COMPUTATION AND ANALYSIS OF THE POLARIZATION DEGREE FOR BREMSSTRAHLUNG AT PERIPHERAL SCATTERING

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An economic technique for calculation of polarized bremsstrahlung process is proposed, assuming typical atomic momentum transfer $q \ll m$. The adopted approach is based on the natural reduction of the matrix element to the form $V^{\alpha}\gamma^{\alpha} + A_5\gamma^5$. Polarization distribution in the fully differential cross-section is analyzed. It is found that at a given momentum transfer to the atom polarization in the plane of small radiation angles is oriented along circles passing through two common points. It is shown that with angular selection of radiated photons carried out, even with momentum transfers to the atom being integrated over, for particular radiation angles polarization may stay as high as 100%. Without angular selection of photons, only by control of the recoil, it is impossible to gain radiation polarization above 50%.

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1. INTRODUCTION

Radiation from ultra-relativistic electrons is naturally collimated along the forward direction, with the opening angle being inversely proportional to the electron energy, but up to energies ~ 10 GeV, corresponding to typical radiation angles $\sim 10^{-4}$ rad, angular distribution of radiation can be resolved. If that is done in practice, photon polarization effects necessarily come into play. In particular, the connection between the polarization degree and the angular distribution is important for purposes of preparation of polarized photon beams. The issue of polarization account may arise also at investigation of spatial evolution of electromagnetic showers. Calculations of differential cross-section for the bremsstrahlung process were conducted in various frameworks [1], [2], but acquired reputation of rather cumbersome a subject. Presentation of the final results in the literature does not help gaining detailed intuition.

The conditions of radiation from ultra-relativistic electrons in matter are such that small momentum transfers to atoms dominate ($q \sim r_B^{-1} \ll m$). Under such conditions the expression for bremsstrahlung cross-section, called dipole approximation, is similar to Compton scattering cross-section from the standpoint of the equivalent photon method. As a matter of fact, the latter method is usually used to derive characteristics of bremsstrahlung, averaged over directions of emission and over polarizations of final, and also (through azimuthal integration in momentum transfers) over polarizations of initial quanta. Nevertheless, it can be extended to the polarized case,

provided one traces correspondence of polarizations (that is traditionally achieved via a transition between reference frames [1]).

From the technical side, the case of Compton scattering has the advantage that photon polarization vectors in it might be chosen orthogonal simultaneously to two momenta in the problem (out of three momenta not bound by 4-momentum conservation). That is expected to substantially simplify the procedure of calculation with the account of polarizations and, in the conventional technology of cross-section calculation via computation of a spur from the squared matrix amplitude [3], this is indeed the case. Despite the simplifications gained, calculations for this process still are of formidable complexity. This is in mark contrast with the concise final result, and should be blamed on nothing but poor efficiency of the method of straightforward spurring.

As was discussed in [4], a more adequate method of calculation of QED-processes, also offering access to fermion polarization observables, is the evaluation of spin amplitudes in a matrix basis chosen by some convenience reasons.

In the present paper, firstly, it is desired to apply the spin amplitude approach to the specific case of Compton scattering. It appears that in this particular case the most convenient spin matrix choice is different from what may be appropriate in other cases. With the choice adopted in the present article, the whole procedure of differential cross-section calculation with the account of initial and final photon polarizations becomes fairly elementary. Issuing from the representation for the differential cross-section for

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Compton scattering in a gauge- and Lorenz-invariant form, it is straightforward further on to pass to the differential cross-section of bremsstrahlung in laboratory frame, corresponding to conditions of peripheral scattering, i.e. dipole approximation. This provides an alternative to explicit implementation of the equivalent photon method.

The second objective of the present work is to discuss the features of polarization distribution as a function of the radiation angle at fixed transverse direction of momentum transfer to the atom. The characteristic feature is that the polarization is aligned along circles, passing through two knot points in the space of radiation angles. One of the circles has its centre coinciding with the origin - and that has an important consequence: upon averaging over momentum transfers to the atom at radiation direction fixed, only contributions with identical direction of polarization are summed up. And since at small radiation frequencies the polarization in the doubly differential cross-section is close to 100%, after the averaging it remains near so.

