SOLITARY MODEL OF THE CHARGE PARTICLE TRANSPORT IN COLLISIONLESS PLASMA

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The one-dimensional MHD solitary model of charged particle transport in plasma is developed. It is shown that self-consistent electric field of ion-acoustic solitons can displace charged particles in space, which can be a reason of local electric current generation. The displacement amount is order of a few Debye lengths. It is shown that the current associated with soliton cascade has pulsating nature with DC component. Methods of built theory verification in dusty plasma are proposed.

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1. INTRODUCTION

There have been numerous observations of small-scale solitary structures in space plasma by Cluster [1], Polar [2], Fast [3] and other spacecrafts. Often such structures are associated with ion and electron beams. Some works reported about two magnetosphere regions - inward current region and outward current region with high population by solitary potential structures [2]. At the same time, in [1] it was pointed on insufficient temporal instrument resolution for experimental study of charge particle beams, associated with solitons. The aim of this study is to show, that solitons can be current carriers themselves.

2. THEORETICAL MODEL

Let's consider the 1D MHD model of nonmagnetized collisionless plasma with two populations of charged particles: hot electrons and cold ions [4].

In absence of dissipation cold ions can be described by the MHD equation system

$$\frac{\partial \upsilon_i}{\partial t} + \upsilon_i \frac{\partial \upsilon_i}{\partial x} = -\frac{e_i}{m_i}\frac{\partial \varphi}{\partial x} \qquad (1)$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i \upsilon_i}{\partial x} = 0 \qquad (2)$$

where m_i, n_i, υ_i, e_i, - mass, density, velocity, and charge of the ions. Electric field is supposed to be electrostatic: $\varepsilon = -\partial\varphi/\partial x$.

Transforming to the stationary wave frame so that all variables depend on the single independent variable $s=x-V\cdot t$, where V – the soliton velocity, and taking into account that $\partial/\partial x=\partial/\partial s$, $\partial/\partial t=-V\times\partial/\partial s$, we get from (2)

$$\frac{\partial}{\partial s} n_i(\upsilon_i - V) = 0 \quad \text{or} \quad n_i(\upsilon_i - V) = n_{0i}V\,, \qquad (3)$$

where n_{0i} is an initial ion density. Now we can transform (1) as follows:

$$\frac{\partial}{\partial s}\left(\frac{n_{0i}^2 V^2}{2n_i^2} + \frac{e_i\varphi}{m_i} - \frac{V^2}{2}\right) = 0 \qquad (4)$$

or

$$\frac{V^2}{2N_i^2} + \frac{e_i\varphi}{m_i} - \frac{V^2}{2} = 0\,, \qquad (5)$$

where $N_i=n_i/n_{0i}$ is normalized ion density.

Let's introduce dimensionless potential - $\Phi = e_i\varphi / T_e$ and Mach number – $M_i=V/C_s$, where $C_s=(T_e/m_i)^{1/2}$ – ion-acoustic (IA) velocity [4], T_e – electron temperature. Now we can rewrite (5) as

$$\frac{M_i^2}{2}\left(\frac{1}{N_i^2} - 1\right) + \Phi = 0\,\cdot \qquad (6)$$

Equation (6) has next solution:

$$N_i = \sqrt{\frac{(M_i)^2}{(M_i)^2 - 2\cdot\Phi}}\,, \qquad (7)$$

which is valid if $$\Phi < \frac{(M_i)^2}{2}. \qquad (8)$$

C_s is far less than thermal electron velocity. Therefore, it is possible to consider electron density to be approximated by Boltzmann distributions

$$N_e = \frac{n_e}{n_{0e}} = \exp(\Phi)\,. \qquad (9)$$

To close the system we use Poisson's equation:

$$\frac{d^2\varphi}{ds^2} = \frac{n_0 e}{\varepsilon_0}(N_{e1} - N_{i1})\,;\; n_0 = n_{0e} = n_{0i}\,, \qquad (10)$$

or in dimensionless form

$$\frac{d^2\Phi}{dS^2} = N_e - N_i\,, \qquad (11)$$

where $S=s/\lambda_D$, $\lambda_D=(\varepsilon_0 T_e/e^2 n_0)^{1/2}$ – Debye length.

