Section H. PHYSICS OF QUANTUM LIQUIDS

RELAXATION CAUSED BY ONE PHONON DECAY INTO THREE IN SUPERFLUID HELIUM

I.N. Adamenko[1], Yu.A. Kitsenko[2], K.E. Nemchenko[1], V.A. Slipko[1], and A.F.G. Wyatt[3]

[1]Karazin Kharkov National University, Kharkov, Ukriane;
[2]National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine;
e-mail: ykitsenko@kipt.kharkov.ua;
[3]School of Physics, University of Exeter, Exeter, UK

The analytical relation for the rate of one phonon decay into three is obtained. Starting from the relation obtained, the rate of spontaneous decay in the first order of perturbation theory is found. It is shown, that processes of one phonon decay into three provide a fast establishment of equilibrium in anisotropic and isotropic phonon systems. It allows us to relate the momentum region in which processes of one phonon decay into three are permitted to subsystem of low-energy phonons.

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1. INTRODUCTION

Liquid helium is the unique medium in which a number of interesting macroscopic quantum phenomena are observed. One of them is the creation of high-energy phonons with energy close to 10 K by a pulse of low-energy phonons with typical temperature of 1 K (see Refs. [1-3]).

The theory of this phenomenon has been developed in Refs. [4,5]. As it followed from the theory, phonons of superfluid helium break up into two subsystems:

1. Subsystem of low-energy phonons (l -phonons) with $p < p_c$. The establishment of equilibrium in this subsystem is due to three-phonon processes ($1 \leftrightarrow 2$) with typical time $\tau_{3pp} = \nu_{3pp}^{-1}$ (see Refs. [6,7]).

2. Subsystem of high-energy phonons (h -phonons) with $p > p_c$. Here decay processes are forbidden by conservation laws of energy and momentum and the fastest are the four-phonon processes ($2 \leftrightarrow 2$) with typical time $\tau_{4pp} = \nu_{4pp}^{-1}$ (see Refs. [4,5,8]).

There is strong inequality

$$\tau_{3pp} << \tau_{4pp} \tag{1}$$

between τ_{3pp} and τ_{4pp} . As a result the equilibrium in l -phonon subsystem occurs very quickly in contrast to h -phonon subsystem where it happens rather slowly.

The difference in group velocities and rather weak connection between h - and l -phonons results to h-phonon leaving through a rear wall of l -phonon pulse and forming of h -phonon pulse which comes to the detector after l -phonon pulse.

However, in Refs. [9,10] it was shown, that three-phonon processes took place not up to p_c but to momentum $p_{3pp} = \sqrt{4/5}\,p_c$ and therefore phonons with momenta

$$p_{3pp} < p < p_c \tag{2}$$

should be related to h -phonon subsystem. But in momentum region (2) processes of one phonon decay into a greater number of phonons (decay processes) are still allowed and if the rates of these processes appear one order of magnitude with ν_{3pp} then these phonons should be related to l -phonon subsystem. It would seem that it is not essential to what subsystem these phonons should be related, as the momentum range (2) is rather small. However, as it has been shown in Ref. [11], the rate of four-phonon processes is very sensitive to a numerical value of momentum which delimitates l - and h -phonon subsystems. Therefore the calculation of the rates of decay processes is of undoubtful interest as it will allow us to answer the question to what subsystem the mentioned momentum range should be related. Here we consider one of these processes such as the process of one phonon decay into three ($1 \leftrightarrow 3$).

2. PROCESSES OF ONE PHONON DECAY INTO THREE

Conservation laws of energy and momentum which should be satisfied in process of one phonon decay into three can be written as

$$\mathbf{p}_1 = \mathbf{p}_2 + \mathbf{p}_3 + \mathbf{p}_4, \tag{3}$$

$$\varepsilon_1 = \varepsilon_2 + \varepsilon_3 + \varepsilon_4, \tag{4}$$

where $\mathbf{p}_i$ is the momentum of i -th phonon participating in the process and ε_i is its energy which we write as

$$\varepsilon_i \equiv \varepsilon(p_i) = cp_i\left(1 + \psi(p_i)\right). \tag{5}$$

