CIRCULAR POLARIZATION OF THE HIGH ENERGY ELECTRON BREMSSTRAHLUNG WITH ACCOUNT OF THE SECOND BORN APPROXIMATION

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The estimation of contribution of the second Born approximation into circular polarization of bremsstrahlung from linearly polarized electrons is obtained. Account of this contribution could be important for correct interpretation of results of measurements of fine effects due to weak interactions.

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1. INTRODUCTION

It is well known [1-3] that the bremsstrahlung emitted by polarized electrons could possess nonzero circular polarization. It is, particularly, an interesting fact because the particles produced in weak processes turned out to be polarized. Circular polarization of bremsstrahlung was considered in many papers (see article [2] and references therein) in the frameworks of the first Born approximation of quantum electrodynamics. However, it could be necessary to account for the second Born approximation contribution for correct interpretation of the results of measurements of fine effects due to weak interactions. It is of interest also that the second Born approximation leads to the dependence of radiation characteristics (the cross section and polarization) on the charge sign of the radiating particle.

2. BREMSSTRAHLUNG MATRIX ELEMENT AND CROSS SECTION

The cross section of bremsstrahlung in an external field is determined by formula [3]

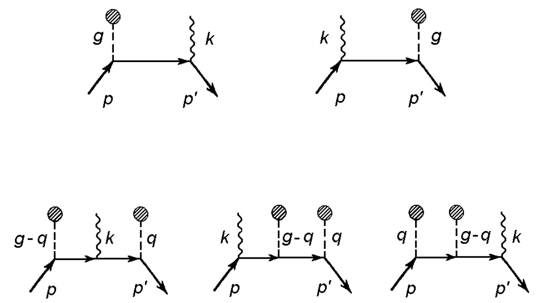
$$d\sigma = \frac{e^2}{4(2\pi)^4 \omega\varepsilon\varepsilon'} \delta(\varepsilon - \varepsilon' - \omega) \left| M_{fi} \right|^2 d^3p' d^3k \,, \quad (1)$$

where e is the electron charge, (ε, $\mathbf{p}$) and (ε', $\mathbf{p}'$) are the energy and momentum of the initial and final particles, (ω, $\mathbf{k}$) are the frequency and wave vector of the photon emitted, $\delta(\varepsilon - \varepsilon' - \omega)$ is the delta-function expressing the energy conservation under radiation, and M_{fi} is the matrix element of the radiation process. Since the main contribution to the bremsstrahlung cross section is made by small values of the momentum $\mathbf{g} = \mathbf{p} - \mathbf{p}' - \mathbf{k}$ transferred to the external field (the last is assumed to be stationary and potential), $g \ll m$, it is convenient to express the matrix element as a function of the transferred momentum. This permits to make an expansion in the matrix element by the powers of small parameter g / m.

According to the rules of diagram techniques [3] the squared absolute value of the matrix element in (1) could be written in the form

$$\left| M_{fi} \right|^2 = \left| M_1 \right|^2 |U_g|^2 + 2U_g \,\mathrm{Re} \int M_1 M_2^* U_q U_{g-q}^* \frac{d^3q}{(2\pi)^3} \,, \quad (2)$$

where U_g is Fourier component of the particle potential energy in the external field, M_1 and M_2 are the matrix elements that determine contributions of the first and the second Born approximations (see Figure).



Feynman diagrams corresponding to the first and the second Born approximations in the description of bremsstrahlung in the external field

Discriminating in the propagator in M_1 the dependence on the longitudinal and transverse components of the transferred momentum $\mathbf{g}$ in an explicit form we could write

$$M_1 = \bar{u}' Q_1 u \,, \quad (3)$$

where

$$Q_1 = \hat{e} b - \frac{\hat{e}\hat{g}\gamma_0}{2\varepsilon\sigma_g} - \frac{\gamma_0 \hat{g}\hat{e}}{2\varepsilon'\tau_g} \,,$$

$g_\mu = (0, \mathbf{g}) = p_\mu - p'_\mu - k_\mu$ is the transferred 4-momentum, p_μ, p'_μ, k_μ are the 4-momenta of the initial and final electrons and photon, e_μ is the photon's polarization vector, $\hat{p} = p_\mu \gamma^\mu$, γ_μ are Dirac matrices, v and v' are the velocities of the initial and final electrons. The values b, σ_g and τ_g are determined by the formulae

$$b = g_{||}\left(\frac{1}{\sigma_g} - \frac{1}{\tau_g}\right), \quad \sigma_g = g_{||} - \frac{\mathbf{g}^2}{2p},$$

$$\tau_g = g_{||} + \mathbf{n}_\perp \mathbf{g} + \frac{\mathbf{g}^2}{2p'},$$

where $\mathbf{n} = \mathbf{p}'/|\mathbf{p}'|$ is the unit vector along the momentum $\mathbf{p}'$ direction, and $\mathbf{n}_\perp$ is the component of this vector orthogonal to $\mathbf{p}$.