Dependence $\Phi(S)$, as solution of equation (11) can be fined either numerically or in small amplitude approach. In the last case we can write

$$\Phi(S) = \Phi_0 \cdot Sech^2\left(\sqrt{\frac{A_2}{2}}\cdot S\right)\cdot \qquad (12)$$

Here $\Phi_0=-A_2/A_3$ is soliton amplitude, with $A_2=1/2(1-1/M_i^2)$, $A_3=1/2(1/3-1/M_i^3)$ [5].

Dependence of IA soliton amplitude on M_i is represented in Fig. 1.

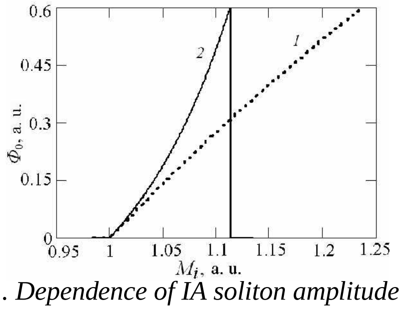

Fig. 1. Dependence of IA soliton amplitude on M_i; 1 – numerical result, 2 – small amplitude approach

Let's introduce soliton width Δ as width of area with $|\Phi|>|\Phi_0|/e$. We can find $\Delta/2$ by solving (12), if assume that the left part is equal Φ_0/e

$$\Delta = \sqrt{2/A_2}\cdot \operatorname{arctanh}\left(\sqrt{1-1/e}\right). \qquad (13)$$

For numerical calculation can be used the expression

$$\Delta = 2\cdot\int_{\Phi_0/e}^{\Phi_0} \frac{2}{\sqrt{-2\cdot U(\Phi)}}\,d\Phi\,, \qquad (14)$$

here $U(\Phi) = -0.5(d\Phi/dS)^2$ - Sagdeev's pseudopotential ($U(\Phi)$ can be obtained analytically according to [5]). Dependence of width on M_i is represented in Fig. 2. It is evident, that the amplitude increase leads to a decrease of width Δ.

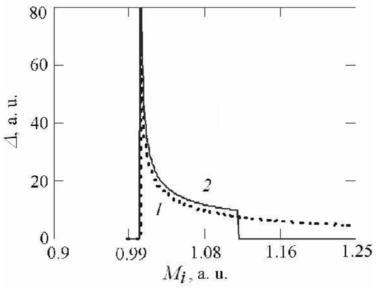

Fig. 2. Dependence of soliton width Δ on M_i;
1 – numerical result, 2 – small amplitude approach

3. MAIN RESULTS

Thermal velocity of ions is far less than IA velocity. Under effect of electric field inside of soliton the ions will get an ordered velocity. Using (2) and velocity summation rule $\upsilon'_i = \upsilon_i - V$ this velocity can be determined as

$$\upsilon'_i = -V\frac{1}{N_i}\,. \tag{15}$$

Here υ'_i is velocity of selected substance sample in moving co-ordinates. Using (2), velocity in fixed co-ordinates can be presented as follows

$$\upsilon_i = V\left(1 - \frac{1}{N_i}\right). \tag{16}$$

Expressions for ion current density are written as

$$j_i = e_i n_i V\left(1 - \frac{1}{N_i}\right), \tag{17a}$$

$$j'_i = -e_i n_i V\frac{1}{N_i} = -e_i n_{0i} V\ , \tag{17b}$$

where j_i, j'_i – current density relative to fixed and moving co-ordinates correspondingly.

Normalizing (17) to $|e_i| n_0 C_s$, we obtain expressions for normalized current densities as follows

$$J_i = M_i (N_i - 1), \tag{18a}$$

$$J'_i = -M_i\,. \tag{18b}$$

Fig. 3 shows the dependences of the normalized electric field potential Φ, dimensionless electric field $E=-d\Phi/dS$, charged particle densities N_i, N_e on S and dependence of velocity υ_i normalized to C_s on x/λ_D for IA soliton at M_i=1.05.