Here $c = 2.38 \cdot 10^4$ cm/s is the velocity of sound and $\psi(p)$ is a function which describes a deviation of a spectrum from linearity which is small ($|\psi(p)| << 1$) but nevertheless it completely determines the mechanisms of phonon interactions. Here and below we shall use the simple analytical approximation of function $\psi(p)$ obtained in [7] which is valid for $p \le p_c$

$$\psi(p) = 4\psi_{\max}\frac{p^2}{p_c^2}\left(1-\frac{p^2}{p_c^2}\right). \quad (6)$$

Here $\psi_{\max}$ is the maximum value of function $\psi(p)$ reached when $p = p_c/\sqrt{2}$. In case of saturated vapour pressure $\psi_{\max} = 0.046$ and $\tilde{p}_c = cp_c/k_B = 10$ K.

From conservation laws (3) — (4) taking (5) into account we obtain a relation on angles between phonons with momenta $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$, $\mathbf{p}_4$

$$\zeta_{12} = \frac{(2p_1 - 2p_2 + \phi)\phi - 2p_3p_4\zeta_{34}}{2p_1p_2}, \quad (7)$$

where $\phi = f_1 - f_2 - f_3 - f_4$, $f_i = p_i\psi(p_i)$, $\zeta_{ij} = 1-\cos\theta_{ij}$, and θ_{ij} is the angle between phonons with momenta $\mathbf{p}_i$ and $\mathbf{p}_j$.

Having put ζ_{34} and ζ_{12} equal to zero in (7) and taking (6) into account we derive the boundaries of regions in which processes of one phonon decay into three can take place

$$p_{3\pm}(p_1, p_2) = \frac{1}{10}\Big(5p_1 - 5p_2$$
$$\pm\sqrt{5}\sqrt{-15p_1^2 + 10p_1p_2 - 15p_2^2 + 12p_c^2}\Big), \quad (8)$$
$$p_3(p_1, p_2) = p_1.$$

From the positivity of radicand in Eq. (8) the restrictions on momentum p_2 could be obtained:

$$\max(0, p_{2-}) < p_2 < \min(p_1, p_{2+}), \quad (9)$$

where

$$p_{2\pm}(p_1) = \frac{1}{3}\left(p_1 \pm 2\sqrt{2}\sqrt{p_{\max}^2 - p_1^2}\right), \quad (10)$$

$$p_{\max} = \sqrt{\frac{9}{10}}p_c. \quad (11)$$

From Eqs. (8) – (11) it follows, that the momenta of phonons participating in $1 \to 3$ processes can change in ranges (see also Ref. [12]):

$$0 < p_{2,3,4} < \sqrt{3/5}\,p_c = p_{\min}, \quad (12)$$

$$0 < p_1 < p_{\max}. \quad (13)$$

At saturated vapour pressure $\tilde{p}_{\min} = 7.75$ K and $\tilde{p}_{\max} = 9.48$ K.

We note that when $p_1 < p_{3pp}$ the processes of one phonon decay into three can proceed at rather big angles between momenta of interacting phonons while when $p_1 > p_{3pp}$ the mentioned processes are small-angle.

Interaction of phonons in superfluid helium is described by Landau Hamiltonian which we write as (see, for example, [13])

$$\hat{H}_{ph} = \hat{H}_0 + \hat{V}_3 + \hat{V}_4. \quad (14)$$

Here $\hat{H}_0$ is a Hamiltonian of noninteracting phonons and terms $\hat{V}_3$ and $\hat{V}_4$ describe the interaction of phonons caused by the third and the fourth orders of small deviations of a system from an equilibrium state accordingly.