Neglecting the terms of order m^2/ε^2 and m^2/ε'^2 the formula for M_2 could be derived to the form [4,5]

$$M_2 = \bar{u}'\left\{\left[Q_1 - \frac{\omega}{\varepsilon'\tau_g}\left(\hat{e} + \frac{\gamma_0 \hat{g}\hat{e}}{2\varepsilon'}\right)\right]\frac{\mathbf{q}_\perp \mathbf{q}'_\perp}{2\varepsilon\sigma_q \sigma_{q'}} + Q_2\right\}u, \quad (4)$$

where $q'_\mu = g_\mu - q_\mu$ and

$$Q_2 = -\frac{\hat{e}\hat{g}\hat{q}_\perp}{4\varepsilon^2 \sigma_g \sigma_q} - \frac{\hat{q}'_\perp \hat{g}\hat{e}}{4\varepsilon'^2 \tau_g \tau_{q'}} + \frac{\gamma_0 \hat{q}'_\perp \hat{e}\hat{q}_\perp \gamma_0}{4\varepsilon\varepsilon'\tau_{q'}\sigma_q}.$$

The cross section itself also could be expressed through the transferred momentum (and the angle ϑ between the vectors $\mathbf{k}$ and $\mathbf{p}$). Transformation to new variables is described in [6]. After that the differential cross section gets the form

$$d\sigma = \frac{e^4}{(2\pi)^4}\frac{\varepsilon'}{\varepsilon}\left|M_{fi}\right|^2 \frac{\delta}{m^2}\frac{d\omega}{\omega}\frac{dy}{\sqrt{1-y^2}}d^3 g, \quad (5)$$

where $\delta = \omega m^2/2\varepsilon\varepsilon'$. The variable y is connected to ϑ by the relation

$$(\varepsilon\vartheta/m)^2 = f + y\sqrt{a}, \quad -1 \le y \le 1, \quad (6)$$

where

$$a = \frac{4g_\perp^2}{m^2\delta}\left(g_{||} - \delta - \frac{g_\perp^2}{2\varepsilon}\right),$$

$$f = \frac{1}{\delta}\left(g_{||} - \delta - \frac{g_\perp^2}{2\varepsilon} + \frac{g_\perp^2 \delta}{m^2}\right),$$

$g_{||}$ and $\mathbf{g}_\perp$ are the components of $\mathbf{g}$ that parallel and orthogonal to the momentum $\mathbf{p}$ of the projectile particle. Eq. (6) determines the possible values of the radiation angle ϑ under given values of $g_{||}$ and and $g_\perp$. One could conclude from the condition of positiveness of the value a under radical in (6) that

$$g_{||} \ge \delta + g_\perp^2/2\varepsilon.$$

The formulae that describe the bremsstrahlung cross section averaged over polarizations of initial electron and summed over polarizations of final particles were obtained in [4, 5].

3. ACCOUNT OF POLARIZATION EFFECTS

Remember how to take into account the polarization of interacting particles. Matrix element of a process with single electron in initial and final states has the form $M_{fi} = \bar{u}'Qu$. The squared absolute value of such matrix element is equal to (see, e.g., [3])

$$\left|M_{fi}\right|^2 = M_{fi}M_{fi}^* = \mathrm{Sp}\left\{u'\bar{u}'Qu\bar{u}\,\bar{Q}\right\},$$

where

$$\bar{Q} = \gamma_0 Q^+ \gamma_0.$$

If an electron is in a mixed (partially polarized) state, the products of bispinor amplitudes are to be replaced by the corresponding density matrices:

$$u\bar{u} \to \rho, \quad u'\bar{u}' \to \rho'.$$

The density matrix for polarized electron is described by Eq. (29.13) from [3]:

$$\rho = \frac{1}{2}(\hat{p} + m)[1 - \gamma_5 \hat{s}], \quad (7)$$

where

$$s_\mu = \left(\frac{\mathbf{p}\cdot\boldsymbol{\zeta}}{m}, \boldsymbol{\zeta} + \mathbf{p}\frac{\mathbf{p}\cdot\boldsymbol{\zeta}}{m(m+\varepsilon)}\right), \quad s_\mu p_\mu = 0,$$

ζ is the double average value of the spin vector in the electron rest frame (in pure state $|\zeta| = 1$, in mixed one $|\zeta| < 1$). It is easy to see that $s_0 = \frac{\mathbf{s}\cdot\mathbf{p}}{\varepsilon}$, and that in ultrarelativistic case $\mathbf{s} \approx \frac{\boldsymbol{\zeta}\cdot\mathbf{v}}{m}\mathbf{p}$.

