Section G. PHASE TRANSITIONS IN CONDENSED MATTER

SPONTANEOUS SYMMETRY BREAKING IN THE THEORY OF MODULATED STRUCTURES

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The parametrical evolution of the one-component order parameter under the spontaneous symmetry breaking in the Michelson model is considered. A critical behavior of a system with the critical point which has the properties of both the Lifshitz point of arbitrary order and the multicritical point is investigated. Critical dimension of such systems was found. The scale variational invariance of the model is discussed.

PACS: 64.60.Kw, 64.60.-i

1. INTRODUCTION

Spontaneous symmetry breaking is one of the most important ideas of the modern physics. It arises in the various fields of physics and gives one the possibility to describe the phenomena which, at first sight, have nothing common from the general point of view. In particular the notion of spontaneous symmetry breaking is a background of the Landau theory of the phase transitions (PT) of the second kind which allow one to describe the critical phenomena in the different physical systems in a general way. Therefore the problem of determination of the field of validity of the Landau theory is one of the most important problems in the theory of critical phenomena. The important criterion of the validity of the Landau theory is the dimension of the space. If the dimension of the physical space is lager than certain value called critical dimension (CD) then the Landau theory is valid, otherwise one needs to use more complicated methods to investigate the critical phenomena in vicinity of critical point (CP). One of the problems discussed in this paper is the CD of the systems which allow the CP with the properties of both the multicritical points and Lifshitz points. The parametrical evolution of a one-component order parameter (OP) in the Michelson model is also discussed.

2. PARAMETRICAL EVOLUTION OF ORDER PARAMETER

In order to describe the phase transition in a system with one-component OP, which allows the incommensurable structures of OP, the thermodynamic potential may be written in following form:

$$\Phi = \Phi_0 + \int_0^L \left\{ (\varphi'')^2 - g\varphi^2 (\varphi')^2 - \gamma (\varphi')^2 + q\varphi^2 + \frac{p}{2}\varphi^4 + \frac{1}{3}\varphi^6 \right\} dx. \quad (1)$$

Here L is a size of a system, φ' is a spatial derivative of OP $\varphi(x)$ and g, γ, q, p are the material parameters.

The spatial distribution of OP is defined by the solution of the following Eulier – Poisson variational equation for the functional (1):

$$\varphi^{IV} + g\left(\varphi^2\varphi'' + 2\varphi(\varphi')^2\right) + \gamma\varphi'' + q\varphi + p\varphi^3 + \varphi^5 = 0. \quad (2)$$

Eq. (2) is a fourth order nonlinear differential equation. General solution of the equation (2) is unknown to this day, but there are methods to obtain a partial solution of eq. (2). One of the possibilities to analyze the equations of this type is to represent φ'^2 as function of $\varphi(x)$:

$$\varphi'^2 = V(\varphi), \quad (3)$$

and substitute it into eq. (2). In particular, if the function $V(\varphi)$ is the polynomial of third order of $\varphi(x)$ then the solution of eq. (2) is expressed through the Jacobi elliptic functions. Furthermore, if the functional (1) has some additional symmetries then the substitution (3) allows one to find a soliton-like distribution of OP.

We will carry out the qualitative analysis of the parametrical evolution of OP for the model (1) in the case $g>0$ and:

$$V(\varphi) = a_0 + a_1\varphi^2 + a_2\varphi^4. \quad (4)$$

Here the coefficients a_0, a_1, a_2 are the functions of g, γ, q, p.

I. In high-symmetric phase $\varphi(x) = 0$. Initial state has translational symmetry (it is invariant under the transformation $x \to x + x_0$, where x_0 is an arbitrary translation in the coordinate space), and initial states are invariant under the transformation $\varphi \to -\varphi$ and $x \to -x$. This situation is illustrated in Fig 1. All the figures presented consist of 2 graphs, the first one shows the dependence of φ'^2 on φ, the second one illustrates the dependence of OP on the coordinate x.

