WHISTLER WAVES SELF-FOCUSING IN LABORATORY AND IONOSPHERIC PLASMAS IN DENSITY TROUGHS

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Stationary self-focusing of whistler waves, which propagate along magnetic field with frequencies below half electron-cyclotron frequency is considered in the framework of two-dimensional generalized nonlinear Schroedinger equation. It takes into account electrostatic wave component of whistler waves and nonlocal nonlinearity caused by plasma heating during intense whistler wave propagation, which may be essential in laboratory plasma and in ionospheric experiments. Necessary conditions for stationary nonlinear self-trapping in self-sustained waveguides are found and their stability confirmed both analytically and numerically.

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1. INTRODUCTION

Whistler or helicon wave is one of the most frequently observed waves in magnetized laboratory plasmas, in the ionosphere and the magnetosphere of the Earth and in the plasma of solids. In spite of intensive investigations since the beginning of the last century the nonlinear properties of the whistler waves are still not well understood. As known, the stationary whistler waveguide formation in density troughs has been experimentally demonstrated in laboratory plasma both below [1-3] and above [1,2] half electron cyclotron wave frequency ($\omega_c/2$). In these experiments plasma heating during intense wave propagation have caused density troughs (wells) along wave beam propagation. It is well known, that at $\omega=\omega_c/2$ the sign of group velocity, perpendicular to an external magnetic field $B_0 = B_0 e_z$, changes. For both dispersive regimes (for ω both below and above $\omega_c/2$) in linear approximation wave beam spreads out due to diffraction [4]. One can expect, however, that due to plasma density changing, induced by intense HF wave, refractive index may vary in such a way that to compensate linear wave diffraction.

Conventional model of self-focusing of the wave propagating in z-direction bases on nonlinear Schrödinger equation (NSE):

$$i\partial_z \Psi + \hat{D}\Psi + \Psi \hat{N}(|\Psi|^2) = 0\,, \qquad (1)$$

where only the lowest order dispersive (diffractive) effect is taken into account: $\hat{D}\Psi = D\Delta_\perp \Psi$ (here $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is Laplacian operator), and refractive index is supposed to depend on wave intensity $|\Psi|^2$ linearly ($\hat{N}(|\Psi|^2) = B|\Psi|^2$). The problem is that such oversimplified model fails to explain the experiments. Actually, in the case $\omega<\omega_c/2$ ($DB<0$) thermal nonlinear effect increases beam spreading, and in the case $\omega>\omega_c/2$ ($DB>0$) self-focusing is so strong that may give rise to wave beam collapse. It was shown in [4] that taking into account the saturation of nonlinearity and higher order dispersion (polarization) effects one can explain formation of stable whistler waveguids in both regimes. Other higher order nonlinear effect, which may be even more essential than saturation of nonlinearity [5,6], is the nonlocal thermal wave response caused by thermal conductivity. For not too high intensities, in the so-called weak nonlocality limit, this effect can be described by additional term of the form $C\Delta_\perp|\Psi|^2$ [7]. Nonlocal nonlinearity may be also very important to describe two-dimensional (2d) Langmuir solitons [8] and 2d upper-hybrid (UH) wave structures [9], 2d structures in molecular lattices [10, 11] in atomic Bose-condensate [12], and power optical beam propagation in vapors accompanying atom diffusion [13]. For whistler wave propagation with frequencies $\omega>\omega_c/2$ a saturation of nonlinearity [4, 14] or defocusing nonlocal nonlinear effect [8,12] may arrest wave collapse. Therefore, each of these supplementary effects, even taken separately, let to explain a formation of coherent 2d structure. However, below half electron frequency a formation of stationary whistler waveguides in troughs may be possible only due to competition of nonlinear effects with higher-order dispersion. The same is true for formation of UH 2d structures in anomalous dispersion region ($\omega_{pe}^2 < 3\omega_c^2$), which was shown in [9].