2. CALCULATION OF AMPLITUDES AND CROSS-SECTION FOR COMPTON SCATTERING

We are considering the process of electron scattering on an atom[1], accompanied by emission of a single photon. Denoting by p and p' 4-momenta of the initial and the final electrons, q and q' - the momentum transfer to the atom and the momentum of the emitted photon, the 4-momentum conservation and mass

shell conditions for them read as:

$$p + q = p' + q', \quad p^2 = p'^2 = m^2, \quad q'^2 = 0.$$

For q^2 there is no strict condition, but

$$q^2 \sim r_B^{-2} \sim e^4 m^2 \ll m^2, \tag{1}$$

r_B being Bohr radius. This estimate relates mainly to q components orthogonal to p . The component q_z parallel to p is small, as long as typical denominators emerging in Feynman diagrams are of order $pq \simeq Eq_z \sim m^2$, and E is large. Then $q_z r_B \ll 1$, owing to which condition the matrix element for the whole process factorizes into Born-level radiation matrix element $M_{rad}(q_\perp, q')$ and the exact elastic scattering amplitude $A_{scat}(q_\perp)$:

$$T_{fi} = A_{scat}(q_\perp) \sqrt{4\pi} e M_{rad}(q_\perp, q'),$$
$$d\sigma_{scat} = |A_{scat}|^2 \frac{d^2 q_\perp}{(2\pi)^2},$$
$$M_{rad} = \bar{u}_p \left(\frac{\hat{e}'^*(\hat{p}+\hat{q}+m)\hat{e}}{2pq} - \frac{\hat{e}(\hat{p}-\hat{q}'+m)\hat{e}'^*}{2pq'} \right) u_p,$$
$$d\sigma_{rad} = \frac{1}{2E} |T_{fi}|^2 \frac{d^2 q_\perp}{(2\pi)^2 2E'} \frac{d^3 q'}{(2\pi)^3 2q'_0}, \tag{2}$$

(the elastic scattering amplitude through small angles can be regarded as independent from the electron polarization). We are interested in $M_{rad}(q_\perp, q')$ modulus squared and averaged over initial electron and summed over final electron polarizations. By the straightforward Feynman's method, this quantity is to be computed as

$$\left\langle |M_{rad}|^2 \right\rangle = \frac{1}{2} Sp \, (\hat{p}'+m) \left(\frac{\hat{e}'^*(\hat{p}+\hat{q}+m)\hat{e}}{2pq} - \frac{\hat{e}(\hat{p}-\hat{q}'+m)\hat{e}'^*}{2pq'} \right) (\hat{p}+m) \left(\frac{\hat{e}^*(\hat{p}+\hat{q}+m)\hat{e}'}{2pq} - \frac{\hat{e}'(\hat{p}-\hat{q}'+m)\hat{e}^*}{2pq'} \right). \tag{3}$$

However, at that one needs to calculate a spur from a polynomial of 8th degree in γ-matrices. Typically, even relying on properties of photon crossing symmetry and advantages of the gauge choice (see (4) below), that entails calculations along 2-3 pages (cf. [3]). A more efficient approach would be prior reduction of the spin matrix to some "minimal" form, embarking on the condition of initial and final electron bispinors belonging to the mass shell.

2.1. Deduction of basic amplitudes

As usual, the computations of Compton scattering are strongly facilitated in the gauge

$$ep = eq = 0 = e'p = e'q'. \tag{4}$$

To start with, commute in the matrix element (2) $\hat{p}$ through $\hat{e}$, $\hat{e}'$ to a position neighboring to u_p:

$$M_{rad} = \bar{u}_{p'} \left(\frac{\hat{e}'^*(\hat{p}+\hat{q}+m)\hat{e}}{2pq} - \frac{\hat{e}(\hat{p}-\hat{q}'+m)\hat{e}'^*}{2pq'} \right) u_p$$
$$\cong \bar{u}_{p'} \left(\frac{\hat{e}'^*\hat{q}\hat{e}}{2pq} + \frac{\hat{e}\hat{q}'\hat{e}'^*}{2pq'} \right) u_p. \tag{5}$$

Feynman in [3] started squaring from this modified representation, but it is still 8-th order in γ-matrices, and we shall proceed a little further. With the application of the standard formula

$$\gamma^\alpha \gamma^\beta \gamma^\gamma = g^{\alpha\beta}\gamma^\gamma - g^{\alpha\gamma}\gamma^\beta + g^{\beta\gamma}\gamma^\alpha + i\varepsilon^{\nu\alpha\beta\gamma}\gamma^\nu\gamma^5,$$