II. The PT to the phase MS1 takes place when $q = \gamma^2/4$. The OP $\varphi(x) \approx 0$, therefore the spatial distribution of OP may be found in the linear approximation from eq. (2) (see Fig. 2):

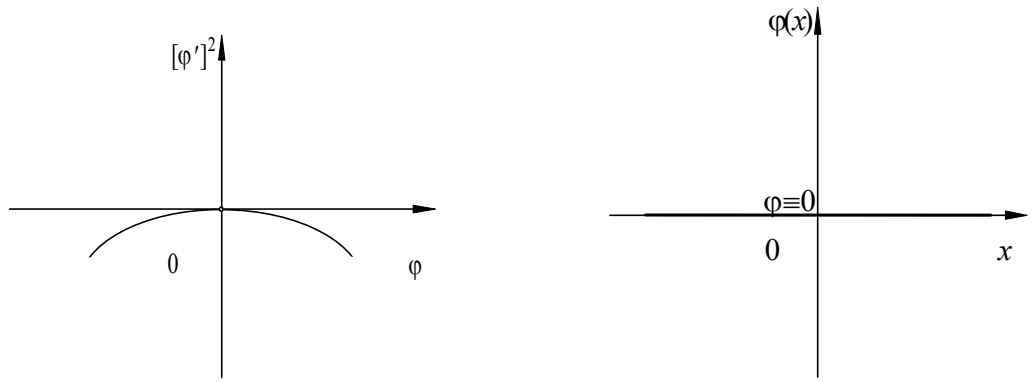

Fig. 1. *Initial high-symmetric phase*

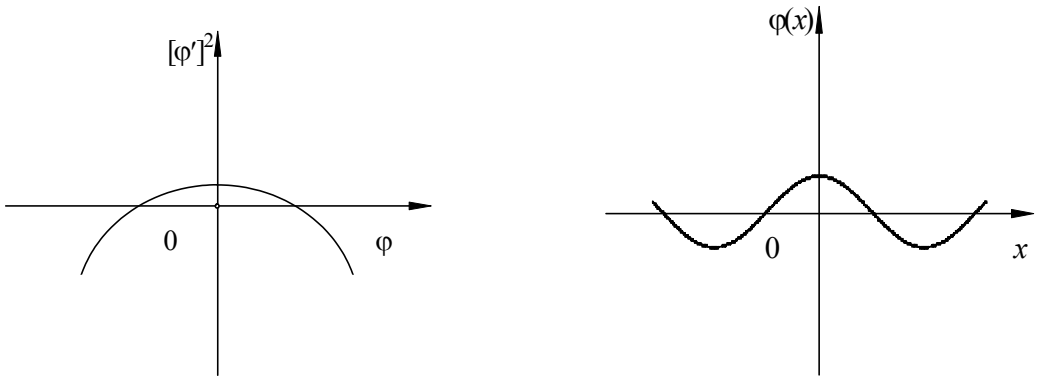

Fig. 2. *Modulated structure MS1*

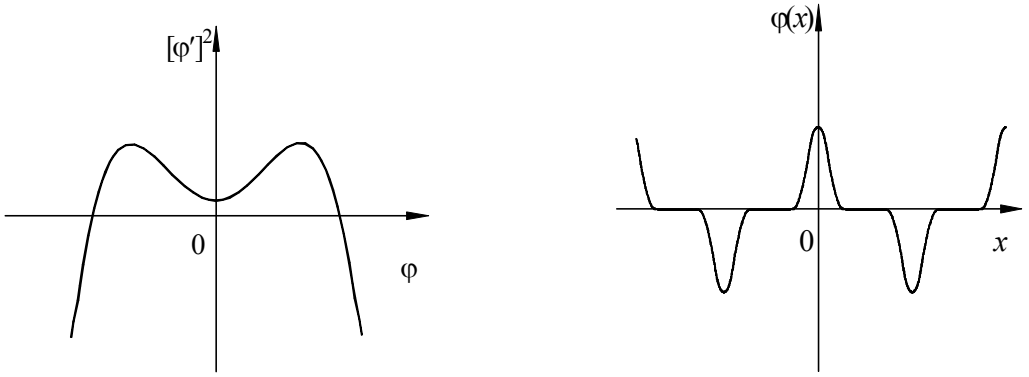

Fig. 3. *The evolution of phase MS1*

$$\varphi(x) = a_c \cdot \mathrm{cn}(b_c x, k_c) \approx \bar{a} \cdot \cos(cx) + \bar{\bar{a}} \cdot \cos(3cx) + \ldots, \quad (5)$$

Here the parameters $\bar{a}$, $\bar{\bar{a}}$, c are certain functions of initial parameters a_c, b_c, k_c. The OP φ is a periodic function.