The probability density of process of one phonon decay into three in the correspondence with [14] can be written as

$$W(\mathbf{p}_1|\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4) = \frac{2\pi}{\hbar}V^2|H_{fi}|^2\frac{1}{(2\pi\hbar)^6}. \quad (15)$$

Here V is a volume of a system and H_{fi} is an amplitude of the process of one phonon decay into three which can be written in the form

$$H_{fi} = \delta_{\mathbf{p}_1;\mathbf{p}_2+\mathbf{p}_3+\mathbf{p}_4}\frac{\sqrt{p_1p_2p_3p_4}}{8\rho V}M. \quad (16)$$

Here $\rho = 0.145$ g/cm^3 is a density of helium and

$$\begin{aligned} M &= M_{12}^{(2)} + M_{13}^{(2)} + M_{14}^{(2)} \\ &M_{12}^{(4)} + M_{13}^{(4)} + M_{14}^{(4)} + M_4 \end{aligned} \quad (17)$$

is a matrix element consisting of seven terms which can be written as

$$\begin{aligned} M_{12}^{(2)} &= \frac{\varepsilon_{1-2}}{\varepsilon_1 - \varepsilon_2 - \varepsilon_{1-2}} \\ &\times(2u - 1 + \mathbf{n}_3\mathbf{n}_4 + \mathbf{n}_3\mathbf{n}_{3+4} + \mathbf{n}_4\mathbf{n}_{3+4}) \\ &\times(2u - 1 + \mathbf{n}_1\mathbf{n}_2 + \mathbf{n}_1\mathbf{n}_{1-2} + \mathbf{n}_2\mathbf{n}_{1-2}), \end{aligned} \quad (18)$$

$$\begin{aligned} M_{12}^{(4)} &= -\frac{\varepsilon_{1-2}}{\varepsilon_1 - \varepsilon_2 + \varepsilon_{1-2}} \\ &\times(2u - 1 + \mathbf{n}_3\mathbf{n}_4 - \mathbf{n}_3\mathbf{n}_{3+4} - \mathbf{n}_4\mathbf{n}_{3+4}) \\ &\times(2u - 1 + \mathbf{n}_1\mathbf{n}_2 - \mathbf{n}_1\mathbf{n}_{1-2} - \mathbf{n}_2\mathbf{n}_{1-2}), \end{aligned} \quad (19)$$

$$M_4 = 4\left\{(u-1)^2 + w\right\}, \quad (20)$$

where $\mathbf{n}_i = \dfrac{\mathbf{p}_i}{p_i}$, $\varepsilon_{i-j} = \varepsilon(|\mathbf{p}_i - \mathbf{p}_j|)$, $u = \dfrac{\rho}{c}\dfrac{\partial c}{\partial \rho} = 2.84$, $w = \dfrac{\rho^2}{c}\dfrac{\partial^2 c}{\partial \rho^2} = 0.188$. The other terms of Eq. (17), can be obtained from the mentioned by replacement of corresponding subscripts. We note, that M_4 corresponds to the first order of perturbation theory on $\hat{V}_4$, and the others correspond to the second order of perturbation theory on $\hat{V}_3$.

The first three terms in the right-hand side of Eq. (17) are resonant. These terms give the main contribution to amplitude (16). In momentum range where three-phonon processes are allowed their denominators can vanish giving the essential divergence in matrix element. This divergence can be eliminated by taking the final lifetime of phonon caused by three-phonon processes into account.

3. THE KINETIC EQUATION FOR PROCESSES OF ONE PHONON DECAY INTO THREE

The kinetic equation describing change of distribution function $n_1 \equiv n(\mathbf{p}_1)$ of phonon with momentum $\mathbf{p}_1$ due to $1 \leftrightarrow 3$ processes can be written as

$$\frac{dn_1}{dt} = \frac{1}{3!}I_d(\mathbf{p}_1) + \frac{1}{2}I_c(\mathbf{p}_1). \quad (21)$$

Here

$$I_{d,c}(\mathbf{p}_1) = \int W_{d,c}n_{d,c}\delta\left(\varepsilon_{d,c}^{\Sigma}\right)\delta\left(\mathbf{p}_{d,c}^{\Sigma}\right)d\Gamma, \quad (22)$$

where $d\Gamma = d^3 p_2 d^3 p_3 d^3 p_4$,

$$W_d = W(\mathbf{p}_1|\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4)\; W_c = W(\mathbf{p}_4|\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3), \quad (23)$$

$$n_d = n_2 n_3 n_4 (1+n_1) - n_1(1+n_2)(1+n_3)(1+n_4), \quad (24)$$

$$n_c = n_4(1+n_1)(1+n_2)(1+n_3) - n_1 n_2 n_3 (1+n_4), \quad (25)$$

$$\varepsilon_d^\Sigma = \varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4, \mathbf{p}_d^\Sigma = \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4, \quad (26)$$

$$\varepsilon_c^\Sigma = \varepsilon_4 - \varepsilon_1 - \varepsilon_2 - \varepsilon_3, \mathbf{p}_c^\Sigma = \mathbf{p}_4 - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3. \quad (27)$$

