HOW OLD SUPERGRAVITY IS: THIRTY FIVE YEARS OR MORE?

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These notes is aimed at comparing two different approaches to theory of gravity with a fermionic gauge field, which possesses the invariance under special transformations called supersymmetry. One of these approaches is the approach by Ferrara-Freedman-Nieuwenhuizen-Deser-Zumino (FFNDZ) of 1976; the other one is that of Volkov-Soroka (VS) of 1974. The analysis is based on the standard concept of realizing Supergravity as a gauge theory for the super-Poincare group. We deliberately sacrifice the rigor of the proposed consideration in compare to the pioneering papers on D=4 N=1 Supergravity to make our presentation simple as much as possible. In effect we emphasize the differences between the above mentioned approaches. We keep out of making rigorous conclusions throughout the paper, and suggest the reader to get the own answer to the question posed in the title.

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FOREWORD

This paper is based on the authors' talk at the Special Research Scientific Council of Kharkov Institute of Physics and Technology in July of 2005 to Commemorate the 80th Anniversary of Academician D.V. Volkov. My notes would never have appeared in print if it were not an accident this summer. Suddenly and very unexpectedly for us Vyacheslav Aleksandrovich Soroka passed away. He was a great man, good friend and colleague. Involved in many activities, related to studies of various aspects of modern theoretical physics, he took an active and direct part in studies of supersymmetric models, which was resulted, in particular, in the Supergravity invention. That is why I took the liberty to publish these notes, which, I hope, will be useful in dating the Supergravity age.

1. INTRODUCTION

There is a conventional wisdom that the foundation of Supergravity, or more precisely the simple N=1 D=4 Supergravity, took place in 1976 and began with two seminal papers by Ferrara-Freedman-Nieuwenhuizen [6] and by Deser-Zumino [7] (FFNDZ). This point of view is widespread [1], [2] and without no doubt is true. However, the history of Science learns us that every significant innovation has its own, perhaps dramatic, pre-history, when a little gap in insight does not allow predecessors to make a final step towards a new discovery. The history of Science also learns us that sometimes a discovery of one person is erroneously ascribed to another one as it happened for instance with the Kamerlingh Onnes superconductivity, which was in fact observed by Gilles Holst [3]. It is not the unique example of course, so in the light of the above it is natural to ask could it be something like that with the Supergravity invention?

The latter question gains a new insight once I recall that in the Former Soviet Union the Supergravity foundation is ascribed to D.V. Volkov and V.A. Soroka in view of [4], [5]. I make the special accent on the dates of these papers: 1973 and 1974. Hence, if the results of these two papers by Volkov-Soroka (VS) are correct, one should at least get a little doubt on the real date of the Supergravity foundation.

To figure out the (in)correctness of the VS approach we have to trace the way of reasoning by VS and that of FFNDZ back and to compare the formulations to each other. I should emphasize that such a comparison of two formulations (more precisely, VS to FFNDZ) has been done, see e.g. [8], [9, 10], [11]. However, it is not so easy to realize the arguments of these papers since they appeal to the clear understanding the VS construction. Roots of misunderstanding are mainly twofold: The VS construction is based on the non-linear realization of the super-Poincare group with its subsequent gauging, the approach which is not so common, popular and well-known for the present days audience (see however [12]); and the notation in [8]-[10], borrowed in part from the original papers [4, 5], is either far from the modern notation accepted in Supergravity, or denotes different objects. Therefore, to make things clear we have to reformulate old results in a modern and commonly accepted fashion.

I would not stick to any "preferred" point of view in the analysis below. Rather I suggest the reader to be a referee and to get the own answer to the question posed in the title.

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2. SUPERGRAVITY – WHAT'S THIS?