The invariance under the reflection ($\varphi \to -\varphi$) is broken in phase MS1, and the translation invariance is reduced to the operation $x \to x + \bar{x}_0$, where $\bar{x}_0$ is value divisible by the period of OP, and the average value of the OP $\left(\varphi(x)\right)_T = 0$.

Thus the OP is locally not invariant under the reflection, but globally it preservs the invariance under the transformation $\varphi \to -\varphi$.

Therefore the periodical distribution of OP $\varphi(x)$ is the reaction of the system on appearance of locally nonzero OP and it restores the broken symmetry $\varphi \to -\varphi$. Appearance of a local order not at $q = 0$ (as in a homogeneous case), but at $q = \gamma^2/4 > 0$, is an example of a "thermodynamic" mechanism of a shift of the point of PT.

Sign reversal of parameter a_1 in eq.(4) leads to the arising of the new peculiar properties of the dependence of $\varphi'^2(x)$ on φ (Fig. 3). Such a behavior of the OP realized in the model of an elliptic cosine if $0.5 < k^2 < 1.0$.

III. Let us consider the point of PT from the phase MS1 to the phase MS2. The value of $\varphi'^2(x)$ vanishes at $\varphi = 0$ (Fig. 4).

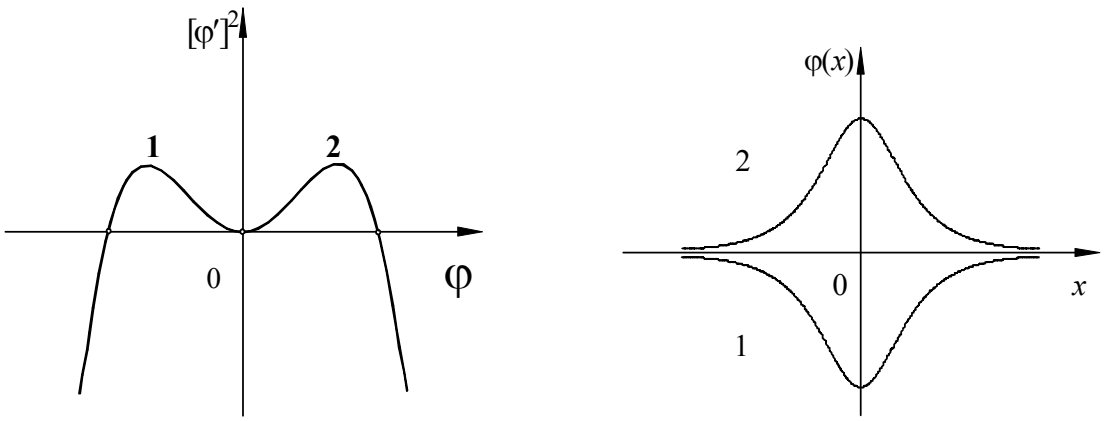

Fig. 4. *The point of the PT from MS1 To MS2*

In this situation $a_0 = 0$ and $a_1 \neq 0$ in eq. (4). The distribution of the OP has the following form:

$$\varphi(x) = a_c \cdot \mathrm{cn}(b_c x, 1)$$
$$= \frac{\hat{a}}{\mathrm{ch}(wx)} = a_d \cdot \mathrm{dn}(b_d x, 1), \qquad (6)$$

here the parameters $\hat{a}, w, a_d, b_d$ are certain functions of the material parameters. and the period of the OP is $T = \infty$.

Translation invariance of $\varphi(x)$ is completely breaking $(\varphi(x))_T \neq 0$.