Solitons have positive polarity according to considered model (Fig. 3a). As seen in Fig. 3b, the charged particle densities (both electrons and ions) are higher in soliton field as compared with ones in non-perturbed state. According to (16) the normalized ion velocity υ_i/C_s will be positive and its direction will be coincident with one of soliton moving (Fig. 3c). It follows that the IA soliton field of positive polarity produces an ion transfer in the soliton moving direction only. As result is the ion current pulse J_i, with direction coinciding with one of υ_i (Fig. 4). Thus, IA solitons can be recognized as unidirectional pulse of ion current (Fig. 4). Electron current can not be found in such approximation.

Let's enter variables $X=x/\lambda_D$, $\tau=t\cdot\omega_i$ where ω_i – ion plasma frequency. Knowing the dependence $\upsilon_i(S)$, where $S=(x-V\cdot t)/\lambda_D=X-M_i\cdot\tau$, the amount of ion transfer by soliton field can be written as follows

$$X - X_0 = \int \upsilon_i(X,\tau)d\tau\ , \tag{19}$$

where X_0 – initial position of physically small volume occupied by ions. Solution of (19) is presented by curve in Fig. 5. As seen, ions are displaced on a few λ_D from its initial positions in the direction of soliton moving. This result is agreed with dependence $\upsilon_i(x)$ (Fig. 3c). In fact, υ_i is positive only. This corresponds to the ion displacement in the positive direction. Let's note that the ion velocity before and after interaction is equal zero (Fig. 3c.).

Obtained result can be verified in kinetics approximation at solution of motion equation of selected ion in soliton electric field $m_i d^2x/dt^2=e_i\varepsilon(x,t)$, where $\varepsilon(x,t)=-d\varphi/dx$ – soliton electric field.

Motion equation can be written in dimensionless form:

$$\frac{d^2X}{d\tau^2} = -\frac{d\Phi(X,\tau)}{dX}. \tag{20}$$

$\Phi(X,\tau)$ can be obtained at substitution of $S=X-M_i\tau$ in (12). The solution (20) at $X=0|_{\tau=0}$ is the same one represented in Fig. 5.

4. DISCUSSIONS

Analysis of developed model allows to conclude that the IA solitons produce the spatial transfer of ions. At that appeared currents are pulse-like (Fig. 4).

Soliton-like structures in space plasma are often registered in form of long cascades which have tens of the solitons moving in series [1, 2]. An ion current should have a direct (DC) component in this case. Fig. 6 represents modeling results of a cascade of IA solitons corresponding to numerical solution (11).

Average current value J_i for described case is $\overline{J_i} = \frac{1}{\tau_{max}}\cdot\int_0^{\tau_{max}} J_i dt = 0.026$, where τ_{max}=710. For comparison, according to normalization assumed in (18a) the same current corresponds to ion beam with concentration n_{beam}=0.026$\cdot n_{0i}$, moving with near acoustic velocity $\upsilon_{beam}\approx C_s$.

Solitons can be amplified at presence of charged particle beams [6]. So, developed model creates new mechanism of plasma current transformation, namely, direct current of beam can transform into the pulse current of solitons excited by initial beams.

Time resolution of experimental apparatus for registration of mentioned process should be order of magnitude of ω_i^{-1}, since it can be shown that the ion current pulse duration is tens of ω_i^{-1}. It is possible to test experimentally the resume of considered model in dusty plasma. Dusty-acoustic soliton properties arising in the three-component dusty plasma are analyzed in [7]. Plasma frequency and acoustic velocity, corresponding to a heavy dust component are less on many orders of magnitude than mentioned values for ion plasma component. At the same time their spatial scales are commensurable quantities. Therefore, experimental registration of temporal parameters of dust acoustic solitons can be easy realized at lab conditions.

