ORIGIN OF TRANSVERSAL DISPLACEMENT OF THE PLASMA FLUX MOVING IN A CURVILINEAR MAGNETIC FIELD

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It is shown that transversal displacement of the plasma flux propagating across toroidal magnetic field, may be interpreted as a result of conversion (by means of Lorentz force) of the energy obtained by electrons in a field of polarization force and in a field of forces aroused due to magnetic field curvature, into kinetic energy of the transversal movement of the plasma as a whole. The same interpretation is valid also for the drift motion of a single particle.

PACS: 52.77.-J

1. INTRODUCTION

It is known that plasma jet (or plasmoid), after passing through a quarter torus magnetic duct, shifts a little (~1 cm) in the $\boldsymbol{B}\times\boldsymbol{R}$ direction [1–3], where $\boldsymbol{R}$ is a radius of the magnetic field $\boldsymbol{B}$ line of force, Fig.1.

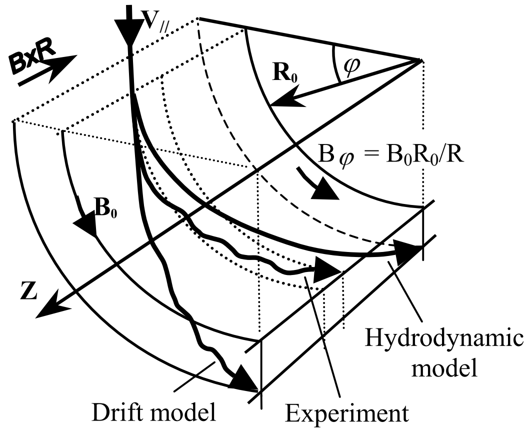

Fig.1

It should be noted that this shift is opposite to the plasma displacement predicted by a drift model [4–7] and it does not agree with the flux-tube model [8,9], which foresees only a radial displacement. An attempt to explain this shift by a simple entrainment of unmagnetized ions by the electrons with a speed of its electric ($\boldsymbol{E}\times\boldsymbol{B}$) drift in the radial polarization field E_R, gives too much value ~ 25 cm [10,11]. Furthermore, if the ions are magnetized, the model [10,11] gives no $\boldsymbol{B}\times\boldsymbol{R}$ displacement, while the experiment confirms its existence [1,5]. This contradiction have been overcame in [12,13], where the time scales compared with the electron Larmor period have been taken into account. During of such time intervals, the electron component of the plasma drifts mainly with a mean velocity $V_e = c\,F_e/eB$ perpendicular to some force F_e, whereas the ions, having a much greater Larmor period, pass only a small part of its Larmor orbit, deflecting in the direction to which the force F_i acts on ions. Relative displacement of the ion and electron components in the plasma moving along a curvilinear magnetic field gives rise to self-consistent polarization field E (see Fig. 2) with the components $E_\perp$ and E_R (parallel to $\boldsymbol{B}\times\boldsymbol{R}$ and to $\boldsymbol{R}$, correspondingly) which, in the coordinate system moving on the circular trajectory with the ions, depend approximately on the time t in the following way [12,13]:

$$E_\perp(t) \approx \left(\frac{M}{eZ}\frac{V_{i//}^2}{R} + \frac{2mV_{e//}^2}{eR}\right)\sin\left(\frac{eZ}{M}\frac{B}{\eta c}\right)t\,, \tag{1}$$

$$E_R(t) \approx \left(\frac{M}{eZ}\frac{V_{i//}^2}{R} + \frac{2mV_{e//}^2}{eR}\right)\cos\left(\frac{eZ}{M}\frac{B}{\eta c}\right)t - \frac{M}{eZ}\frac{V_{i//}^2}{R}\,, \tag{2}$$