IV. Arising of the new phase MS2 (Fig. 5). The OP distribution may be described by the following equation:

$$\varphi(x) = a_d \cdot \mathrm{dn}(b_d x, k_d), \; 0.0 < k_d^2 < 1.0 . \qquad (7)$$

As one can see $(\varphi(x))_T \neq 0$. This solution is invariant under translations $x \to x + \bar{x}_0$. If the structures with the values of the OP $-\varphi$ and $+\varphi$ are physically equivalent then the both of them can be realized as separate region.

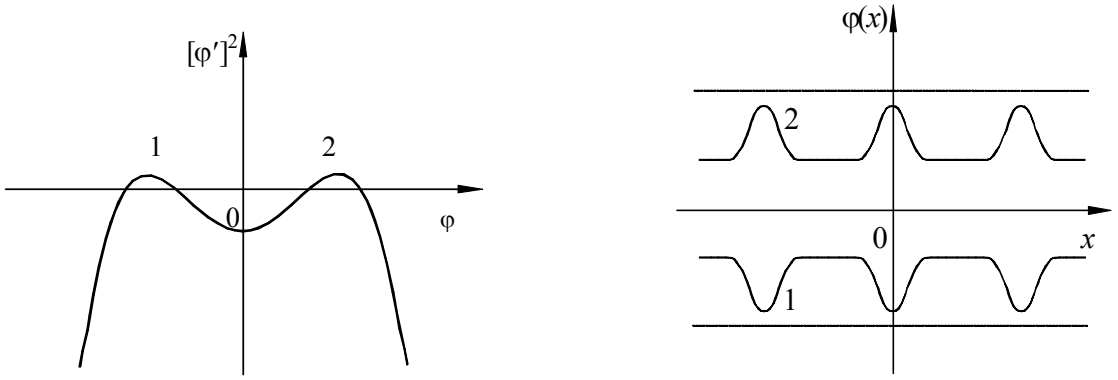

Fig. 5. *Phase MS2*

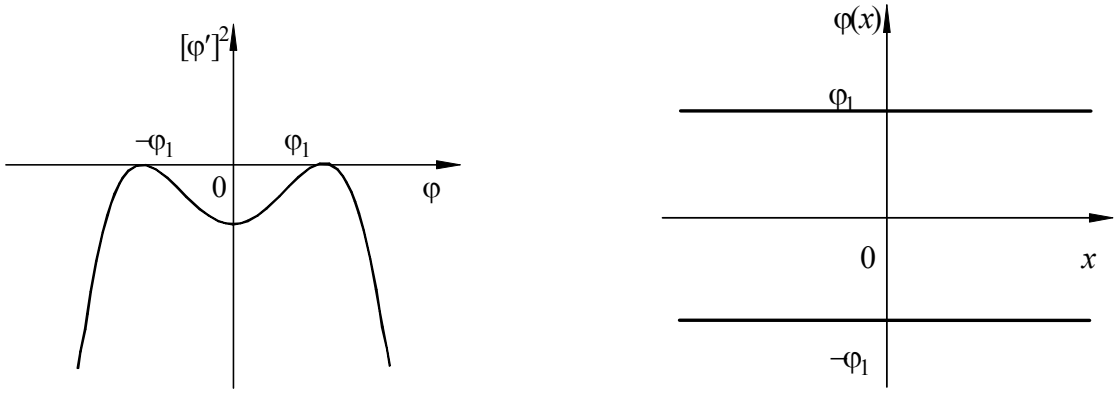

Fig. 6. *The final low-symmetric phase*

V. Low-symmetric phase. The system turns into the state illustrated by Fig. 6. The translation invariance of the OP ($x \to x + x_0$) is completely restored. The values of the OP are $\varphi(x) = \pm\varphi_1$, where φ_1 is a nonzero constant.

We note that the numerical calculations in the approximation $\varphi'^2 = \sum_{i=0}^{3} a_i \varphi^{2i}$ confirm the results of the qualitative analysis.

