FEATURES OF DYNAMICS OF THE INSTABILITIES AT PRESENCE MULTIPLICATIVE FLUCTUATION

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Dynamics of systems, which parameters are subject to random influences, is considered. The random affect can be as external, and be caused by own chaotic dynamics of system. The important features of dynamics of such systems are allocated: their behavior has an intermittence character; in these systems (even linear) the properties characteristic for a stochastic resonance can be shown; instability, which are caused by noise, practically are not stabilized by nonlinearities.

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The real physical systems are subject to effect of fluctuations. There is class of systems, on which the effect of fluctuations results in development of parametrical instability. It happens when the fluctuations are multiplicative, i.e. in a random way change parameters of an investigated oscillatory system. Frequently multiplicative fluctuations result in that, all moment are unstable. And the increment of each following moment is more than previous. In this case process of instability has an intermittence character. It means, that the process is characterized seldom, but very intensive casual bursts. The probability of occurrence of such emissions is small. The following time intervals of this burst are casual. Let's note, that this case is a seldom example of systems dynamics, when the maximum moments get the concrete physical contents. Really, in common cases for the description of physical systems it is enough to know a behavior of two first moments.

We shall formulate the most important features of behavior of systems, instability in which induced by noise.

1. Dynamics of a linear system, which has an intermittence character is similar to nonlinear dynamics. As a simplest example on Fig.1 the dependence of amplitude of a linear oscillator is represented. The frequency is a subject to random disturbances. The equation of such oscillator is:

$$\ddot{x}+\omega^{2}\cdot\left(1+\xi(t)\right)\cdot x=0, \qquad (1)$$

where $\xi(t)$-white noise.

All moments of such oscillator (beginning with second) are unstable. The increments of each following are more than previous. To the equation (1) the analysis of large number of plasma systems is reduced. From Fig.1 it is visible, that dynamics of such linear oscillator has an intermittence character. This dynamics is similar to dynamics generated by nonlinear processes. To the equation (1) the analysis of the large number of plasma systems is reduced. All moments of such oscillator (since second) are unstable. Increments of each following moment it is more previous. From a Fig. 1 it is visible, that dynamics such linear oscillator has alternated character. Such feature of linear dynamics should be meant, as it is similar to dynamics caused by nonlinear processes.

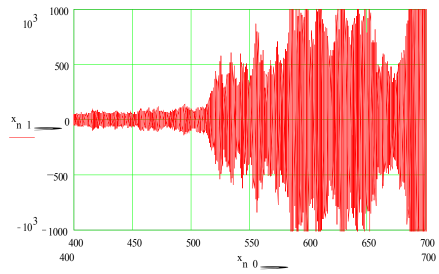

Fig.1. Example of the intermittence of the unstable linear oscillator

Let's give the brief proof of the formulated statements. For definiteness we shall consider, that the function ξ has the following properties:

$$\left\langle \xi(t)\right\rangle =0, \quad \left\langle \xi(t)\cdot\xi(t_1)\right\rangle =2D\delta\left(t-t_1\right) \qquad (2)$$

Here angular brackets mean statistical averaging on casual ensemble ξ . Using variation technique (look, for example, [1]) from the equation (1) it is possible to receive the following system of the $\left(n+1\right)$ ordinary differential equations for definition of the moments about displacement ($x(t)$) and speeds ($y=\dot{x}$):

$$\frac{d}{dt}\left\langle y^{p}\cdot x^{n-p}\right\rangle =-p\cdot\omega^{2}\left\langle y^{p-1}\cdot x^{n-p+1}\right\rangle + \\ +p\left(p-1\right)\cdot\omega^{4}D\left\langle y^{p-2}\cdot x^{n-p+2}\right\rangle + \\ +(n-p)\left\langle y^{p+1}\cdot x^{n-p-1}\right\rangle , \qquad (3)$$

where $p=0...n$.