(5) naturally reduces to a basic-matrix form:

$$M_{rad} = \bar{u}_{p'} \left(V^\alpha \gamma^\alpha + A_5 \gamma^5 \right) u_p. \tag{6}$$

The coefficients in the decomposition are

$$V^\alpha = \frac{e^\alpha q e'^* - q^\alpha e e'^*}{2pq} + \frac{e'^{\alpha*} q' e - q'^\alpha e e'^*}{2pq'},$$
$$A_5 = -i\varepsilon^{\mu\alpha\beta\gamma} \frac{p^\mu}{m} e^\alpha e'^{\beta*} G^\gamma,$$
$$\left(G^\gamma = \frac{q^\gamma}{2pq} - \frac{q'^\gamma}{2pq'} \right)$$

and it was utilized that vector G, as well as e, e', is orthogonal to p,

$$Gp = 0,$$

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[1] A composite and lower-dimensional scattering system will be addressed in Sec. 3.1.

so,

$$\varepsilon^{\nu\alpha\beta\gamma}e^{\alpha}e'^{\beta*}G^{\gamma} = \frac{p^{\nu}}{m}\cdot\varepsilon^{\mu\alpha\beta\gamma}\frac{p^{\mu}}{m}e^{\alpha}e'^{\beta*}G^{\gamma},$$

whereas action of $p^{\nu}\gamma^{\nu}\gamma^5/m$ on u_p gives $-\gamma^5$.

One can still add to the vector V^{α} an arbitrary vector, proportional to $(p-p')^{\alpha}$. It is advantageous to tune it so that V be orthogonal to momentum p:

$$\begin{aligned}
V^{\alpha} \to V_p^{\alpha} &= \frac{e^{\alpha}qe'^* - q^{\alpha}ee'^*}{2pq} + (p-p')^{\alpha}\frac{ee'^*}{(p-p')^2} + \frac{e'^{\alpha*}q'e - q'^{\alpha}ee'^*}{2pq'} + (p-p')^{\alpha}\frac{ee'^*}{(p-p')^2}\\
&= e^{\alpha}e'^*G - e'^{\alpha*}eG - G^{\alpha}ee'^*\frac{pq+pq'}{m^2-pp'}.
\end{aligned}$$

Now V_p^{α} and A_5 together have 4 independent components, as it should be for parametrization of a matrix describing transition between two spin-1/2 on-shell states.

2.2. Computation of the differential cross-section, averaged over fermion polarizations

Substituting (6) to (3), the spur is calculated easily:

$$\left\langle |M_{rad}|^2\right\rangle = \frac{1}{2}Sp\,(\widehat{p}' + m)\left(V_p^{\alpha}\gamma^{\alpha} + A_5\gamma^5\right)(\widehat{p} + m)\left(V_p^{\alpha*}\gamma^{\alpha} + A_5^*\gamma^5\right) = 2(m^2 - pp')\left\{|V_p|^2 - |A_5|^2\right\}.$$

The entries thereat are evaluated to be

$$|V_p|^2 = |e^{\alpha}e'^*G - e'^{\alpha*}eG|^2 + G^2\,|ee'^*|^2\left(\frac{pq+pq'}{m^2-pp'}\right)^2,$$

$$\begin{aligned}
-|A_5|^2 = \begin{vmatrix} |e|^2 & ee' & eG \\ (ee')^* & |e'|^2 & e'^*G \\ e^*G & e'G & G^2 \end{vmatrix} &= |e|^2\,|e'|^2\,G^2 + 2\mathcal{R}e\,ee'\cdot e'^*G\cdot e^*G - |e|^2\,|e'G|^2 - |e'|^2\,|eG|^2 - G^2\,|ee'|^2\\
&= -|e^{\alpha}e'^*G - e'^{\alpha*}eG|^2 + |e|^2\,|e'|^2\,G^2 - G^2\,|ee'|^2.
\end{aligned}$$