Here basic generalized NSE (GNSE) including fourth order dispersive effects and nonlocal thermal self-focusing effects for description of whistler wave propagation is derived. It is demonstrated that there may exist at least two soliton branches with the same number of quanta, but different spatial scales. Their stability has been analyzed both analytically and numerically against small and finite perturbations. In the framework of our model, we predict an existence of new type of whistler wave beam stationary structures, which have curved wave front (their phase varies nonlinearly in perpendicular to beam propagation plane). In linear approximation, such wave beams would either converge or spread out.

2. BASIC EQUATIONS

To describe intense whistler propagation along magnetic field, taking into account their mixed polarization and thermal nonlinear effects, we use the GNSE in the form:

$$i\partial_z \Psi + (D + P\Delta_\perp)\Delta_\perp \Psi + \Psi(B + C\Delta_\perp)|\Psi|^2 = 0\,. \qquad (2)$$

It includes dispersive effects of the second, as well as of the fourth order (terms proportional to D and P respectively) and combination of cubic nonlinearity with nonlocal nonlinearity (terms proportional to B and

C). (Derivation of the GNSE (2) see in the Sec. 5.)

Any localized wave packet envelope $\Psi(r,z)$ conserves the following finite integrals of motion during its evolution in z direction: (i) number of quanta (or "energy"):

$$N = \int \Psi \, d^2\vec{r} \qquad (3)$$

(ii) momentum, (iii) angular momentum, which are equal to zero for radially-symmetric solitons, and (iv) Hamiltonian:

$$H = D\int |\nabla_\perp \Psi|^2 d^2\vec{r} - P\int |\Delta_\perp \Psi|^2 d^2\vec{r} - \frac{B}{2}\int |\Psi|^4 d^2\vec{r} + \frac{C}{2}\int (\nabla_\perp |\Psi|^2)^2 d^2\vec{r} = DI_D - PI_P - \frac{B}{2} I_B + \frac{C}{2} I_C. \qquad (4)$$

For whistler wave with $\omega<\omega_c/2$ propagating in density troughs one should put $D>0$, $P>0$, $B<0$, $C<0$ (see Sec. 5). As was demonstrated in Ref. [9], the Hamiltonian (4) is bounded functional at $C \le 0$), which guarantees an existence of stable background soliton solution:

$$\Psi(r,z) = \psi(r) e^{i\Lambda z}, \qquad (5)$$

where Λ is the propagation constant or the nonlinear shift of z-component of wave number. The function describing soliton radial profile $\psi(r)$ satisfies the ordinary differential equation, which after rescaling transformations $r = \rho\sqrt{P/D}$, $\Lambda = \lambda D^2/P$, $\psi(r) = U(\rho)\sqrt{D^2/(|B|P)}$ may be rewritten as follows:

$$-\lambda U + \Delta_\rho U + \Delta_\rho^2 U - U|U|^2 + \sigma U \Delta_\rho |U|^2 = 0, \qquad (6)$$

where

$$\sigma = CD/PB, \quad \Delta_\rho = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}.$$

Below the two-parameter (with parameters λ and σ) soliton family is investigated analytically and numerically.

3. VARIATIONAL ANALYSIS

As known, solution of the GNSE (2) corresponds to the extremum of the action functional: $\delta\int \Im \, d^2\vec{r} dz = 0$, where $\Im$ is the Lagrangian density for Eq. (2). Taking into account variation (with z) of the phase and of the spatial scale of arbitrary wave packet we choose the normalized trial function in the form:

$$\Psi(\vec{r},z) = \frac{\mu(z)\sqrt{N}}{\sqrt{2\pi \ln 2}} f(\mu(z)r) e^{i\gamma(z)\ln\cosh(\mu r) + i\Phi(z)}, \qquad (7)$$

which satisfies the reduced variational problem with Lagrangian

$$L = \frac{i}{2}\int \left\{\Psi^* \partial_z \Psi - \Psi \partial_z \Psi^*\right\} d^2\vec{r} - H, \qquad (8)$$

where H is the Hamiltonian (4) with the trial function (7). Introducing the new longitudinal distance variable:

$$\eta = \frac{1}{N}\int_0^z \mu^2(\alpha) d\alpha,$$

one can obtain after Ritz optimization the set of canonical equations describing evolution (with z) of parameters of cylindrically symmetric localized wave packet:

$$\frac{d\beta}{d\eta} = \frac{\partial H}{\partial \mu}, \quad \frac{d\mu}{d\eta} = -\frac{\partial H}{\partial \beta}, \qquad (9)$$

where $\beta = \gamma\mu$ and Hamiltonian in variational approach with the trial function (7) is:

$$H = \frac{N}{\ln 2}[DI_d(\mu^2 + \beta^2) - P(\mu^2 + \beta^2)(I_{p\mu}\mu^2 + I_{p\beta}\beta^2) - \frac{B}{2} I_b N\mu^2 + \frac{C}{2} I_c N\mu^4], \qquad (10)$$

where we introduced the following integrals:

$$I_d = \int_0^{+\infty} f^2(\xi)\xi \tanh^2\xi \, d\xi = 0.398,$$

$$I_{p\mu} = \int_0^{+\infty} [\Delta_\xi f(\xi)]^2 \xi \, d\xi = 0.642,$$

$$I_{p\beta} = \int_0^{+\infty} \xi f^2(\xi) \tanh^4\xi \, d\xi = 0.289,$$

$$I_b = \frac{1}{2\pi \ln 2}\int_0^{+\infty} f^4(\xi)\xi \, d\xi = 0.0678,$$

$$I_c = \frac{1}{\pi \ln 2}\int_0^{+\infty} f^2(\xi) f'^2(\xi)\xi \, d\xi = 0.1,$$

here radial profile is chosen to be $f(\xi) = 1/\cosh\xi$. Obviously, the stationary soliton solution with parameters μ_0 and β_0, which do not change with z, corresponds to the stationary point of the Hamiltonian. These 2d solitons correspond to a formation of the stationary self-induced whistler waveguide. The trial function (7) with $\gamma \ne 0$ takes into account possible phase front curvature of the ducted whistler wave beam. Indeed, the set of equations

$$\left.\frac{\partial H}{\partial \mu}\right|_{\mu=\mu_0, \beta=\beta_0} = 0, \quad \left.\frac{\partial H}{\partial \beta}\right|_{\mu=\mu_0, \beta=\beta_0} = 0, \qquad (11)$$

has two types of the solutions: (i) the ordinary soliton solution with zero phase curvature parameter $\beta_0 = 0$ and

$$\mu_{\text{ord}}^2 = \frac{DI_d - BI_b N/2}{2(PI_{p\mu} - CI_c N/2)}$$

(ii) the soliton (wave beam) with curved phase front (CPF) having parameters

$$\mu_{\text{chirp}}^2 = \frac{D\tilde{I}_d - BI_b N/2}{2(P\tilde{I}_p - CI_c N/2)}, \quad \beta_{\text{chirp}}^2 = \frac{DI_d - PI_{p\mu}\mu_{\text{chirp}}^2}{2PI_{p\beta}},$$

where $\tilde{I}_d = -I_d(I_{p\mu} - I_{p\beta})/2I_{p\beta}$,

$\tilde{I}_p = -(I_{p\mu} - I_{p\beta})^2/4I_{p\beta}$.

Below we consider mostly case $D>0$, $P>0$, $B<0$, $C<0$, which corresponds to a stationary waveguide formation.

It is important to find the stable solitons among a manifold of obtained variational solutions. A soliton is stable if it corresponds to extremum (minimum or maximum) of the Hamiltonian (10). In this case any deviation from the extremum point (μ_0, β_0) would lead to change of the Hamiltonian, which is impossible because of its conservation. Obviously, soliton, which corresponds to a saddle point of the Hamiltonian, is unstable.