Let's show that since the second moment, all moments are unstable. And increment of each following moment is more than previous. As is known [2], the behavior of such system is characterized by intermittence. This statement more easily to prove for unstable oscillator $\left(\omega^{2}<0\right)$.

For the proof it is convenient system of the equations to copy in a vector kind. For this purpose we shall enter function $Y_p=\left\langle y^{p}\cdot x^{n-p}\right\rangle$. Using this function, is possible to present system of the equations (3) as:

$$\frac{dY_p}{dt} = A_{p,i} \cdot Y_i \ , \qquad (4)$$

where $p, i = 0,1,2,.....n$;

$$A = \begin{vmatrix} 0 & n-1 & 0 & 0 \ ....0 & 0 & 0 \\ -\omega^2 & 0 & n-2 & 0. \ ....0 & 0 & 0 \\ 2\omega^4 D & -2\omega^2 & 0 & 0 \ ....0 & 0 & 0 \\ .... & .... & .... & .... & & \\ 0 & 0 & 0 & 0.......(n-1)(n-2)D\omega^4 & (n-1)\omega^2 & 0 \end{vmatrix}$$

Matrix A at $\omega^2 < 0$ is a non-negative matrix. Under the theorem of Perron [3] the greatest positive own meaning of such matrix grows at increase of any of elements $A_{p,i}$ of this matrix. Increase of fluctuation intensity D, and also increase number n of the moments, result in increase of these elements.

For a case, when $\omega^2 > 0$, the roots (λ_i) characteristic equation of system (3) can be easily determined at $D = 0$. In this case characteristic equation for moment with number n will look like

$$\prod_{i=1}^{n} \left(\lambda - \lambda_i \right) = 0 \qquad (5)$$

If fluctuation intensity not equal zero ($D \neq 0, \ D << 1$), than characteristic equation (5) we can to rewrite as:

$$\prod_{i=1}^{n} \left(\lambda - \mathsf{D}_i \right) = D \cdot P\left(\lambda \right) \qquad (6)$$

where $P\left(\lambda \right)$ -is unknown polynom.

The solution of the characteristic equation (6) is possible to search as $\mathsf{D}_i = \lambda_i + \delta$, where λ_i - are roots of the characteristic equation (5), δ - small deviation value of this roots, which arise as result the fluctuation appearing. The expressions for these additives are easy to find:

$$\delta \sim \frac{D \cdot P\left(\lambda \right)}{\prod_{i=1}^{n} \left(-\mathsf{D}_i \right)} \qquad (7)$$

From (3) it is possible to receive

for $p = 0$: $\left(\lambda + \delta \right) \left\langle x^n \right\rangle = n \cdot \left\langle y \cdot x^{n-1} \right\rangle \qquad (8)$

for $p = 1$:
$$\begin{aligned} &\left(\lambda + \delta \right) \left\langle y \cdot x^{n-1} \right\rangle = -\omega^2 \cdot \left\langle x^n \right\rangle + \\ &+ \left(n-1 \right) \cdot \left\langle y^2 \cdot x^{n-2} \right\rangle \end{aligned} \qquad (9)$$

for $p = 2$:
$$\begin{aligned} &\left(\lambda + \delta \right) \left\langle y^2 \cdot x^{n-2} \right\rangle = -2\omega^2 \cdot \left\langle y \cdot x^{n-1} \right\rangle + \\ &+ 2\omega^4 D \left\langle x^n \right\rangle + \left(n-2 \right) \cdot \left\langle y^3 \cdot x^{n-3} \right\rangle \end{aligned} . \qquad (10)$$

From equation (8) find $\left\langle y \cdot x^{n-1} \right\rangle$ and substitute it in (9):

$$\frac{\left(\lambda + \delta \right)^2}{n} \left\langle x^n \right\rangle + \omega^2 \cdot \left\langle x^n \right\rangle = \left(n-1 \right) \cdot \left\langle y^2 \cdot x^{n-2} \right\rangle . \qquad (11)$$