For nonpolarized electron $\rho = \frac{1}{2}(\hat{p} + m)$. Substitution of this formula is equivalent to averaging over the electron's polarizations. Since we need to calculate the cross section of the process with arbitrary polarization of final electron, let us put $\rho' = \frac{1}{2}(\hat{p}' + m)$ and multiply the result by the factor 2 that would be equivalent to summation over final electron polarizations.

The polarization of final photon is present in the value Q as 4-vector e_μ^*, and in $\bar{Q}$ as e_μ. So, in the squared absolute value of the matrix element we get the tensor $e_\mu^* e_\nu$. For description of the case of arbitrary partially polarized state this tensor ought to be replaced by the density matrix [3]:

$$e_\mu^* e_\nu \to -\frac{1}{2}g_{\mu\nu} + \frac{\xi_1}{2}\left(e_\mu^{(1)}e_\nu^{(2)} + e_\mu^{(2)}e_\nu^{(1)}\right)$$
$$+ i\frac{\xi_2}{2}\left(e_\mu^{(1)}e_\nu^{(2)} - e_\mu^{(2)}e_\nu^{(1)}\right) \quad (8)$$
$$+ \frac{\xi_3}{2}\left(e_\mu^{(1)}e_\nu^{(1)} - e_\mu^{(2)}e_\nu^{(2)}\right).$$

Here ξ_1, ξ_2, ξ_3 are the Stocks parameters, $e_\mu^{(1)}$ and $e_\mu^{(2)}$ are the polarization 4-vectors.

Note that here ξ is the polarization discriminated by the detector. Polarization of the final photon as well could be easily found if we know the squared absolute value of the matrix element as a function of parameters ξ :

$$\left|M_{fi}\right|^2 = \alpha + \boldsymbol{\beta}\cdot\boldsymbol{\xi}. \quad (9)$$

Then the polarization of the final photon will be described by Stocks parameters [3]

$$\xi^{(f)} = \frac{\boldsymbol{\beta}}{\alpha}. \quad (10)$$

Since the *circular* polarization degree is of our interest, we need to calculate only the values α and β_2, that have their origin from the substitution of the first and the third terms of the photon density matrix, respectively.

Substituting the expression for the matrix element in the form (2) with account of Eqs. (3), (4) for M_1 and M_2 and (7), (8) for density matrices into Eq. (5) for the radiation cross section, we obtain (after simple but rather awkward calculations) for the values α and β_2, determined in (9), integrated over the variables y and $g_{||}$ [1]:

$$\int_\delta^\infty dg_{||}\int_{-1}^{1}\frac{dy}{\sqrt{1-y^2}}\alpha = 2\pi\frac{\mathbf{g}_\perp^2}{\delta}\left\{\left[\frac{2}{3}+\frac{\omega^2}{2\varepsilon\varepsilon'}\right]U_g^2\right.$$

$$\left.+\frac{1}{\varepsilon}\frac{1}{(2\pi)^3}U_g I(g_\perp)\left[\frac{2}{3}+\frac{\omega^2}{2\varepsilon\varepsilon'}+\frac{\omega}{2\varepsilon'}\left(\frac{2}{3}+\frac{\omega^2}{2\varepsilon\varepsilon'}\right)\right]\right\};$$

$$\int_\delta^\infty dg_{||}\int_{-1}^{1}\frac{dy}{\sqrt{1-y^2}}\beta_2$$

$$=2\pi\frac{\mathbf{g}_\perp^2}{\delta}(\boldsymbol{\zeta}\cdot\mathbf{v})\left\{\left[\frac{\omega(\varepsilon+\varepsilon')}{2\varepsilon\varepsilon'}-\frac{1}{3}\frac{\omega}{\varepsilon}\right]U_g^2\right.$$

$$+\frac{1}{2\varepsilon}\frac{1}{(2\pi)^3}2U_g I(g_\perp)$$

$$\left.\times\left[\frac{\omega(\varepsilon+\varepsilon')}{2\varepsilon\varepsilon'}-\frac{1}{3}\frac{\omega}{\varepsilon}-\frac{1}{2}\frac{\omega}{\varepsilon'}\left(\frac{2}{3}\frac{\omega}{\varepsilon}+\frac{1}{2}\frac{\omega}{\varepsilon'}\right)\right]\right\},$$

where for the screened Coulomb potential

$$U(r)=\frac{Z|e|e}{r}e^{-r/R},\quad U_g=\frac{4\pi Z|e|e}{\mathbf{g}^2+R^{-2}}$$

as the potential of atom in which the electron radiates,

$$I(g_\perp)=(4\pi)^2Z^2e^4\frac{\pi^2}{g_\perp}$$

$$\times\left\{-\frac{4g_\perp/R}{g_\perp^2+4R^{-2}}-\frac{\pi}{2}-\arcsin\frac{g_\perp^2-4R^{-2}}{g_\perp^2+4R^{-2}}\right\}. \quad (11)$$

This result is justified with the accuracy up to the terms of order of m^2/ε^2, m^2/ε'^2 and $m^2/\omega\varepsilon$.