We note that the first term of Eq. (21) corresponds to phonon with momentum $\mathbf{p}_1$ decay into three and the inverse process and the second term corresponds to combining of a phonon with momentum $\mathbf{p}_1$ with other two phonons and the process inverse to it.

We consider that in all momentum range (13) phonons are in equilibrium. In this case their equilibrium distribution function in the accordance with Refs. [6, 15] can be written as

$$n^{(0)}(\mathbf{p}_i) = \left\{ \exp\left(\frac{\varepsilon_i - \mathbf{p}_i \mathbf{u}}{k_B T} \right) - 1 \right\}^{-1}. \quad (28)$$

Here $\mathbf{u} = \mathbf{N}c(1-\chi)$ is a drift velocity, which is defined by the unit vector $\mathbf{N}$ directed along the total momentum of phonon system (an anisotropy axis of phonon system) and parameter of anisotropy χ. In isotropic phonon systems $\chi = 1$. In case corresponding to experiments [1-3] phonon pulses are strongly anisotropic phonon systems with $\chi << 1$.

To obtain the relaxation rate caused by $1 \leftrightarrow 3$ processes we change the equilibrium number of phonons with momentum $\mathbf{p}_1$ on a small value δn_1 at equilibrium distribution of the others phonons. In this case Eqs. (24) and (25) can be rewritten as

$$n_d = -\frac{\delta n_1}{1+n_1^{(0)}} \left(1+n_2^{(0)}\right)\left(1+n_3^{(0)}\right)\left(1+n_4^{(0)}\right), \quad (29)$$

$$n_c = -\frac{\delta n_1}{1+n_1^{(0)}} n_2^{(0)} n_3^{(0)} \left(1+n_4^{(0)}\right). \quad (30)$$

We define the relaxation rate caused by $1 \leftrightarrow 3$ processes by equality

$$\nu_{1\leftrightarrow 3} = -\frac{1}{\delta n_1}\frac{d\delta n_1}{dt}. \quad (31)$$

Substituting (29) and (30) into (21) and taking (31) into account we have

$$\nu_{1\leftrightarrow 3} = \nu_d + \nu_c, \quad (32)$$

where

$$\nu_d = \frac{1}{3!}\frac{1}{1+n_1^{(0)}} \int d\Gamma \delta\left(\varepsilon_d^\Sigma\right) \delta\left(\mathbf{p}_d^\Sigma\right) W_d \times \left(1+n_2^{(0)}\right)\left(1+n_3^{(0)}\right)\left(1+n_4^{(0)}\right), \quad (33)$$

$$\nu_c = \frac{1}{2}\frac{1}{1+n_1^{(0)}} \int d\Gamma \delta\left(\varepsilon_c^\Sigma\right) \delta\left(\mathbf{p}_c^\Sigma\right) W_c \times n_2^{(0)} n_3^{(0)} \left(1+n_4^{(0)}\right), \quad (34)$$

We note, that in the momentum range (2) $n_1^{(0)} << 1$. Thus the definition of the decay rate in Ref. [14] actually coincides with the mentioned above.