From (11) at $n >> 1$ we will receive:

$$\left\langle x^n \right\rangle = \frac{\left(n-1 \right)}{\omega^2} \cdot \left\langle y^2 \cdot x^{n-2} \right\rangle . \qquad (12)$$

Having substituted (12) in the equation (10) it is possible to write down:

$$\begin{aligned} &\left(\lambda + \delta \right) \left\langle y^2 \cdot x^{n-2} \right\rangle = -2\omega^2 \cdot \left\langle y \cdot x^{n-1} \right\rangle \\ &+ \left(n-2 \right) \cdot \left\langle y^3 \cdot x^{n-3} \right\rangle + 2\omega^2 \left(n-1 \right) D \left\langle y^2 \cdot x^{n-2} \right\rangle \end{aligned} \qquad (13)$$

From the equation (13) it is visible, that

$$\delta = 2\omega^2 \left(n-1 \right) \cdot D . \qquad (14)$$

Thus, we have shown, that addition (δ) to the roots of the characteristic equation (6) grows with increase number of the correlation moment (n), and consequently, increment of each following moment is more than previous.

2**.** It is important that the casual change of parameters can occur as a result of dynamic chaos development. An example of such dynamics is the charged particle movement in an external magnetic field and in a field of an external electromagnetic wave. As it is known [4], at enough large wave amplitude the movement of particles becomes chaotic. In this case the value of particle energy randomly varies. The energy of particles, which are moving in external constant magnetic field, is one of basic parameters for dynamics. The casual change of this parameter can result in development of instability with features, which we have described above.

As an example we shall consider the most simple configuration, when the external flat electromagnetic wave with components E_y, H_z is spread in a perpendicular direction to a vector of an external homogeneous constant magnetic field (H_0). This magnetic field is directed along an axis z . It is visible, that the polarization of a flat wave is such, that the component of a magnetic field of a wave is complanar to an external constant magnetic field. Except it we shall consider, that at the initial moment of time the particles has not the component of speed directed along a constant magnetic field ($v_z = 0$). In this case equations describing dynamics of particles in such fields, most simple and can be written as:

$$\begin{cases} \dot{x} = p_x / \gamma \\ \dot{p}_x = \omega_H \cdot \left[\alpha \cdot \cos\left(x - \tau \right) + 1 \right] \cdot p_x / \gamma \\ \dot{p}_\gamma = -\omega_H \cdot \left[\alpha \cdot \cos\left(x - \tau \right) + 1 \right] \cdot p_x / \gamma + \\ + H \cdot \cos(x - \tau) \\ p_z = 0 \end{cases} \qquad (15)$$

Where $\alpha \equiv H_{\omega} / H_0$ – ratio of wave magnetic field intensity to intensity of an external magnetic field; $\gamma^2 = 1 + p_x^2 + p_\gamma^2$, $\omega_H = eH_0 / mc \cdot \omega \cdot \gamma$

In (15) we used the following dimensionless variables: $x \to kx$, $\tau \to \omega t$, $p \to p/mc$, $H = eH_{\omega} / mc\omega$ dimensionless wave force parameter. At moderate wave field strengths the more effective interaction of a particle with a field, as is known, occurs at performance conditions for cyclotron resonances: $1 = n\omega_H$, $n = 1,2,3...$ The dynamics of the particles in these conditions was studied in work [4]. There was shown, that if the intensity of a field satisfied to an inequality

$$K \equiv \frac{4}{\gamma}\sqrt{H \cdot p_\perp \cdot J_n'(\mu)} << \omega_H , \qquad (16)$$

where, $p_\perp^2 = p_x^2 + p_\gamma^2$, $\mu = p_\perp / \gamma \cdot \omega_H$, $J_n(\mu)$ – nth-order Bessel function, J_n' - derivative of Bessel function on argument.

If (16) satisfy than the particles movement are limited by the isolated cyclotron resonances area on a phase plane. Dynamics of particles in these conditions are regular. If external wave field intensity is large enough, so that $K > \omega_H$, than cyclotron resonances are overlapped and dynamics of particles become chaotic.