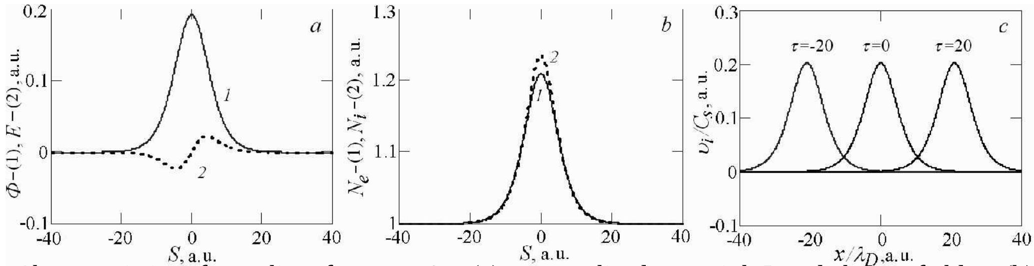

Fig. 3. Characteristics of IA soliton for M_i=1.05: (a) – normalized potential Φ and electric field E; (b) –ion and electron densities;(c) – normalized ion velocity υ_i/C_s for three time points $\tau=t\cdot\omega_i$ (ω_i – ion plasma frequency)

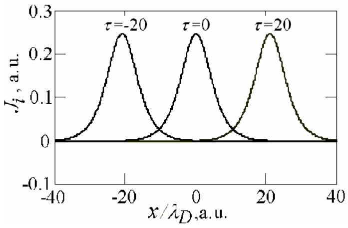

Fig. 4. Dependence of normalized ion current on x/λ_D for three time points $\tau=t\cdot\omega_i$

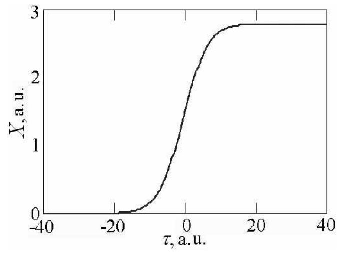

Fig. 5. Solution of (19) for M_i=1.05, at $X=0|_{\tau=0}$

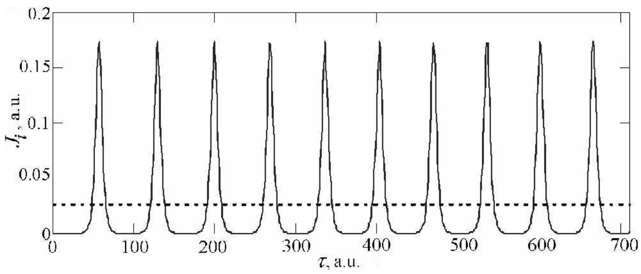

Fig. 6. Dependence of current J_i of IA soliton sequence on τ according to (18a). Dashed curve corresponds to average value of normalized current $\overline{J_i}$

In conclusion, we underline the fundamental amount of this work – new property of solitons is found concluding in transfer of charged particles. It can be important for understanding of nonlinear plasma dynamics.

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СОЛИТОННАЯ МОДЕЛЬ ПЕРЕНОСА ЗАРЯЖЕННЫХ ЧАСТИЦ В БЕССТОЛКНОВИТЕЛЬНОЙ ПЛАЗМЕ

Л.В. Симончик, Ф.М. Трухачев

Построена одномерная солитонная МГД-модель переноса заряженных частиц и генерации токов в плазме. Показано, что в самосогласованном электрическом поле ионно-звуковых солитонов положительной полярности, ионы перемещаются на расстояние порядка нескольких радиусов Дебая в направлении движения солитона. Промоделирован практически важный случай – движение каскада солитонов. Анализ результатов свидетельствует о том, что ток при этом имеет импульсный характер со значительной постоянной составляющей. Предложен способ экспериментальной проверки построенной модели в пылевой плазме.

СОЛІТОННА МОДЕЛЬ ПЕРЕНОСУ ЗАРЯДЖЕНИХ ЧАСТОК У БЕЗЗІШТОВХУВАЛЬНІЙ ПЛАЗМІ

Л.В. Симончик, Ф.М. Трухачов

Побудована одновимірна солітонна МГД-модель переносу заряджених часток і генерації струмів у плазмі. Показано, що в самоузгодженому електричному полі іонно-звукових солітонів позитивної полярності, іони переміщаються на відстань порядку декількох радіусів Дебая в напрямку руху солітона. Промодельований практично важливий випадок – рух каскаду солітонов. Аналіз результатів свідчить про те, що струм при цьому має імпульсний характер зі значною постійною складовою. Запропоновано спосіб експериментальної перевірки побудованої моделі в пиловій плазмі.