ON LIMITS OF INDUCED RADIATION CONCEPT IN FELs

M.A. Gorbunov, A.N. Lebedev
P.N. Lebedev Physical Institute of RAS
E-mail: lebedev@sci.lebedev.ru; ujh@mail.ru

We discuss a restriction of the induced radiation concept in classical beam systems due to accompanying spontaneous radiation (radiation friction). For short wave-length FELs spontaneous radiation renders a noticeable influence on phasing of particles which is the base mechanism of induced radiation in classical systems. It leads to an essential restriction on the radiating system length and gain which cannot be compensated by an increase in the beam current.

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1. ON THE INDUCED RADIATION CONCEPT

The natural tendency to increase in frequency and spectral brightness of microwave electronic devices brought to the appearance of so-called free electron lasers (FELs). Nowadays they are claiming with confidence to VUF and X-ray domains where the potential of quantum lasers are quite ambiguous. The large spectral brightness requires, of course, the development of induced radiation which traditionally is treated as a quantum effect. At the same time the particle motion is purely classical, meaning the distance between energy levels (if any) is much lesser than the radiated photons energy. Under these conditions the induced radiation concept should be based on phasing of particles in the inducing radiation field rather than on the well known Einstein probabilities of radiation and absorption of separate quanta. This phasing should lead to coherency of particles radiation obeying classical electrodynamics laws.

The mechanism of particles phasing is similar to that of phase stability in accelerators. Particles with energy exceeding the resonant value provide radiation, while the slower particles absorb the energy. The essential role in this mechanism is played by the phase slipping of particles with respect to the wave.

The role of the active medium in short-wave FELs is played by an electron beam of high energy, the larger the smaller is the wave length. The classical phasing mechanism decreases at high energy because very relativistic particles hardly slip along the wave due to independency of their velocity of energy. Still, the existing concept of the induced phasing supposes that the effect can be reached if the interaction length is large enough. Actually, all the projects of X-ray and even γ frequency domains free electron lasers are based on this belief.

However, following this way one comes to an odd conclusion: any two particles of the same energy must radiate coherently independently of separation if the interaction length is large enough. The obvious absurdity of this situation forces to look for principal physical limitations of the induced radiation development in beam systems.

We deliberately ignore here so-called technical limitations such as random perturbation of motion, diffraction phenomena, finite beam emittance an so on. All of them in principle could be avoided improving beam and undulator qualities. More important are general physical limitations of the induced radiation mechanism described above and evaluations of their influence on future projects.

In our opinion there are at least the following limiting effects:

1. Quantum fluctuations of the radiation power influencing the wave-particle phasing.
2. Schottke noise in the electron beam of low density destroying the beam presentation as an active media.
3. Radiation friction leading to misphasing.
4. Temporal non-coherence.
5. Dynamical chaos and modes competition.

The first effect apparently is of importance at low field amplitudes when there are less than one photon per phase space cell and/or for very large energy of the emitted quantum. Anyway, it should be discussed within the frames of quantum electrodynamics.

The second one seems to be important for SASE regimes, i.e., for amplification of proper noises. At intuition level one can accept the criterion of coherent radiation as large enough “cooperative” number of particles, i.e. their number in the λ^3 volume (of course, in the beam frame). This would give the possibility to consider the beam as an active continuous medium.

2. RADIATION FRICTION

We consider here the third item only. The well known approximation of a self-consistent field ignores the proper field of an individual particle on the background of a far-distance radiation field of other particles; i.e. the radiation damping due to spontaneous radiation is neglected. The proper field is really very small at large distances but can be quite essential in the nearest neighborhood. Furthermore it acts on the particle continuously and does not depend on the phase of the particle in the self-consisted field. The same approximation one makes treating alternatively induced radiation as an instability of a space-charge proper wave.