4. THE RATE OF PROCESSES OF ONE PHONON DECAY INTO THREE

Taking (15)-(17) into account we rewrite the relation (33) in spherical coordinates

$$\nu_d = \frac{Kp_1}{1+n_1^{(0)}} \int M^2 \delta\left(\varepsilon_d^\Sigma\right) \delta\left(\mathbf{p}_d^\Sigma\right) \times p_2^3 p_3^3 p_4^3 \left(1+n_2^{(0)}\right)\left(1+n_3^{(0)}\right)\left(1+n_4^{(0)}\right) \times dp_2 d\varphi_2 d\zeta_2 dp_3 d\varphi_3 d\zeta_3 dp_4 d\varphi_4 d\zeta_4. \quad (35)$$

Here $\zeta_i = 1 - \frac{\mathbf{p_i N}}{p_i}$, $K = \frac{1}{3 \cdot 2^{12} \pi^5 \hbar^7 \rho^2}$.

Having made the integration in (35) with the help of δ-functions we get

$$\nu_d = \frac{2Kp_1}{c \cdot \left(1+n_1^{(0)}\right)} \int dp_2 dp_3 d\zeta_2 d\zeta_3 d\varphi_2 \times \left\{M_+^2 + M_-^2\right\} \frac{p_2^3 p_3^3 p_4^2}{\sqrt{R}} \left(1+n_2^{(0)}\right)\left(1+n_3^{(0)}\right)\left(1+n_4^{(0)}\right), \quad (36)$$

where

$$M_\pm = M\left(\cos\varphi_3 = \cos\varphi_3^{(\pm)}, \cos\varphi_4 = \cos\varphi_4^{(\pm)}\right), \quad (37)$$

$$\cos\varphi_3^{(\pm)} = \frac{\alpha_1\left(A + p_{3\perp}^2 - p_{4\perp}^2\right) \mp \alpha_2\sqrt{R}}{2Ap_{3\perp}}, \quad (38)$$

$$\cos\varphi_4^{(\pm)} = \frac{\alpha_1\left(A - p_{3\perp}^2 + p_{4\perp}^2\right) \pm \alpha_2\sqrt{R}}{2Ap_{4\perp}}, \quad (39)$$

$$\alpha_1 = p_{1\perp} - p_{2\perp}\cos\varphi_2 \;\; \alpha_2 = p_{2\perp}\sin\varphi_2, \quad (40)$$

$$R = 4p_{3\perp}^2 p_{4\perp}^2 - \left(A - p_{3\perp}^2 - p_{4\perp}^2\right)^2, \quad (41)$$

$$A = p_{1\perp}^2 + p_{2\perp}^2 - 2p_{1\perp}p_{2\perp}\cos\varphi_2, \quad (42)$$

$$p_4 = p_1 + p_2 - p_3 - \phi \;\; p_{i\perp} = p_i\sqrt{2\zeta_i - \zeta_i^2}, \quad (43)$$

$$\zeta_4 = \frac{p_1\zeta_1 + p_2\zeta_2 - p_3\zeta_3 - \phi}{p_4}. \quad (44)$$

The rate of processes of one phonon decay into three defined by relation (36) consists of two compounds. The first of them is connected with a spontaneous decay of a phonon with momentum $\mathbf{p}_1$, and the other is caused by stimulation of a phonon with momentum $\mathbf{p}_1$ decay due to the presence of phonon system, i.e. nonzero functions $n_i^{(0)}$. As in prevailing momentum range distribution functions $n_i^{(0)}$ are much less than the unity we will be interested in the rate of spontaneous decay.

We shall find the decay rate ν_d^I caused by the first order of perturbation theory on $\hat{V}_4$. In this case $M_+ = M_- = M_4$ and we can get an exact analytical expression for the rate ν_d^I:

$$
\nu_d^I = N \cdot \begin{cases} g_1(p_1), & p_1 \le p_{\min}, \\ g_2(p_1), & p_{\min} < p_1 < p_{3pp}, \\ g_3(p_1), & p_{3pp} \le p_1 < p_{\max}, \\ 0, & p_1 \ge p_{\max}. \end{cases} \tag{45}
$$