Under integration of the expressions obtained over $d^2g_\perp$ there arises logarithmic divergence under large $g_\perp$ in the first Born approximation, and linear divergence in the second one. That divergence is connected to the approximation $g_\perp << m$ used above. Hence for estimation of the contribution of the second Born approximation into the radiation polarization one needs to integrate over $g_\perp$ from 0 to m introducing the cut-off on the upper limit.

Substituting the result of such integration into (10) we get the following formula for the circular polarization degree:

$$\xi_2^{(f)}=(\boldsymbol{\zeta}\cdot\mathbf{v})\frac{\omega\left(\varepsilon+\frac{1}{3}\varepsilon'-F\omega\left[\frac{5}{6}+\frac{\varepsilon}{\varepsilon'}\right]\right)}{\varepsilon^2+\varepsilon'^2-\frac{2}{3}\varepsilon\varepsilon'}, \quad (12)$$

where we have denoted

$$F=\frac{1}{(2\pi)^3}\frac{\int_0^m I(g_\perp)U_g g_\perp^2\cdot g_\perp dg_\perp}{\varepsilon\int_0^m U_g{}^2 g_\perp^2\cdot g_\perp dg_\perp}. \quad (13)$$

It is easy to see that in the first Born approximation the formula (12) manifests agreement with well known result [2].

Substituting $I(g_\perp)$ in the form (11) into (13), we obtain the following estimate for the value of F:

$$F\sim-\frac{e}{|e|}\frac{\pi}{2}Z\alpha\frac{m/\varepsilon}{\ln(mR)}, \quad (14)$$

where $\alpha\approx 1/137$ is the fine structure constant. For example, the silicon atom has $Z = 14$, $\ln(mR)\approx 4$, and for electrons of energy 100 MeV

$$F\sim-2\cdot10^{-4}e/|e|.$$

Account of the second Born approximation leads to the dependence of the polarization on the radiating particle charge sign. It is easy to see from (12), (14) that the degree of circular polarization of radiation from positrons exceeds the same from electrons by the relative value

$$\pi Z\alpha\frac{m\omega/\varepsilon}{\ln(mR)}\frac{\frac{5}{6}+\frac{\varepsilon}{\varepsilon'}}{\varepsilon+\frac{1}{3}\varepsilon'}.$$

For soft photons ($\omega << \varepsilon$) that relative difference is of order of $Z\alpha\, m\omega/\varepsilon^2$.

4. CONCLUSION

We see that the contribution of the second Born approximation into circular polarization of the photons emitted by polarized electrons of high energy on an amorphous target is rather small. However, such small correction could be important for correct interpretation of the results of measurement of fine effects due to weak interactions.

It should be mentioned also that the relative contribution of the second Born approximation could substantially grow in the case of radiation of the electrons in oriented crystal due to coherent effects [6], like it takes the place for the bremsstrahlung cross section summed over polarizations of particles participating in the process [4,5].

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[1] Since in a field of an individual atom the characteristic value of $g_\perp\sim R^{-1}>>\delta$ appreciably exceeds $g_{||}\sim\delta$, $g_{||}$-dependence of U_g can be neglected [5,6].

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ЦИРКУЛЯРНАЯ ПОЛЯРИЗАЦИЯ ТОРМОЗНОГО ИЗЛУЧЕНИЯ ЭЛЕКТРОНОВ ВЫСОКОЙ ЭНЕРГИИ С УЧЕТОМ ВТОРОГО БОРНОВСКОГО ПРИБЛИЖЕНИЯ

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Получена оценка вклада второго борновского приближения в циркулярную поляризацию тормозного излучения линейно-поляризованными электронами. Учет этого вклада может быть важным для корректной интерпретации результатов измерения тонких эффектов, обусловленных слабыми взаимодействиями.

ЦІРКУЛЯРНА ПОЛЯРИЗАЦІЯ ГАЛЬМОВНОГО ВИПРОМІНЮВАННЯ ЕЛЕКТРОНІВ ВИСОКОЇ ЕНЕРГІЇ З УРАХУВАННЯМ ДРУГОГО БОРНІВСЬКОГО НАБЛИЖЕННЯ

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Отримано оцінку внеску другого борнівського наближення до циркулярної поляризації гальмівного випромінювання лінійно-поляризованими електронами. Урахування цього внеску може бути важливим для коректної інтерпретації результатів вимірювання тонких ефектів, що обумовлені слабкими взаємодіями.