As known [1,2], phasing of particles quasi-synchronous with the wave in the simplest case is governed by the equations:

$$\frac{d\Delta}{dz} = g\cos\varphi\,; \qquad \frac{d\varphi}{dz} = \alpha\Delta\,. \qquad (1)$$

They describe the energy transfer from the particle to wave and vice versa. Here Δ is a deviation of the dimensionless energy γ from the resonant value, g is the maximal energy which could be transferred on the unit length, φ is the particle phase remaining constant at the exact resonance. The coefficient of phase slippage will be taken in the form $\alpha \approx 2\pi/\lambda\gamma^3$ typical for a short wave undulator FEL.

To take radiation friction into account one may modify (1) adding a term describing spontaneous radiation. This term should be phase independent and hence is acting along all the interaction length, leading the particle out of synchronism

$$\frac{d\Delta}{dz} = g\cos\varphi - w; \quad \frac{d\varphi}{dz} = \alpha\Delta . \qquad (2)$$

This is true, of course, for any quasi-synchronous particle, but for simplicity we shall neglect *w(Δ)* dependency. It should be emphasized that spectral and angular characteristics of spontaneous radiation have nothing common with the inducing wave

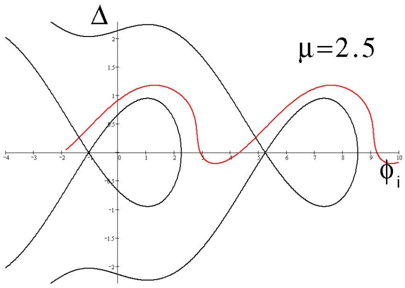

Fig.1. The phase portrait of the beam for w≠0

The value of *g* determines the system acceptance, i.e. the area of the phase plane where particles are captured by the wave and according to the phase stability principle neither get energy nor loose it in the average. The value of *w* defines the gap between these domains. Particles falling in the gap loose their energy mainly for spontaneous radiation.

It was proved (see, e.g. [1,2]) that for initially uniform phase distribution the maximal induced radiation is provided by particles with small initial energy detuning providing the phase shift about 2.6 at the total interaction length. Fig.1 shows that for small *g* and large *w* this shift can be reached for rather short time (if at all) because the particles fall through the gap mentioned above. Analogous situation takes place in cyclic accelerators where too fast increase in the magnetic field can not be supported by a fixed accelerating voltage and particles can not be captured. Moreover, for $g < w$ phase stability vanishes.

As a rule, parameters *g* and *w* are small enough to permit perturbation methods assuming *g*=const. The later is justified also by our aim to find limitation of induced radiation.

Expansion over power of *g* has a simple physical meaning. The zero approximation includes the spontaneous losses only. The first approximation vanishes in the average because it describes acceleration/deceleration of particles uniformly distributed over random phases. The next approximation gives induced radiation proportional to higher powers of *g*.

As an independent variable we use below the value of the kinematic phase shift $\mu(z) = \alpha\Delta_i z$, proportional to initial detuning Δ_i and distance *z*. Then, in the first approximation

$$\Delta = \Delta_i - 2\Delta_i W\mu; \quad \varphi = \varphi_i + \mu - W\mu^2; \quad W = \frac{w}{2\alpha\Delta_i^2} . \qquad (3)$$

The next approximation

$$\Delta = \Delta_i - 2\Delta_i\mu + \frac{g}{\alpha\Delta_i}\int_0^\mu \cos\left(\varphi_i + \mu' - W\mu'^2\right)d\mu' , \qquad (4)$$

gives induced phase shift

$$\varphi = \varphi_i + \mu - W\mu^2 + \frac{g}{\alpha\Delta_i^2}\left[\cos\varphi_i \cdot C - \sin\varphi_i \cdot S\right], \qquad (5)$$

where

$$\begin{matrix} C \\ S \end{matrix} = \int_0^\mu (\mu - \mu') \begin{matrix} \cos \\ \sin \end{matrix} \left(\mu' - W\mu'^2\right)d\mu' .$$