Here

$$
N = \frac{\left\{(u-1)^2 + w\right\}^2 \psi_{\max}}{265720500\sqrt{3}\pi^3 \hbar^7 \rho^2 p_c^4 c}, \tag{46}
$$

$$
g_1(p_1) = \frac{492075\sqrt{3}}{64064} p_1^{11}\left(117 p_c^2 - 136 p_1^2\right), \tag{47}
$$

$$
\begin{aligned}
g_2(p_1) = {} & \frac{9 g_{21}(p_1)}{1601600}\sqrt{p_{3pp}^2 - p_1^2} \\
& + p_1^3\left(p_{\max}^2 - p_1^2\right)^2 g_{22}(p_1)\cdot\left(\frac{\pi}{2}\right. \\
& \left. + \arctan\left(\frac{\left(35 p_1^2 - 27 p_c^2\right) p_1}{45\left(p_1^2 - p_{\min}^2\right)\sqrt{p_{3pp}^2 - p_1^2}}\right)\right),
\end{aligned} \tag{48}
$$

$$
g_3(p_1) = \pi \cdot p_1^3 \left(p_{\max}^2 - p_1^2\right)^2 g_{22}(p_1), \tag{49}
$$

where

$$
\begin{aligned}
g_{21}(p_1) = {} & 2254448500 p_1^{12} \\
& - 83596342875 p_1^{10} p_c^2 + 121429933650 p_1^8 p_c^4 \\
& - 88262447850 p_1^6 p_c^6 + 33618986820 p_1^4 p_c^8 \\
& - 6231375360 p_1^2 p_c^{10} + 423263232 p_c^{12},
\end{aligned} \tag{50}
$$

$$
\begin{aligned}
g_{22}(p_1) = {} & 93500 p_1^6 - 157950 p_1^4 p_c^2 \\
& + 87480 p_1^2 p_c^4 - 15309 p_c^6.
\end{aligned} \tag{51}
$$

Big numerical coefficients in expressions (46) - (51) are caused by repeated integration of high powers of momentum.

We note, that the obtained relation for the rate is valid for all pressures up to 19 bar at which the dispersion becomes normal and decay processes are forbidden by conservation laws.

In Fig. 1 the rate ν_d^I calculated with a help of Eq. (45) is shown. We see, that the rate sharply vanishes near $p_1 = p_{\max}$. Such behavior of the rate could be explained by factor $\left(p_{\max}^2 - p_1^2\right)^2$ in (49).

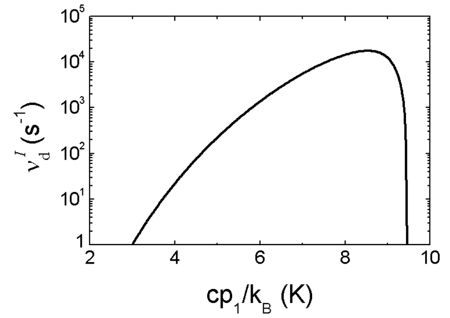

Fig. 1. *The rate of spontaneous phonon decay into three in the first order of perturbation theory on $\hat{V}_4$ calculated from Eq. (45)*

The integration in case of the second order of perturbation theory on $\hat{V}_3$ cannot be exactly made analytically due to the complexity of integrand in (36). However in this case the integration can be made numerically. The result of this integration in momentum range (2) is shown in Fig. 2 (curve 2). As it follows from the comparison of curve 2 in Fig. 2 with (45), the main contribution in the momentum range (2) is due to the second order of perturbation theory on $\hat{V}_3$.

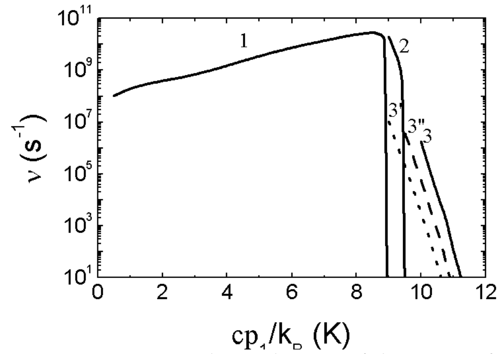

Fig. 2. *Momentum dependences of the rates of three-phonon processes $1 \to 2$ (curve 1), processes of one phonon decay into three $1 \to 3$ (curve 2), h-phonon creation for three values of delimitating momentum p_d equal to p_c, p_{3pp} and $p_{\max}$ (curves 3, 3′ and 3″). Calculations were made with $\theta_1 = 0$, $\chi = 0.02$ and $T = 0.041$*