It is convenient to investigate soliton properties at fixed parameter $\sigma = CD/PB$. Results of the variational analysis may be summarized as follows:

1. $\sigma < \sigma_c = \widetilde{I}_p I_b / \widetilde{I}_d I_c = 0.301$ (see Fig. 1 a). If $\mu_0^2 < \mu^2 < \mu_{cr}^2$ ordinary solitons (with $\beta_0 = 0$) are unstable. They became stable if $N > N_{cr}$ ($\mu^2 > \mu_\infty^2$), where

$$N_{cr} = \frac{2DI_d}{BI_b} \frac{\sigma_c/\sigma_o}{1-\sigma/\sigma_{cr}}, \quad \sigma_o = \frac{I_{p\mu} I_b}{I_d I_c} = 1.093,$$

$$\sigma_{cr} = \sigma_o - \sigma_c,$$

$$\mu_0^2 = \frac{DI_d}{2PI_{p\mu}}, \quad \mu_\infty^2 = \frac{BI_b}{2CI_c}, \quad \mu_{cr}^2 = \frac{DI_d}{2PI_\beta}.$$

   In this case solitons with CPF are always unstable.
2. $\sigma_c < \sigma < \sigma_{cr}$ (see Fig. 1 b). The *N*-dependence of parameter μ^2 for ordinary solitons and for solitons with CPF changes dramatically. As before, ordinary solitons are stable at $N > N_{cr}$. Furthermore, a pair of stable solitons with CPF (corresponding to parameter values $\beta = \pm\beta_0$) appears at $N_0 < N < N_{cr}$, where $N_0 = 2D\widetilde{I}_d / BI_b$.
3. $\sigma_{cr} < \sigma < \sigma_o$ (see Fig. 1.c.) Ordinary solitons become unstable. Stable soliton branch corresponds to solitons with CPF.
4. $\sigma > \sigma_o$ (see Fig. 1.d) This case is similar to the previous one, but the branch of unstable ordinary solitons appears for $\mu^2 > \mu_\infty^2$.

As was demonstrated in Ref. [4] for the cubic-quintic (CQ) media and in Ref. [9] for media with nonlocal nonlinearity, soliton profile has pronounced oscillating tails if *DP*>0. The variational analysis, which takes into account oscillations of soliton profile as well as radial variation of the soliton phase, has been performed in Ref. [9] using trial function of the form:

$$\psi_0(r) = hJ_0(\kappa r) e^{-\frac{1}{2}\mu_1^2 r^2 (1+i\mu_2)}, \tag{12}$$

where $J_0(x)$ is the Bessel function of zero order. Soliton solution gives an extremum to Hamiltonian at fixed number of quanta. It was demonstrated, that oscillating nature of soliton is crucial for right description of solitons especially nearby the threshold value of number of quanta for soliton existence, where the scale of the oscillations κ^{-1} becomes of the order of $\sqrt{P/2D}$ when $\mu_1^2 \to 0$, so that the third variational parameter $\mu_3 = \kappa/\mu_1$ tends to infinity. Note that, the variational analysis with the trial function (7), predicts an existence of a soliton branch with nonlinear spatial scale of order of $\sqrt{C/B}$. This gives rise to the view, that in the case under consideration coexist two branches of the solitons with the same number of quanta but different spatial scales.

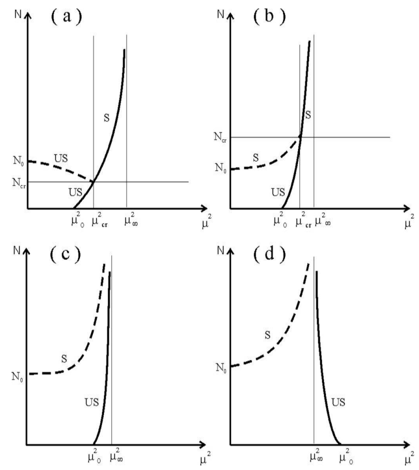

Fig. 1. Variational parameter μ^2 vs numbers of quanta N. Solid curves represent ordinary solitons, dashed curves – solitons with CPF. "US" indicates unstable branch, "S" corresponds to the stable solitons