In sum, after cancellation of terms $\pm|e^{\alpha}e'^*G - e'^{\alpha*}eG|^2$,

$$|V_p|^2 - |A_5|^2 = |e|^2\,|e'|^2\,G^2 - G^2\,|ee'|^2 + G^2\,|ee'^*|^2\left(\frac{pq+pq'}{m^2-pp'}\right)^2, \qquad G^2 = \frac{m^2-pp'}{2pq\cdot pq'},$$

$$\left\langle |M_{rad}|^2\right\rangle = \frac{1}{pq\cdot pq'}\left\{\left(|e_p|^2\,|e'_p|^2 - |e_pe'_p|^2\right)(pq-pq')^2 + |e_pe'^*_p|^2\,(pq+pq')^2\right\}. \tag{7}$$

(In the final formula an explicit subscript p at polarization vectors is introduced emphasizing the employed gauge). In what follows, we will be mainly interested in the case of linearly polarized initial photons. Then, the final photon polarization is also linear. For those conditions one can set $|e_pe'^*_p| = |e_pe'_p| = (e_pe'_p)$ and add in (7) the two like terms:

$$\left\langle |M_{rad}|^2\right\rangle = e_p^2e'^2_p\frac{(pq-pq')^2}{pq\cdot pq'} + 4(e_pe'_p)^2. \tag{8}$$

This is the renowned Klein-Nishina's formula for linearly polarized initial and final photons [3], [5].

To apply formula (8) to bremsstrahlung in laboratory frame, where the scatterer atom is at rest, it should first be rendered a gauge-invariant appearance. To this end, substitute for e_p, e'_p expressions $e_p = e - q\frac{ep}{pq}$, $e'_p = e' - q'\frac{e'p}{pq'}$, where e, e' are polarization vectors in an arbitrary gauge:

$$\left\langle |M_{rad}|^2\right\rangle = \left(e - q\frac{ep}{pq}\right)^2\left(e' - q'\frac{e'p}{pq'}\right)^2\frac{(pq-pq')^2}{pq\cdot pq'} + 4\left\{\left(e - q\frac{ep}{pq}\right)\left(e' - q'\frac{e'p}{pq'}\right)\right\}^2. \tag{9}$$

Thereupon, this formula can be applied in the laboratory frame.

3. APPLICATION TO THE BREMSSTRAHLUNG IN LABORATORY FRAME

For bremsstrahlung in the laboratory frame $e = (1,\mathbf{0})$, $e' = (0,\mathbf{e}')$, $q = (0,-\mathbf{q})$, $q' = (\omega,\mathbf{k})$.

At that, combinations e_p, e'_p in components equal

$$e - q\frac{ep}{pq} \simeq \left(1, 1, \frac{\mathbf{q}_{\perp}}{q_z}\right), \tag{10}$$

$$e' - q'\frac{e'p}{pq'} \simeq \left(e_z'\omega\frac{E}{E'q_z}, e_z'\left(1+\omega\frac{E}{E'q_z}\right), \mathbf{e}_\perp' + \mathbf{k}_\perp\frac{\mathbf{e'p}}{\mathbf{E'q_z}}\right). \tag{11}$$

In terms of orthogonal components, the combination $\mathbf{e'p}$ entering (11) equals $\mathbf{e'p} \simeq \frac{E}{\omega}(\mathbf{e'k} - \mathbf{e}_\perp'\mathbf{k}_\perp) = -\frac{E}{\omega}\mathbf{e}_\perp'\mathbf{k}_\perp$.

Apparently, in scalar products present in (9), the temporal and the longitudinal spatial components of vectors (10-11) do not essentially contribute, except in $e_p'^2$, which is easier calculated in the Lorenz-invariant fashion, with the use of $e'q' = 0$, $q'^2 = 0$: $e_p'^2 = e'^2 = -1$. So,

$$e_p^2 \simeq -\frac{\mathbf{q}_\perp^2}{q_z^2},$$
$$e_p'^2 = -1,$$
$$e_p e_p' \simeq \frac{|\mathbf{q}_\perp|}{q_z}\left(-\mathbf{n_{q_\perp}} + \frac{(\mathbf{k}_\perp\mathbf{n_{q_\perp}})}{E'q_z}\frac{E}{\omega}\mathbf{k}_\perp\right)\mathbf{e}',$$

with
$$\mathbf{n_{q_\perp}} = \frac{\mathbf{q}_\perp}{|\mathbf{q}_\perp|}.$$

Next, the kinematical combinations required are

$$\frac{(pq - pq')^2}{pq \cdot pq'} \simeq \frac{(qq')^2}{pq \cdot pq'} \simeq \frac{(\omega q_z')^2}{Eq_z \cdot E'q_z} = \frac{\omega^2}{EE'},$$