5. MOMENTUM THAT DELIMITATES *l*- AND *h*-PHONON SUBSYSTEMS

As it has been already told in introduction the important question is where we should delimitate l- and h-phonon subsystems. To answer this question we start from Fig. 2. In Fig. 2 momentum dependences of the rates of three-phonon processes (curve 1), processes of one phonon decay into three (curve 2), four-phonon processes of high-energy phonons creation with momenta $p > p_d$ for three different values of momentum p_d which delimitates l- and h-phonon subsystems (curves 3, 3′, 3″) are represented.

From Fig. 2 it can be seen, that the rates of three-phonon processes ($1 \to 2$) and processes of one phonon decay into three ($1 \to 3$) are comparable and appear much greater than the rate of four-phonon processes of high-energy phonons creation. As a result the momentum range (2) should be related to l-phonon subsystem in which equilibrium occurs quickly in contrast to h-phonon subsystem with $p > p_c$ where the decay processes are forbidden and equilibrium occurs slowly.

Thus there is a question where we should relate phonons with momenta from $p_{\max}$ up to p_c. The calculation of the rates of decay processes in this range is rather difficult problem as the order of integrals in this case increases strongly. It is possible only to state, that due to decay processes the time of establishment of equilibrium in this momentum range will be less than in the range of $p > p_c$ where the decay processes are forbidden. Besides this we must take into account that, as it can be seen from Fig. 2, the rate of four-phonon processes decreases quickly enough with increasing of momentum. As a result the momentum p_d, that delimitates phonons of superfluid helium into two subsystems with different relaxation times can be considered to be equal to p_c, as it was supposed in Refs. [4,5,8].

6. CONCLUSION

In paper the processes of one phonon decay into three in anisotropic and isotropic phonon systems of superfluid helium are investigated. The restrictions on momenta of phonons which can participate in the mentioned processes are obtained.

The general relation (35) for the rate ν_d of processes of one phonon decay into three is derived. With a help of numerical integration of relation (35) the numerical value of the rate ν_d caused by the second order of perturbation theory on $\hat{V}_3$ is found. Starting from the general relation (35) the analytical relation (45) for the rate of spontaneous decay in the first order of perturbation theory on $\hat{V}_4$ is derived.

The question about the value of momentum p_d, which delimitates l- and h-phonon subsystems is considered and it is shown, that p_d is equal to p_c.

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РЕЛАКСАЦИЯ, ОБУСЛОВЛЕННАЯ ПРОЦЕССАМИ РАСПАДА ОДНОГО ФОНОНА НА ТРИ В СВЕРХТЕКУЧЕМ ГЕЛИИ

И.Н. Адаменко, Ю.А. Киценко, К.Э. Немченко, В.А. Слипко, A.F.G. Wyatt

Получено аналитическое выражение для частоты процессов распада одного фонона на три. Исходя из полученного выражения, найдена частота самопроизвольного распада в первом порядке теории возмущений. Показано, что процессы распада одного фонона на три обеспечивают быстрое установление равновесия в анизотропных и изотропных фононных системах. Это позволяет область, в которой разрешены процессы распада одного фонона на три, отнести к подсистеме низкоэнергетических фононов.

РЕЛАКСАЦІЯ, ЩО ОБУМОВЛЕНА ПРОЦЕСАМИ РОЗПАДУ ОДНОГО ФОНОНА НА ТРИ У НАДПЛИННОМУ ГЕЛІЇ

І.Н. Адаменко, Ю.О. Кіценко, К.Е. Немченко, В.А. Сліпко, A.F.G. Wyatt

Отримано аналітичний вираз для частоти процесів розпаду одного фонона на три. Виходячи з отриманого виразу, знайдено частоту самовільного розпаду в першому порядку теорії збурень. Показано, що процеси розпаду одного фонона на три забезпечують швидке встановлення рівноваги в анізотропних і ізотропних фононних системах. Це дозволяє область, у якій дозволені процеси розпаду одного фонона на три, віднести до підсистеми низькоенергійних фононів.