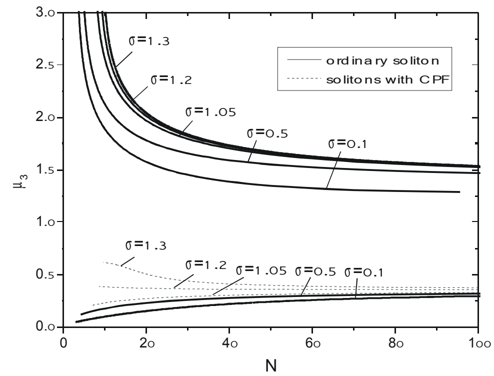

Fig. 2. Variational parameter μ_3 vs number of quanta N. Solids curves for ordinary solitons, dashed curves for solitons with CPF

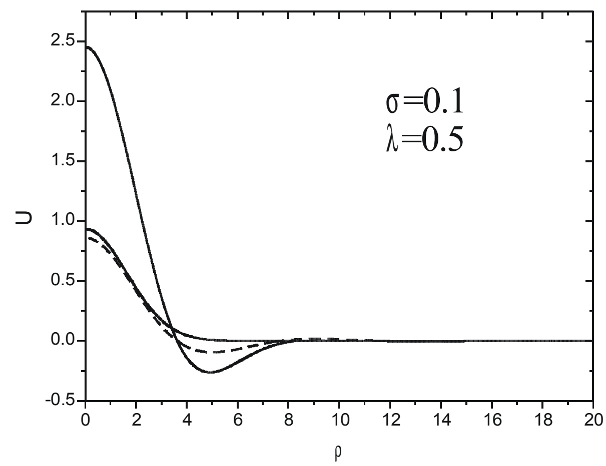

Fig. 3. Radial profiles of the ordinary solitons illustrating coexistence of solution with different spatial scales. Solid curves correspond to variational results, dashed curve – to numerical solution

This prediction has been confirmed by means of more general analysis with trial function (12) having three variational parameters, which confirmed, that in our problem, where there are two different spatial scales of oscilations constructed from coefficients: "linear" $\sqrt{P/D}$ and "nonlinear" $\sqrt{C/B}$. The results of the analysis are presented in Fig. 2.

Stability of all obtained solutions have been verified by means of computer solution of nonstationary GNSE (2) with the initial condition of the form of perturbed soliton solution. Extensive series of simulations confirmed stability of the ordinary solitons and solitons with CPF even with respect to significan perturbations.

4. EXACT SOLUTION OF GNSE WITH CUBIC-QUINTIC AND NONLOCAL NONLINEARITY

It is remarkable, that for whistlers with frequency $\omega=\omega_c/2$ the second-order dispersive term vanishes (coefficient D is equal to zero). Therefore, the fourth-order dispersive term is crucial in this case. If such whistler wave beam propagates in CQ nonlinear media with nonlocal nonlinearity a novel soliton-like solution of Eq. (1), the so-called algebraic soliton, exists if $\hat{N}(|\Psi|^2) = B|\Psi|^2 + K|\Psi|^4 + C\Delta_\perp|\Psi|^2)$, $\hat{D} = P\Delta_\perp^2$ and $P>0$, $B<0$, $K>0$, $C<0$. In contrast with common solitons, which decay at infinity exponentially, this algebraic *exact* solution has asymptotic behavior of the form $\psi(r) \to h/(r/a)^2$. It can be straightforwardly verified that stationary solution (5) with radial profile

$$\psi(r) = \frac{h}{1+(r/a)^2}, \tag{13}$$

is an exact solution of the GNSE if D=0, $\lambda = 0$, and $C^2/PK = 3$. In this case amplitude at the soliton center and characteristic soliton width are defined as follows:

$$h = \sqrt{3|B|/K}, \quad a = \sqrt{8C/3|B|}.$$

Note, that algebraic soliton has zero nonlinear frequency shift and appears exceptionally under *combined* influence of fourth-order dispersive term, CQ an nonlocal nonlinearities.