$$E'q_z \simeq p'q \simeq pq' = E\omega - p_z k_z$$
$$\simeq E\omega - \left(E - \frac{m^2}{2E}\right)\left(\omega - \frac{\mathbf{k}_\perp^2}{2\omega}\right) \simeq \frac{\omega}{2E}m^2 + \frac{E}{2\omega}\mathbf{k}_\perp^2,$$

or, introducing the radiation angle $\theta_\mathbf{k} = \mathbf{k}_\perp/\omega$ and ratios $\gamma = E/m \gg 1$, $x_\omega = \omega/E$, $0 \le x_\omega \le 1$,

$$q_z = \frac{mx_\omega}{2(1-x_\omega)}\left(\frac{1}{\gamma} + \gamma\theta_\mathbf{k}^2\right).$$

Substituting all the ingredients into (9), one arrives at the expression

$$\left\langle|M_{rad}|^2\right\rangle = 4\mathbf{q}_\perp^2\frac{(1-x_\omega)}{m^2\left(\frac{1}{\gamma} + \gamma\theta_\mathbf{k}^2\right)^2}\left\{1 + \frac{4(1-x_\omega)}{x_\omega^2}|\nu\mathbf{e}'|^2\right\}. \tag{12}$$

Here ν is a vector

$$\nu = -\mathbf{n_{q_\perp}} + \frac{2}{\gamma^{-2} + \theta_\mathbf{k}^2}(\theta_\mathbf{k}\mathbf{n_{q_\perp}})\theta_\mathbf{k},$$

along which the radiation polarization orients itself[2]. The absolute value of the polarization amounts

$$P = \frac{\nu^2}{\frac{x_\omega^2}{2(1-x_\omega)} + \nu^2},$$

with

$$\nu^2 = 1 - \frac{4\gamma^{-2}}{(\gamma^{-2} + \theta_\mathbf{k}^2)^2}(\theta_\mathbf{k}\mathbf{n_{q_\perp}})^2$$
$$\equiv \frac{(\gamma\theta_\mathbf{k} + \mathbf{n_{q_\perp}})^2(\gamma\theta_\mathbf{k} - \mathbf{n_{q_\perp}})^2}{(1 + \gamma^2\theta_\mathbf{k}^2)^2}.$$

It is easy to show by solving the differential equation $d\theta_y/d\theta_x = \nu_y(\theta_x, \theta_y)/\nu_x(\theta_x, \theta_y)$, that the curves, in every point tangential to the direction of vector ν, are circles passing through two specific points: $\theta_\mathbf{k} = \pm\mathbf{n_{q_\perp}}/\gamma$ (see Fig. 1). In those two points the polarization turns to zero. In vicinities of those points $\nu^2 \simeq (\gamma\theta_\mathbf{k} \mp \mathbf{n_{q_\perp}})^2$. At distances from them much greater then $\frac{x_\omega}{2\gamma\sqrt{1-x_\omega}}$ polarization is close to 100%.

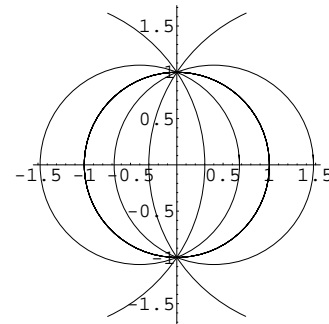**Fig.1.** *Curves in $\gamma\theta_\mathbf{k}$ plane, directing polarization of radiation. All displayed curves are circles. $\mathbf{n_{q_\perp}}$ is a unit vector along the vertical axis*

The doubly differential cross-section in itself is usually not measured in experiment, as long as recoiling atoms are not detected. So, the picture described above serves mainly for intuition purposes. Below we shall discuss two most important cases of A_{scat} dependence on $\mathbf{n_{q_\perp}}$. In both those cases there is a factorization of azimuthal and radial dependences $A_{scat}(\mathbf{q}_\perp) = \Phi(\mathbf{n_{q_\perp}})A(|\mathbf{q}_\perp|)$, the latter being irrelevant for polarization effects in view of homogeneity of M_{rad} dependence on $\mathbf{q}_\perp^2$. Hence, it suffices to discuss averaging of $\left\langle|M_{rad}|^2\right\rangle$ over $\mathbf{n_{q_\perp}}$.

