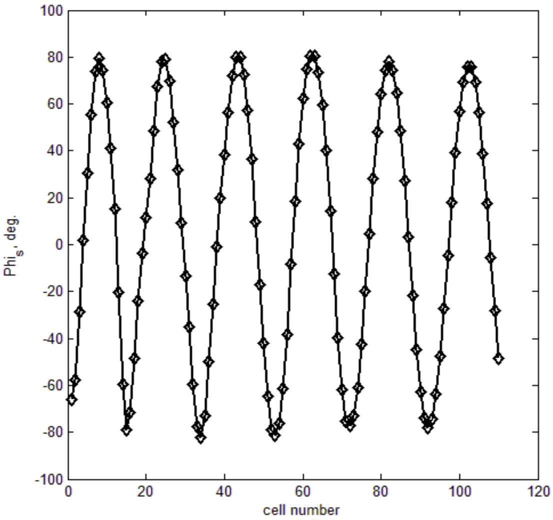

Fig. 1. Synchronous phase sequence

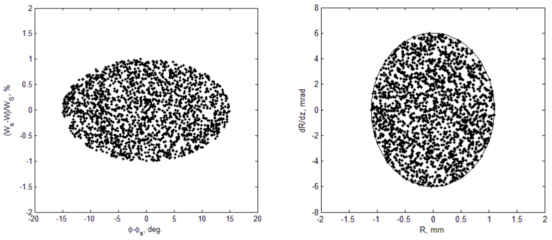

Fig. 2. Initial conditions for longitudinal and transversal phase volumes

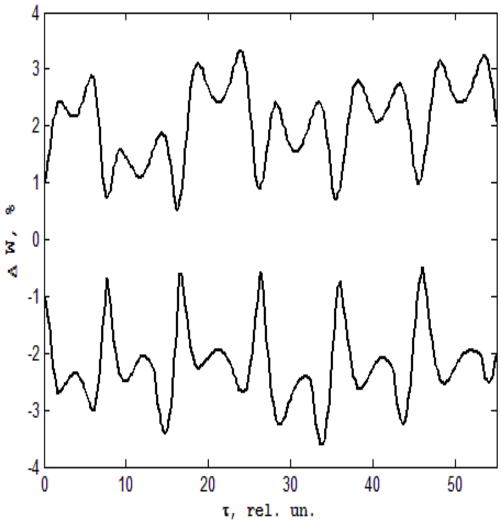

Fig. 3. Energy beam envelopes. I=14 mA

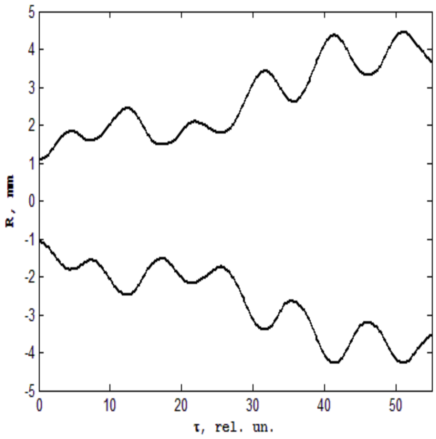

Fig. 4. Radial beam envelopes. I=14 mA

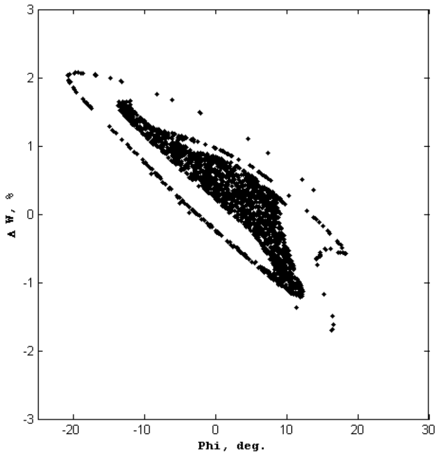

Fig. 5. Output longitudinal emittance. I=14 mA

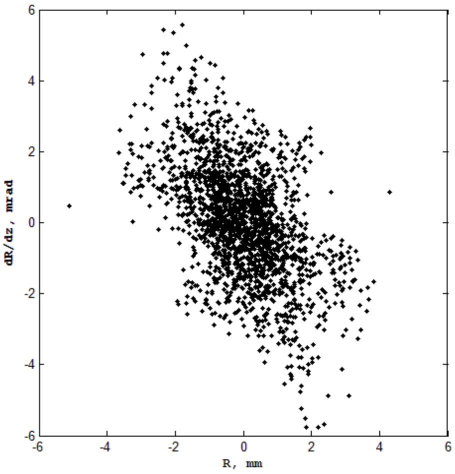

Fig. 6. Output transversal emittance. I=14 mA

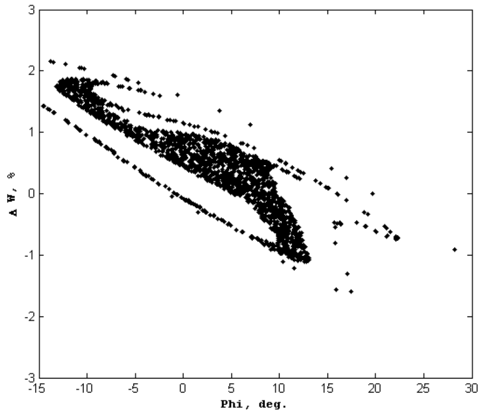

Fig. 7. Output longitudinal emittance. I=16 mA

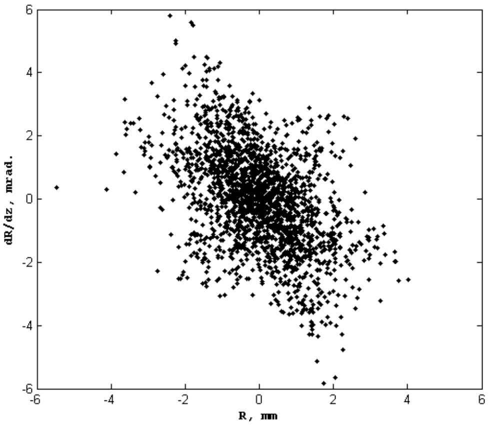

Fig. 8. Output transversal emittance. I=16 mA

CONCLUSIONS

In this paper the problem of intensity beam dynamics optimization in an APF accelerator is considered. To solve this problem we propose the mathematical optimization model. Compared to [1], in this work the space charge was take into account. An analytic representation of the quality functional variation allows us to provide the numerical optimization by other motion parameters and criteria for various focusing periods.