To realize the announced program let us get started with recalling what Supergravity is? I will refer to the standard definition of Supergravity (cf. e.g. [2]): Supergravity is a theory of gauge fields, which is invariant under the local (i.e. with space-time dependent parameters) supersymmetry. Any theory of Supergravity contains a spin 2 gauge field or graviton, – this field is responsible for the gravitational interaction, – together with its spin 3/2 super(symmetric) partner, the so-called gravitino. Supergravity may also include scalars, vector and antisymmetric tensor gauge fields of spin zero and one, together with their superpartners with spin 1/2. A field content of Supergravities is strongly dependent on the number of space-time dimensions, features of the considered theory (simple, i.e. with $N = 1$ local supersymmetry, or extended, i.e. with $N > 1$, Supergravity), gauged or un-gauged Supergravity and so on. However, in all cases the field content should be supersymmetric, that means it has to provide the precise balance of the bosonic and fermionic degrees of freedom, at least on the mass-shell. The off-shell matching the bosonic and fermionic degrees of freedom is also desired, but not necessary.

There are many ways to Supergravity constructing. A general, but not always simple, method of the construction of Supergravity is based on the corresponding superalgebra gauging (see [13] for details). In general, the superalgebra possesses a complicated structure, but it always includes the super-Poincare algebra as a subalgebra. To reach conclusions on VS vs. FFNDZ model, no need to deal with a complicated superalgebra. It is enough to gauge the N=1 D=4 super-Poincare algebra to this end.

3. GAUGING THE SIMPLE SUPER-POINCARE ALGEBRA

We follow the way of N=1 D=4 Supergravity constructing in the spirit of the Yang-Mills theory, as it was done (but in more complicated manner) in [4, 5], and was recently discussed in [14]. The starting point is the N=1 D=4 super-Poincare algebra

$$
\begin{gathered}
[P_a, P_b] = 0, \qquad [J_{ab}, P_c] = \eta_{ac}\ P_b - \eta_{bc}\ P_a, \\
[J_{ab}, J_{cd}] = \eta_{ac}\ J_{bd} - \eta_{bc}\ J_{ad} + \eta_{bd}\ J_{ac} - \eta_{ad}\ J_{bc}, \\
[J_{ab}, Q^\alpha] = -\frac{1}{2}(\gamma_{ab})^\alpha{}_\beta\ Q^\beta, \qquad [P_a, Q_\beta] = 0, \\
\{Q^\alpha, Q_\beta\} = \frac{1}{2}(\gamma^a)^\alpha{}_\beta\ P_a. \qquad (1)
\end{gathered}
$$

The next step is to construct the Yang-Mills-type connections

$$\mathbf{A} = A^A t_A = e^a P_a + \frac{1}{2}\omega^{ab} J_{ab} + Q\Psi, \qquad (2)$$

and their field strengths (curvatures)

$$
\begin{aligned}
\mathbf{F} = F^A t_A = d\mathbf{A} + \mathbf{A}\wedge\mathbf{A} = \\
= T^a P_a + \frac{1}{2}R^{ab}J_{ab} + R^\alpha Q_\alpha, \\
T^a = de^a + \omega^a{}_b\ e^b - \frac{1}{2}\bar\Psi\gamma^a\Psi, \\
R^{ab} = d\omega^{ab} + \omega^a{}_b\ \omega^{cb}, \\
R^\alpha = d\Psi^\alpha + \frac{1}{4}\omega^{ab}(\gamma_{ab})^\alpha{}_\beta\Psi^\beta \equiv D\Psi^\alpha. \qquad (3)
\end{aligned}
$$

Using the superalgebra (1) it is easy to verify that the local transformations of the connections

$$\delta\mathbf{A} = D\lambda = d\lambda + [\mathbf{A}, \lambda\}, \quad \lambda = \rho^a P_a + \frac{1}{2}\kappa^{ab}J_{ab} + Q\epsilon, \qquad (4)$$

which transform curvatures in the covariant way

$$\delta\mathbf{F} \sim [\mathbf{F}, \lambda\}, \qquad (5)$$

can be split on

i) translations

$$\delta_{\text{translations}}\ e^a = D\rho^a, \quad \delta_{\text{translations}}\ \omega^{ab} = 0, \quad \delta_{\text{translations}}\ \Psi^\alpha = 0; \qquad (6)$$

ii) local Lorentz transformations

$$\delta_{\text{Lorentz}}\ e^a = \kappa^a{}_b e^b, \quad \delta_{\text{Lorentz}}\ \omega^{ab} = -D\kappa^a{}_b, \quad \delta_{\text{Lorentz}}\ \Psi^\alpha = \frac{1}{4}\kappa^{ab}\gamma_{ab}\Psi^\alpha; \qquad (7)$$

iii) local supersymmetry transformations

$$\delta_{\text{SUSY}}\ e^a = \frac{1}{2}\bar\epsilon\gamma^a\Psi, \quad \delta_{\text{SUSY}}\ \omega^{ab} = 0, \quad \delta_{\text{SUSY}}\ \Psi^\alpha = D\epsilon^\alpha. \qquad (8)$$