3.1. Planar geometry

In the planar geometry $A_{scat}(\mathbf{q}_\perp)$ exhibits a sharp peak along some particular direction $\mathbf{n_{q_\perp}}$.[3] If one selects photons away from knot directions $\pm\mathbf{n_{q_\perp}}/\gamma$, where polarization is maximal - say, emitted in the middle band of angles $|\theta_\mathbf{k}\mathbf{n_{q_\perp}}| < 1/2\gamma$ (ref. to Fig. 1), the dependence of polarization on x_ω may be estimated from formula (12) upon substitution in it

[2]It may be worth pointing out, that except overall proportionality to $\mathbf{q}_\perp^2$, $\left\langle|M_{rad}|^2\right\rangle$ is also dependent on $\mathbf{n_{q_\perp}}$ despite $\mathbf{q}_\perp^2 \ll m^2$ - unless it is integrated over directions of $\theta_\mathbf{k}$ and summed over $\mathbf{e}'$. That circumstance is often missed in presentations of the equivalent photon method, including treatises [5], [6], though is taken care in [1].

[3]Physically, it may correspond either to electron passage through a thin crystal close to a strong crystalline plane, or to passage through gap of a magnet deflecting electrons to small angles. It should be minded that in those cases actual dimensions of the scattering system exceed the cross-section of electron beam in both transverse directions, so the concept of differential cross-section looses direct physical sense. Nevertheless, values of polarization extracted from it do not depend on A_{scat} and are correct.

$\mathbf{k}_\perp \perp \mathbf{q}_\perp$:

$$\left\langle |M_{rad}|^2 \right\rangle = \frac{4\mathbf{q}_\perp^2(1-x_\omega)}{m^2\left(\frac{1}{\gamma}+\gamma\theta_{\mathbf{k}}^2\right)^2}\left\{1+\frac{4(1-x_\omega)}{x_\omega^2}\left|\mathbf{n}_{\mathbf{q}_\perp}\mathbf{e}'\right|^2\right\}.$$

Then,

$$P \simeq P(x_\omega) = \frac{1}{1+x_\omega^2/2(1-x_\omega)}, \quad (|\theta_{\mathbf{k}}\mathbf{n}_{\mathbf{q}_\perp}| \ll 1/\gamma)$$

independently on $\theta_{\mathbf{k}} \times \mathbf{n}_{\mathbf{q}_\perp}$. As Fig. 2 displays, polarization stays higher then 90% for $x_\omega < 0.35$.
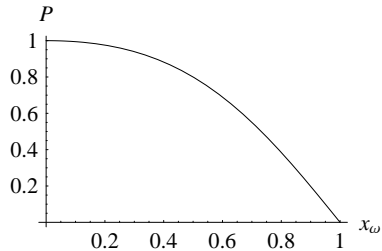
Fig.2. *Polarization of bremsstrahlung at planar scattering and orthogonal radiation $|\theta_{\mathbf{k}}\mathbf{n}_{\mathbf{q}_\perp}| \ll 1/\gamma$*

On the other hand, if angular separation is not attempted (which might be technically challenging at $\gamma > 10^4$), and only the natural collimation due to emission from an ultra-relativistic particle is used, one needs to integrate over radiation angles, or equivalently, photon transverse momenta. Evaluation of the integral of (12) over $d^2\mathbf{k}_\perp$ yields:

$$\int \left\langle |M_{rad}|^2 \right\rangle \frac{d^2k_\perp}{(2\pi)^2} = \int \left\langle |M_{rad}|^2 \right\rangle \frac{\omega^2 d^2\theta_{\mathbf{k}}}{(2\pi)^2}$$

$$=\mathbf{q}_\perp^2\gamma^2\frac{1-x_\omega}{\pi}\left\{x_\omega^2+\frac{2}{3}(1-x_\omega)\left[1+2(\mathbf{n}_{\mathbf{q}_\perp}\mathbf{e}')^2\right]\right\},$$

$$P \simeq P(x_\omega) = \frac{1-x_\omega}{\frac{3}{2}x_\omega^2+2(1-x_\omega)}.$$