The expression (5) can be presented as

$$\varphi = \varphi_i + \mu - W\mu^2 - \frac{g\sqrt{C^2+S^2}}{\alpha\Delta_i^2}\sin\left(\varphi_i - \arctan\frac{C}{S}\right),$$

so that for an energy gradient one has

$$\frac{d\Delta}{dz} = -w + g\cos\left(\psi - \frac{g\sqrt{C^2+S^2}}{\alpha\Delta_i^2}\sin(\psi)\right)\cos\left(\mu - W\mu^2 + \arctan\frac{C}{S}\right)$$
$$- g\sin\left(\psi - \frac{g\sqrt{C^2+S^2}}{\alpha\Delta_i^2}\sin(\psi)\right)\sin\left(\mu - W\mu^2 + \arctan\frac{C}{S}\right),$$

where $\psi = \varphi_i - \arctan(C/S)$ as well as φ_i is an uniformly distributed random value.

After averaging over the interval (-π, π) the second (odd) term vanishes while the first one can be expressed via Bessel function [3]:

$$\frac{\overline{\Delta'} + w}{g} = \cos\left(\mu - W\mu^2 + \arctan\frac{C}{S}\right)J_1\left(\frac{g\sqrt{C^2+S^2}}{\alpha\Delta_i^2}\right). \qquad (6)$$

The corresponding results are presented in the Fig.2 where regions of induced radiation are light and those of absorption are dark. Note that the used method takes into account not only spontaneous radiation reaction but also a non-linear saturation of induced effects. For *w*=0 and small argument of Bessel function it gives the usual formula with $\mu_{opt} \approx 2.6$

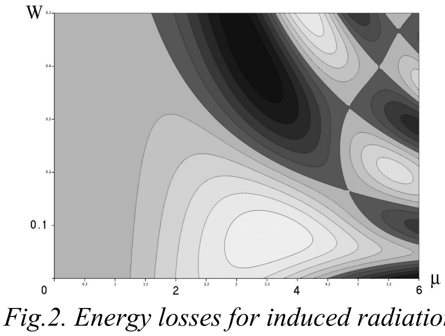

Fig.2. Energy losses for induced radiation

3. EVALUATIONS AND CONCLUSIONS

For moderate amplitude of transverse motion an electron in a plane undulator is equivalent from the standpoint of radiation to a non-relativistic oscillator moving as a whole along *z* with large velocity. The corresponding radiation losses per unit length for harmonic oscillations of period *l* are given by [4,1]:

$$w = \frac{2}{3}(2\pi)^2\frac{r_0}{l^2}\overline{\beta_\perp^2}\gamma^2; \quad r_0 = \frac{e^2}{mc^2} \cong 2{,}8\cdot 10^{-13}\text{cm}. \qquad (7)$$

For common FELs $\overline{\beta_\perp^2}\gamma^2 \cong 1$. So one gets the main parameter of spontaneous radiation

$$W = \frac{(2\pi)^2}{3l^2}\frac{r_0}{\alpha\Delta_i^2} = \frac{(2\pi)^3}{3l^2}\frac{r_0}{\lambda\mu^2}z^2. \quad (8)$$

The first part of this equality gives the idea of a minimal starting field capable for inducing effects. From one hand the Fig.2 says that $W_{max} \cong 0.2$. From the other the optimal detuning is approximately equal to the energy spread of the phase stability region, i.e. $\alpha\Delta_i^2 \cong g$. So one gets a value practically independent of energy

$$g > g_{min} \cong r_0(2\pi)^2/0{,}6l^2 . \quad (9)$$

Due to smallness of r_0 the obtained limitation hardly has a practical meaning but, at least, resolves the paradox formulated above for a beam of a very low density. (So it gives a universal limitation of the input signal). Physically, (9) means that the input signal should be sufficient for compensation of spontaneous radiation.