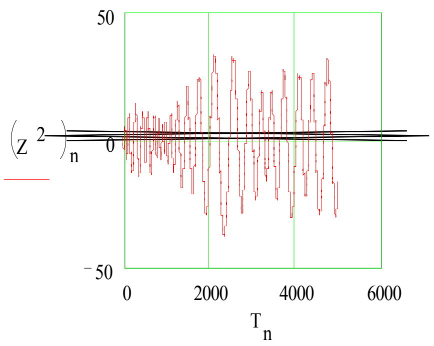

Fig. 2. Dependence of a perpendicular impulse on time

In all these cases dynamics of particles is well enough investigated. However, all received results are fair only for a case, when wave force parameter is small ($H << 1$). If such inequality is not carried out, it is possible only to tell about dynamics of a particle, that it will be chaotic. In details dynamics can be studied only by numerical methods.

In these conditions the system of the equations (15) was analyzed numerically at the following values of parameters: $\omega_H = 0.5$; $H = 0.9$; $\alpha = 1.8$. An example of particle impulse dynamics is represented on Fig. 2. It is visible, that it has typical picture for a alternation. This dynamic besides is characterized by local instability

3. The instability that was induced by noise has features of a stochastic resonance. It can be shown, for example, so. Let parameters of system vary simultaneously under the regular law and under noise law. Let, besides the period of regular perturbation is such, that parametric instability can develop. If the amplitude of periodic change of parameters is insufficiently great for threshold of instability achievement than the introduction of casual modulation of these parameters can result in occurrence of instability. Thus, the energy of external casual influence can promote development of instability in examined system.

As an example we shall consider oscillator dynamics, which is described by the following equation

$$\ddot{x} + \nu \cdot \dot{x} + \omega^2 \left[1 + \xi(t) + A\cos(2\omega \cdot t) \right] \cdot x = 0 . \qquad (17)$$

In this equation A - amplitude of regular parametrical influence, and the function $\xi(t)$ characterizes noise influence. Using this equation, it is easy to find such meanings of parameters, at which action of one regular parametrical force or the influence only noise force does not result in instability. However joint action of these two forces results in development of instability.

And, dynamics of this instability is characterized by intermittence. The large level of noise the large the level of intermittence. As an example in a fig. 3 the

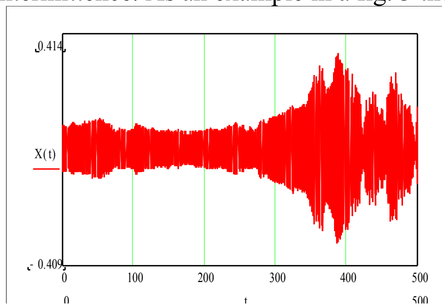

Fig. 3 Change of oscillator amplitude at simultaneous influence regular and Noise force ($\nu = 0,01$; $A = 0,003$; $A_N = 0,11$)

decision of the equation (16) is represented at $\nu = 0,01$; $A = 0,003$; $A_N = 0,11$. Here A_N - amplitude noise influence: $(\xi(nT) = rnd(2A_N) - A_N)$. The action only one regular parametrical forces do not result in development of instability. Really, the increment of parametrical instability $\Gamma \approx A/4$ is less than attenuation. Increment of instability, which develops under action of casual force also less attenuation.

It is necessary note, however, that can to be observed and boomerang effect, when the development of parametrical instability caused by action only of regular force, is broken at addition of casual force.

4. The multiplicate fluctuations play special role at acting on unstable systems. In this case they can by radical change dynamics of system. Practically always in these cases is realized regime with intermittence. Under this condition only on the certain time interval (or distance) regular dynamics is kept. Outside of this interval dynamics is chaotic. The large intensity of noise,

the shortly this interval. It is important, that of instability, that induced by noise, are not stabilized by nonlinearities. Really, in this case nonlinear shifting of frequency is compensated by presence of a wide spectrum of a noise signal.

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