Thus, without separation in $\theta_{\mathbf{k}}$, by means of only keeping $\mathbf{n}_{\mathbf{q}_\perp}$ fixed, polarization can not be obtained higher then 50% (see Fig. 3). On the other hand, we are going to show below, that with separation in $\theta_{\mathbf{k}}$ performed, it is possible to achieve a 100% polarization even at a spherically-symmetric scatterer.
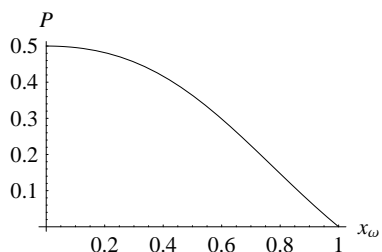
Fig.3. *Polarization of bremsstrahlung at planar scattering, averaged over $\theta_{\mathbf{k}}$ angles*

3.2. Centrally-symmetric scatterer, azimuthal integration over $\mathbf{q}_\perp$

The averaging in (12) over directions of $\mathbf{q}_\perp$ is achieved through the substitution $(n_{\mathbf{q}_\perp})_i(n_{\mathbf{q}_\perp})_k \rightarrow \frac{1}{2}\delta_{ik}$. One gets

$$\left\langle |M_{rad}|^2 \right\rangle \rightarrow \frac{8\mathbf{q}_\perp^2}{m^2}\frac{(1-x_\omega)^2}{(\gamma^{-1}+\gamma\theta_{\mathbf{k}}^2)^2}$$

$$\cdot \left\{\frac{x_\omega^2}{2(1-x_\omega)}+1-\frac{4\gamma^{-2}(\theta_{\mathbf{k}}\mathbf{e}')^2}{(\gamma^{-2}+\theta_{\mathbf{k}}^2)^2}\right\},$$

$$P(x_\omega,\gamma|\theta_{\mathbf{k}}|) = \frac{2\gamma^2\theta_{\mathbf{k}}^2}{\frac{x_\omega^2}{2(1-x_\omega)}(1+\gamma^2\theta_{\mathbf{k}}^2)^2+1+\gamma^4\theta_{\mathbf{k}}^4}.$$
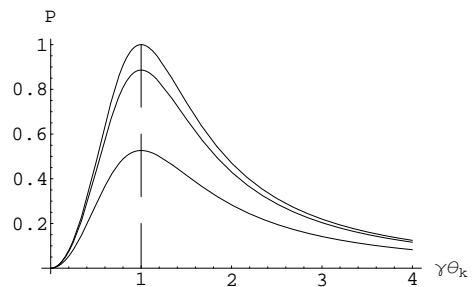
Fig.4. *Polarization of bremsstrahlung on a centrally-symmetric scatterer for $x_\omega = 0; 0.3; 0.6$ (from top to bottom)*

At $x_\omega \rightarrow 0$, $|\theta_{\mathbf{k}}| = 1/\gamma$ polarization reaches 100%, in spite of the angular averaging. That could hardly be expected based on very general reasons only. (The corresponding result was displayed in Fig. 4 of [1] but with no explanation supplied for the backing of such possibility). From our Fig.1 it is apparent that 100% magnitude of polarization may exist because at $|\theta_{\mathbf{k}}| = 1/\gamma$ polarization is oriented along a circle, centered at the origin of the plane and invariant under rotations of $\mathbf{n}_{\mathbf{q}_\perp}$, corresponding to angular averaging.

Thus, observation at some radiation angles of polarization close to 100% does not yet signal existence in the target of some special collective fields, guiding electron motion. Such an effect is also possible in an amorphous medium [4].

4. SUMMARY

The present work has offered formulation of a method for evaluation of differential cross-section of Compton scattering for polarized initial and final photons, based on advanced reduction of the matrix element. As was demonstrated, after the full reduction, the calculation leading to Lorentz-invariant formula (7) consumes only a few lines. To compare with, the derivation starting with squaring of the initial matrix element or expression (5), occupies a few pages. The transition to differential cross-section of

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[4] As usual, at that in order to be able to neglect multiple scattering effects as compared to deflection by emitting radiation, one needs fulfillment of the Landau-Pomeranchuk's type condition $L \ll e^2 L_{rad}$ (see, e.g., [5]), where L is target thickness and L_{rad} the radiation length (centimeters to decimeters for solid targets). Moreover, at x_ω rather small, photon emission angle which is of main interest for us exceeds electron deflection angle by a factor $\frac{1-x_\omega}{x_\omega}$, so the true condition may be $L \ll \left(\frac{1-x_\omega}{x_\omega}\right)^2 e^2 L_{rad}$.

bremsstrahlung in dipole approximation was based on specification of covariant expressions in terms of vector components in laboratory frame, which lifts the necessity to manually implement the equivalent photon method and transit between different reference frames - not very trivial task when dealing with polarization of the emitted photon.