Clearly, the Poincare superalgebra gauging naturally leads to $\delta_{\text{SUSY}}\omega^{ab} = 0$. The same transformation property of the connection follows from the VS papers [4, 5]. However, this result contradicts the FFNDZ supergravity construction [6], [7], that can be treated as a manifestation of the difference between the Yang-Mills and gravitational theories, and calls into question (see for instance [14]) the correctness of the VS approach.

One could stop here, since it seems to be unreasonable to proceed further after observing such a discrepancy. But this is not the end of the game. We have to turn to the corresponding actions, having in mind what we have figured out above.

4. FFNDZ VS. VS: ACTIONS ANALYSIS

Now consider the action of D=4 N=1 supergravity, proposed in [6, 7]

$$S_{FFNDZ} = \int_{\mathcal{M}^4} \epsilon_{abcd}e^a e^b R^{cd} + 4\bar\Psi\gamma_5 e^a\gamma_a D\Psi. \qquad (9)$$

This action *is not invariant under translations* (6) without the additional requirement

$$T^a = de^a + \omega^a{}_b\ e^b - \frac{1}{2}\bar\Psi\gamma^a\Psi = 0. \qquad (10)$$

In effect

1) the connection is not an independent variable anymore

$$\omega^{ab} = \omega^{ab}(e, \Psi); \tag{11}$$

2) transformations of the connection are different from that of followed from the algebraic consideration; in particular

$$\delta_{\rm SUSY}\ \omega^{ab} \neq 0; \tag{12}$$

3) the relative coefficient between two terms of the action (9) is completely fixed by the requirement of the action invariance under the local supersymmetry transformations;

4) the algebra of the gauge transformations is closed off-shell only by use of auxiliary fields;

5) the structure constants of the gauge transformations algebra become the structure functions of fields. That leads to additional drawbacks upon the quantization of the model.

Now let us turn to the corresponding part of the Volkov-Soroka action [4, 5]

$$S_{VS} = \int_{\mathcal{M}^4} \alpha_1 \epsilon_{abcd} E^a E^b R^{cd} + 4\alpha_2 \bar{\psi}\gamma_5 E^a \gamma_a D\psi \tag{13}$$

with arbitrary coefficients α_1, α_2. Aside from usual vielbeins e^a, (13) involves additional 'coordinates' ξ^a entering the 'generalized vielbeins'

$$E^a = e^a + D\xi^a - \frac{1}{2}\bar{\Psi}\gamma^a\theta - \frac{1}{4}D\bar{\theta}\gamma^a\theta, \tag{14}$$

together with Goldstone-type fermionic coordinates θ^α, entering the combination

$$\psi^\alpha = \Psi^\alpha + D\theta^\alpha. \tag{15}$$

That is why the Volkov-Soroka action *is invariant under translations* (6) and the invariance is guaranteed by the following transformations

$$\delta_{\rm translation}\ E^a = 0 \quad \Longleftrightarrow \quad \delta_{\rm translation}\ e^a = D\rho^a,$$
$$\delta_{\rm translation}\ \xi^a = -\rho^a, \quad \delta_{\rm translation}\ \theta^\alpha = 0 \tag{16}$$

The VS action *is manifestly invariant under the local SUSY transformations* (8) as well, that is provided by

$$\delta_{\rm SUSY}\ E^a = 0, \quad \delta_{\rm SUSY}\ \psi^\alpha = 0,$$
$$\delta_{\rm SUSY}\ \xi^a = \frac{1}{4}\bar{\epsilon}\gamma^a\theta, \quad \delta_{\rm SUSY}\ \theta^\alpha = -\epsilon^\alpha. \tag{17}$$

As the result:

a) the connection and the vielbein are independent variables;

b) the local supersymmetry transformations of the connection are not different from that of coming from the superalgebra gauging, i.e.