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ОПТИМИЗАЦИЯ УСКОРЯЮЩИХ СТРУКТУР С ПЕРЕМЕННО-ФАЗОВОЙ ФОКУСИРОВКОЙ

Д.А. Овсянников, В.В. Алцыбеев

Рассматривается оптимизация ускоряющих структур на основе математической модели в эквивалентной бегущей волне. Предлагается новый алгоритм построения изменения синхронной фазы вдоль ускорителя, обеспечивающий эффективный захват заряженных частиц в режим ускорения и их фокусировку. Исследуется математическая модель оптимизации динамики частиц с учетом их взаимодействия. Приводятся результаты численной оптимизации.

ОПТИМІЗАЦІЯ ПРИСКОРЮВАЛЬНИХ СТРУКТУР ІЗ ЗМІННО-ФАЗОВИМ ФОКУСУВАННЯМ

Д.О. Овсянников, В.В. Алцибеєв

Розглядається оптимізація прискорювальних структур на основі математичної моделі в еквівалентній хвилі, що біжить. Пропонується новий алгоритм побудови зміни синхронної фази уздовж прискорювача, що забезпечує ефективне захоплення заряджених частинок у режим прискорення і їхнє фокусування. Досліджується математична модель оптимізації динаміки частинок з урахуванням їх взаємодії. Приводяться результати чисельної оптимізації.