In application to bremsstrahlung, a previously overlooked feature which seems to be worth emphasizing is that polarization as a function of (small) radiation angles at fixed $\mathbf{n_{q_\perp}}$ orients itself along perfect circles, including one centered at the origin. If averaging over $\mathbf{n_{q_\perp}}$ is performed, at the radius of the latter circle $|\theta_\mathbf{k}| = 1/\gamma$ only polarizations with the same orientation add up. Moreover, at small polarization values along the circle are close to 100%, and so the averaged polarization can be close to 100%, too. Actually, polarization stays in excess of 80% in the interval $0.8 < \gamma |\theta_\mathbf{k}| < 1.3$, $x_\omega < 0.3$.

The practical conclusions reached were as follows. If a beam of energetic and polarized γ-quanta needs to be prepared, it is beneficiary to use bremsstrahlung in an amorphous medium at $|\theta_\mathbf{k}| \simeq 1/\gamma$ (not just $\sim 1/\gamma$). The polarization is orthogonal to the radiation plane - as had long been established. The radiation recoil influence is always depolarizing, but for $x_\omega < 0.3$ rather weak. The method is most convenient to use for $\omega \le 10$ GeV.

There is still an interesting question, whether polarization must necessarily be taken into account at computations of electromagnetic shower spatial development in matter. For showers polarizations of γ-quanta are often neglected even at calculations of the spatial picture subsequently used for evaluation of radioemission from the shower as a whole. Taking into account that polarization of bremsstrahlung on a single atom/nucleus is orthogonal to the direction of photon deflection from the shower axis, at each step the development of the shower roughly makes an azimuthal turn to 90°. After a few steps the shower shall become axially symmetric. Therefore, neglect of polarization may prove acceptable in a thick (as compared to the radiation length) target, but not when the shower is short.

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РАСЧЕТ И АНАЛИЗ ВЕЛИЧИНЫ ПОЛЯРИЗАЦИИ ДЛЯ ТОРМОЗНОГО ИЗЛУЧЕНИЯ ПРИ ПЕРИФЕРИЧЕСКОМ РАССЕЯНИИ

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Предложен упрощенный способ вычисления дифференциального сечения поляризованного тормозного излучения при типичной атомной передаче импульса $q \ll m$. Используемый подход основывается на естественной редукции спинового матричного элемента процесса к форме $V^\alpha\gamma^\alpha + A_5\gamma^5$. Анализируется распределение поляризации в полностью дифференциальном сечении. Показано, что если производить угловую селекцию излученных фотонов, то даже после интегрировании по импульсам, переданным атому, для определенных углов излучения поляризация может сохраняться до 100%. Без угловой селекции фотонов, только за счет контроля отдачи, нельзя получить поляризацию излучения выше 50%.

РОЗРАХУНОК ТА АНАЛІЗ СТУПЕНЮ ПОЛЯРИЗАЦІЇ ДЛЯ ГАЛЬМІВНОГО ВИПРОМІНЮВАННЯ В УМОВАХ ПЕРИФЕРИЧНОГО РОЗСІЯННЯ

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Запропоновано спрощений спосіб обчислення диференціального перерізу поляризованого гальмівного випромінення за типової атомної передачі импульсу $q \ll m$. Використовуваний підхід ґрунтується на природній редукції спінового матричного елементу процесу до форми $V^\alpha\gamma^\alpha + A_5\gamma^5$. Аналізується розподіл поляризації у повністю диференціальному перерізі. Показано, що якщо здійснювати кутову селекцію випромінених фотонів, то навіть після інтегрування по импульсах переданих атому, для певних кутів випромінення поляризація може зберігатися до 100%. Без кутової селекції фотонів, тільки за рахунок контролю віддачі, неможливо отримати поляризацію випромінення вищу, ніж 50%.