Direct numerical simulation of the evolution (along axis of propagation) of nonstationary GNSE with an initial profile of the form of algebraic soliton (13) has shown that such wave packet is unstable with respect to collapse. Wave packet envelope contracts, amplitude increases monotonically with z. Note, that the sign of the coefficient C is opposite here ($PC<0$) in comparison with the case considered in Sec. 2 and Sec. 3. This situation corresponds to a strong focusing effect on sufficiently intense wave packet. However, the stability of the algebraic soliton with respect to small perturbations and possibility of its collapse needs more thorough investigation. This problem will be the subject of our future investigation.

5. APPLICATION TO THE PROBLEM OF WHISTLER WAVE SELF-FOCUSING

In linear approximation whistler wave propagation along an external magnetic field can be described by the set of equations for parallel component of electric and magnetic fields:

$$\left(\Delta_\perp + \frac{\omega^2}{c^2}\varepsilon_\perp - k_z^2\right)B_z + \frac{i\omega g}{ck_z}\left(\Delta_\perp + \frac{\omega^2}{c^2}\varepsilon_\parallel\right)E_z = 0 \tag{14}$$

$$\left[-k_z^2\varepsilon_\parallel + \varepsilon_\perp\left(\Delta_\perp + \frac{\omega^2}{c^2}\varepsilon_\parallel\right)\right]E_z + \frac{ik_z\omega}{c}gB_z = 0, \tag{15}$$

where electric and magnetic fields are assumed to be of the form $E(x,y)e^{-i\omega t+ik_z z}$, $B(x,y)e^{-i\omega t+ik_z z}$, and

$$\varepsilon_\perp = -\omega_{pe}^2/(\omega^2-\omega_c^2), \varepsilon_\parallel = -\omega_{pe}^2/\omega^2,$$

$$g = -(\omega_c/\omega)\omega_{pe}^2/(\omega^2-\omega_c^2)$$

are the components of dielectric tensor in cold plasma approximation. For a plane wave with $k_\perp = 0$ the set (14), (15) gives two independent dispersion relations:

$$k_z^2 - \frac{\omega^2}{c^2}\varepsilon_\perp = \pm\frac{\omega^2}{c^2}g, \tag{16}$$

though in the frequency region of whistler waves $\omega_{pi} < \omega < \omega_{Ce}$ only the left hand polarized wave [with negative sign in the left hand side of (16)] describes plane whistler wave with dispersion

$$\omega = \frac{c^2k_z^2\omega_{Ce}}{\omega_{pe}^2 + c^2k_z^2}. \tag{17}$$

The other "wave" has $k_z^2 < 0$. Excluding B_z from the set (15), (16) we obtain one partial differential equation for the parallel component of the electric field E_z:

$$\left\{\left(-k_z^2 + \frac{\varepsilon_\parallel}{\varepsilon_\perp}k_z^2 + \frac{\omega^2}{c^2}(\varepsilon_\perp+\varepsilon_\parallel) - \frac{\omega^2}{c^2}\frac{g^2}{\varepsilon_\perp}\right)\Delta_\perp + \right.$$
$$\left. + \Delta_\perp^2 + \frac{\varepsilon_\parallel}{\varepsilon_\perp}\left(\left(k_z^2 - \frac{\omega^2}{c^2}\varepsilon_\perp\right)^2 - \frac{\omega^4}{c^4}g^2\right)\right\}E_z = 0$$