$$\delta_{\rm SUSY}\ \omega^{ab} = 0;$$

c) the gauge transformation algebra is closed off-shell and without introducing auxiliary fields;

d) the relative coefficients between different terms of the action are not fixed since the action is constructed out the manifestly invariant under the local supersymmetry transformations forms;

e) the transformation property of the 'coordinates' ξ^a, θ^α is the same as the transformation of the Goldstone fields, and they have no impact on physics described by the model;

f) upon eliminating the Goldstone fields, (13) is reduced to (9). Then, arbitrary coefficients of (13) are fixed by the requirement of the local supersymmetry of the action and $\delta_{\rm SUSY}\ \omega^{ab} \neq 0$ is required.

Before turning to final remarks, let me make an additional comment on the structure of the VS action (13). At a first sight it seems non plausible and unnatural that the local supersymmetry does not uniquely fix the relative coefficient between two manifestly invariant under the local supersymmetry transformations terms of the action. However, the requirement of local supersymmetry is not always enough to fix all the parameters entering the Supergravity action. As a counter-example I recall that the supergravity-scalar fields coupling requires introducing a real function of scalar fields, the precise form of which is not uniquely fixed by the supersymmetry invariance arguments (cf. e.g. [15]).

5. FINAL REMARKS

To summarize, we have compared two different approaches to Supergravity by Volkov-Soroka and by Ferrara-Freedman-Nieuwenhuizen-Deser-Zumino. I have emphasized the differences in these approaches, and have established the source of the differences. As I have mentioned at the beginning of the paper, I would not stick to any "preferred" point of view, suggesting the reader to make own conclusions on the main question of the paper. Let me finally note that the Goldstone-type variables ξ^a entering (14), which were appeared in [4, 5], are called now the Poincare coordinates, and the N=1 D=4 Supergravity formulation, which is invariant under the Poincare group translations, is presently known (notably due to the authors of [16]) as the Grignani-Nardelli-Stelle-West formulation [17, 16].

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НАСКОЛЬКО СУПЕРГРАВИТАЦИЯ СТАРА: ТРИДЦАТЬ ПЯТЬ ЛЕТ ИЛИ БОЛЬШЕ?

А.Ю. Нурмагамбетов

Целью данных заметок является сравнение двух различных подходов к теории гравитации с фермионным калибровочным полем, обладающей инвариантностью относительно специальных преобразований, получивших название суперсимметрии. Одним из этих подходов является подход Феррары, Фридмана, Ньювенхойзена, Дезера и Зумино 1976 года; другим является подход Волкова и Сороки, предложенный в 1974 году. Анализ основывается на стандартной реализации супергравитации как калибровочной теории для группы супер-Пуанкаре. Строгость рассмотрения, по сравнению с пионерскими работами по D=4 N=1 супергравитации, сознательно жертвуется в пользу максимального упрощения в представлении материала. Следствием этого является четкое обозначение различий между двумя сравниваемыми подходами. Заметки не содержат никаких категорических выводов; напротив, читателю предлагается получить собственный ответ на вопрос, фигурирующий в названии работы.

НАСКІЛЬКИ СУПЕРГРАВІТАЦІЯ СТАРА: ТРИДЦЯТЬ П'ЯТЬ РОКІВ ЧИ БІЛЬШЕ?

О.Ю. Нурмагамбетов

Метою цих нотаток є порівняння двох різних підходів до теорії гравітації з ферміонним калібровочним полем, що інваріантна щодо спеціальних перетворень, які одержали назву суперсиметрії. Одним з цих підходів є підхід Феррари, Фрідмана, Ньювенхойзена, Дезера і Зуміно 1976 року; іншим є підхід Волкова і Сороки, запропонований в 1974 році. Аналіз грунтується на стандартній реалізації супергравітації як калібровочної теорії для групи супер-Пуанкаре. Строгість розгляду, в порівнянні з піонерськими роботами з D=4 N=1 супергравітації, свідомо жертвується на користь максимального спрощення в поданні матеріалу. Наслідком цього є чітке висвітлення відмінностей між двома порівнюваними підходами. Нотатки не містять ніяких категоричних висновків, навпаки, читачеві пропонується отримати власну відповідь на питання, що фігурує в назві роботи.