A natural idea comes then that even for small exceeding of (9) the induced radiation could be effectively developed by increase in the beam current and, thus, in the amplification coefficient. To clear the point one could compare the distance L_{lim} corresponding to the critical value of W with the radiation length L_r. It follows from (9) and [1] that

$$L_{\lim} = \left[\frac{3l^2\lambda\mu^2}{(2\pi)^3 r_0}n\right]^{1/2}; \; L_r \cong 2\left[\frac{mc^3\gamma^5\lambda S}{2\pi eI}\right]^{1/3}, \quad (10)$$

where I is the total beam current and S is its cross-section by order of magnitude equal to $\pi\lambda^2\gamma^2$. Note that the optimal value L_{lim} should be substituted in μ=2.6 as far as μ was calculated for this value. Furthermore, there is an additional factor n presenting a "cooperative number" of electrons which are close enough to each other to emit coherently even spontaneous radiation (the effect of this partially coherent spontaneous radiation was considered in [7]). Unfortunately, this number corresponding to a frontier between self-consisted and proper fields is rather difficult for evaluation as far as it depends strongly on possible density modulations within the beam. One can only state that it varies from unity in a much rarified beam (as above) up to the number of particles in a λ^3 cube (in the self frame) for very dense bunching. In the self frame the wavelength is $\lambda\gamma$ and density is γ times lesser than in the lab system. So

$$n = \lambda\gamma I/\pi ec . \quad (11)$$

As a result we may suggest the criteria of the applicability of existing induced radiation concept for small beam currents (less than one particle per λ^3 cube)

$$\frac{L_r}{L_{\lim}} \cong 4.4\left(\frac{I_0}{I}\right)^{1/3}\gamma^{4/3}\left(\frac{r_0}{\lambda}\right)^{1/2} < 1; \; I_0 = \frac{mc^3}{e} = 17kA \quad (12)$$

and for a large current and strongly modulated beam

$$\frac{L_r}{L_{\lim}} \cong 2.8\left(\frac{I}{I_0}\right)^{1/6}\gamma^{5/6} < 1. \quad (13)$$

Note that the second expression is almost independent of I. For a non-modulated dense beam one apparently has to take not a number of particles in the λ cube, but a fluctuating part of it only, i.e. $\sqrt{n}$ instead of n.

In spite of a rather low reliability of the evaluations above which are semi-intuitive, the problem of the cooperative number is definitely of importance. In particular, it is essential for strict determination of statistical properties of radiation in the SASE regime, for quantum recoil effects [6] and for general theory of radiation by the dense electron bunches.

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О ПРЕДЕЛАХ ПРИМЕНИМОСТИ КОНЦЕПЦИИ ИНДУЦИРОВАННОГО ИЗЛУЧЕНИЯ В ЛСЭ

М.А. Горбунов А.Н. Лебедев

Рассмотрено ограничение концепции индуцированного излучения в классических пучковых системах, связанное с учётом сопутствующего спонтанного излучения (радиационного трения). Показано, что при укорочении длины волны в ЛСЭ спонтанное излучение оказывает существенное влияние на фазировку частиц. Это приводит к практически важному ограничению на длину излучающей системы, которое не может быть скомпенсировано увеличением тока пучка.

ПРО МЕЖІ ЗАСТОСОВНОСТІ КОНЦЕПЦІЇ ІНДУКОВАНОГО ВИПРОМІНЮВАННЯ В ЛВЕ

М.А. Горбунов А.Н. Лебєдєв

Розглянуто обмеження концепції індукованого випромінювання в класичних пучкових системах, зв'язане з обліком супутнього спонтанного випромінювання (радіаційного тертя). Показано, що при укороченні довжини хвилі в ЛВЕ спонтанне випромінювання впливає на фазування частинок. Це приводить до практично важливого обмеження на довжину випромінюючої системи, що не може бути скомпенсовано збільшенням струму пучка.