3. CRITICAL DIMENSION

One of the most important properties of the models which describe the system with critical phenomena is the existence of the critical dimension (CD). In the φ^4 model which describes the phase transition (PT) in the usual critical point (CP), the CD is equal to 4. However, in the models which describe the PT in the system with both the multicritical and the Lifshitz points the CD differs from 4 and depends on the power of nonlinearity of the model and the order of the Lifshitz point.

To describe the critical phenomena in the systems with the Lifshitz point one needs to take into account the higher gradients of OP. The CD of such models is $d_c = 4\left(p+1\right)$, where $\frac{p}{2}$ is the order of higher gradients. Describing of the critical phenomena in the systems with the multicritical points requires to take into account in effective Hamiltonian the terms with higher nonliniarities of the OP. The CD in this case has the form $d_c = 4\left(N+1\right)/\left(N-1\right)$, where N is the power of the nonliniarity of the model.

In this paper we investigate the critical behavior of the system with the critical point which has the properties of a both the Lifshitz point of the arbitrary order and the multicritical point.

The effective Hamiltonian of the system in the vicinity of the aforementioned critical point may be written as:

$$H = \int d^m x_i d^{d-m} x_c \left\{ \frac{r}{2}\eta^2 + \frac{\gamma}{2}\left(\Delta_i^{\frac{1}{2}}\eta\right)^2 + \frac{\delta}{2}\left(\Delta_c^{\frac{1}{2}}\eta\right)^2 + \frac{\beta}{2}\left(\Delta_i^{\frac{p}{2}}\eta\right)^2 + u\eta^{N+1} \right\}, \quad (8)$$

where η is the one-component OP, d is the dimension of the physical space, r, γ, δ, β are the material parameters. We assume that the physical space may be divided into two subspaces with the dimensions d-m and m. In the first one, denoted by c, there are no wave vectors of modulation. In other one, denoted by i, the wave vectors of modulation are present. We assume that d and m may be considered as continuous variables and, of course, $d>m$. Δ_c and Δ_i are the Laplacian operators in subspaces c and i, respectively. The operators $\Delta^l = \Delta\left(\Delta^{l-1}\right)$, if l is non-integer number, then Δ^l should be understood as the pseudodifferential operators defined with help of the integral Fourier transforms. In the CP $r=\gamma=0$ and the other parameters of the model (8) are positive constants.

We will find the CD of the model (8) from condition of stability of the fixed point of the renormgroup transformation for the Hamiltonian (8). This condition looks in following way:

$$d > d_c = m\left(1-\frac{1}{p}\right) + 2\frac{N+1}{N-1}. \quad (9)$$

If the dimension of physical space is more then d_c than the Landau theory is valid, otherwise it is invalid due to the anomalous increase of the OP fluctuations in the vicinity of CP.

The space with the dimension which coincides with the CD has some interesting properties. In such space the model which describes the PT is renormalizable and allows the variational scale symmetry. The variational scale invariance of the model is the important property which may be useful in the analysis of the corresponding variational equations.

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СПОНТАННОЕ НАРУШЕНИЕ СИММЕТРИИ В ТЕОРИИ МОДУЛИРОВАННЫХ СТРУКТУР

А.В. Бабич, С.В. Березовский, В.Ф. Клепиков

Рассмотрена параметрическая эволюция однокомпонентного параметра порядка при спонтанном нарушении симметрии в модели Михельсона. Для модели, позволяющей описывать фазовые переходы вблизи критических точек, обладающих одновременно свойствами точек Лифшица произвольного порядка и мультикритических точек, найдена критическая размерность. Обсуждается вариационная масштабная симметрия для таких моделей.

СПОНТАННЕ ПОРУШЕННЯ СИМЕТРІЇ В ТЕОРІЇ МОДУЛЬОВАНИХ СТРУКТУР

А.В. Бабіч, С.В. Березовський, В.Ф. Клепіков

Розглянуто параметричну еволюцію однокомпонентного параметра порядку при спонтанному порушенні симетрії в моделі Міхельсона. Для моделі, яка дозволяє описувати фазові перетворення поблизу точок, які водночас мають властивості точок Ліфшиця довільного порядку і мультикритичних точок, знайдено критичну розмірність. Обговорюється варіаційна масштабна симетрія для таких моделей.