One can verify that the coefficient before $\Delta_\perp E_z$ vanishes for $\omega=\omega_c/2$. Evidently, the term proportional to $\Delta_\perp^2 E_z$, becomes of great importance at $\omega \approx \omega_c/2$. At the same time, the role of nonlinear effects also increases. The main nonlinear effect is connected with a variation of electron density, which in its turn causes a variation in refractive index, under intense wave propagation. In many plasmas such as in Earth ionosphere or in laboratory plasma, where intense whistler waves exist, the main nonlinear effect is the thermal effect: plasma density decreases during plasma heating. As was shown experimentally [1], density perturbation $\delta n/n$ in steady-state regime is proportional to $\delta T/T = \theta$ with opposite sign: $\delta n/n=-\gamma\,\theta$ where coefficient γ is of order 1. In local approximation electron temperature grows with electric field intensity as [16]

$$\theta = \frac{\nu^2}{2}\left(\sqrt{1+\frac{4}{\nu^2}\left(E/E_p\right)^2} - 1\right), \tag{18}$$

where

$$\nu^2 = (\omega^2+\nu_{0e}^2)/\nu_{0e}^2, \quad E_p^2 = 3mT\alpha\,(\omega^2+\nu_{0e}^2)/e^2,$$

and $\alpha \sim m/M$ is a part of energy, which electron has lost (in average) during one collision. In the first order of nonlinearity Eq. (18) is reduced to

$$\theta = \left(E/E_p\right)^2 . \qquad (19)$$

At the same time the nonlocal nonlinear effect connected with thermal conductivity may be of importance, especially for rather thin wave beams. Then equation (19) takes the form [6]

$$\theta - \tau_e D_\perp \Delta_\perp \theta = \left(E/E_p\right)^2$$

where $D_\perp$ is the thermal diffusitivity across the external magnetic field, $D_\perp = 4.66 T_e / m_e \omega_c^2 \tau_e$ according to [15]. In weakly nonlocal limit the last equation is reduced to

$$\theta \approx (1 + 5 r_e^2 \Delta_\perp) E^2 / E_p^2 ,$$

where r_e is electron Larmor radius. This approach is justified because the wave beam radius is much larger then r_e.

To restore the equation for a temporal evolution of the wave in a plane, perpendicular to direction of propagation one need to put $\omega \to \omega_0 - i\partial/\partial t + \omega_{nl}$ into the last term of equation for E_z and express ω_{nl} through δn:

$$\omega_{ne}/\omega_0 = -\delta n/n = (1 + 5 r_e^2 \Delta_\perp) E^2/E_p^2 .$$

In such a way we obtain Eq. (1) with

$$D = \frac{\omega_0}{2\omega_{pe}}\left\{\left(\omega_0/2\omega_{pe}\right)^2 - 1\right\},\ P = \frac{1}{8}\left(\omega_0/\omega_{pe}\right)^3 ,$$

$$B = -\left(\omega_0/\omega_{pe}\right)^3 ,\ C = -5\left(\omega_0/\omega_{pe}\right)^3 r_e^2 .$$

For a problem of stationary wave self-focusing one should replace $\partial/\partial t$ by $\partial/\partial z$. In dispersion region under consideration we have $D>0$, $P>0$, $B<0$, $C<0$. Cubic nonlinear term acts as defocusing for large-scale structures (where the fourth order dispersion term is not essential) but as focusing for small-scale structures. The action of nonlocal term is opposite. It follows form the viraial relation obtained in [9] for evolution of effective wave packet width. Due to these features there may coexist two stable soliton branches describing large and small-scale soliton shapes in perpendicular plane. Our consideration explains experimentally observed stationary self-focusing of whistler waves in density troughs in normal dispersive region ($\omega<\omega_c/2$) which is possible only if forth order dispersion effect and higher order nonlinear effect (nonlocal thermal conductivity in the case under consideration) are taken into account simultaneously.

In summary, an existance of stable self-induced whistler waveguides with depressed density in normal dispersive regime ($\omega<\omega_c/2$) is theoretically explained. In the framework of the model, based on the fourth order GNSE with cubic and nonlocal nonlinearity by means of generalized variational analysis it is demonstrated, that two different soliton branches with the same number of quanta but different spatial scales coexist. For nonlinear media with significant nonlocality ($\sigma \geq 1$) a novel coherent structure is predicted – a stationary wave beam with curved wave front, which phase changes nonlinearly in the plane perpendicular to propagation direction. However, the intensity of this structure remains constant along wave beam propagation.

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