where m and e are the mass and charge of electron, M and Z are the mass and degree of ionization of ion, $V_{e//}$ and $V_{i//}$ are the electron and ion velocities parallel to magnetic field $\boldsymbol{B}$, D is a dielectric constant of magnetized plasma, and $\eta = V_{i//}/V_{e//}$. Thus, the displacement of the centers of the ion and electron distributions from a circular trajectory can be easily calculated. In particular, taking into account that ion component shifts in the $\boldsymbol{B}\times\boldsymbol{R}$ direction under direct action of the polarization force $eZE_\perp$, and the electron one moves owing to electric, centrifugal, gradient, and polarization drift, we find for the average velocity $V_\perp$ [12,13]:

$$V_{\perp,i} \approx V_{\perp,e} = \frac{eZ}{M}\int_0^t E_\perp dt \approx \eta\left(-\frac{c}{B}E_R + \frac{2cmV_{e//}^2}{eRB} - \frac{m_e c^2}{eB^2}\frac{dE_\perp}{dt}\right) \approx$$

$$\approx \frac{\eta c}{B}\left(\frac{M}{eZ}\frac{V_i^2}{R} + \frac{2mV_{e//}^2}{eR}\right). \tag{3}$$

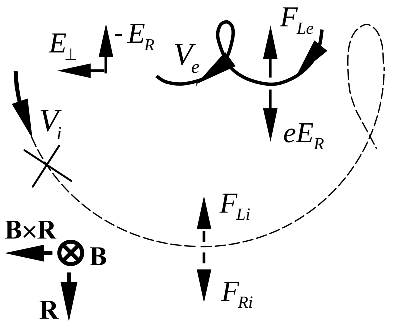

Fig. 2. Lorenz force F_{Le}, acting on the electrons, balances the centrifugal and polarization force $-eE_R$ much sooner than Lorenz force F_{Li}, acting on ions, would balance the centrifugal force acting on it

Substituting well-known parameters of the vacuum-arc plasma [2,8,14–16] into Eq. (3) and multiplying it by $\pi R/2V_{i//}$, we obtain for the displacement of the plasma jet at the exit of quarter torus magnetic duct the value ~1 cm [12,13] that agrees with experimental results [1–3].

In present work we intend to show: 1) velocity (3) can be derived in a different way, without use of the field

$E_\perp$; 2) plasma transversal drift in the $\boldsymbol{B}\times\boldsymbol{R}$ direction has a clear physical sense. Furthermore, the frameworks of applicability of the model proposed in [12,13] and the condition of plasma movement along the toroidal magnetic field are appreciated.

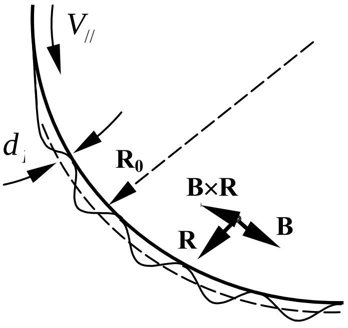

Fig. 3. Radial oscillation of the plasma jet (or a single particle) flying into the curvilinear magnetic field. d_R is a mean deflection of the jet (or the particle leading center) from the circular trajectory

2. ENERGY OF RADIAL DISPLACEMENT OF ELECTRONS

Note that in absence of any force, the leading centers of electrons would follow the magnetic field lines. Appearance of polarization field E_R [see Eq. (2)] and the forces concerned with a magnetic field curvature, cause a drift in the $\boldsymbol{B}\times\boldsymbol{R}$ direction and the mean radial displacement d_R [12,13] of electron leading centers (along with ions) from a circular trajectory with a radius $\mathbf{R_0}$ (see Fig. 3):

$$d_{R,i} \cong d_{R.e} \approx \frac{M}{eZ}\left(\frac{\eta c}{B}\right)^2\left(\frac{M}{eZ}\frac{V_i^2}{R}+\frac{2mV_{e//}^2}{eR}\right). \quad (4)$$

Since the deflection d_R is proportional to the force $F=\frac{M}{Z}\frac{V_i^2}{R}+\frac{2mV_{e//}^2}{R}$, each of electrons obtains the energy

$$W_\perp=\frac{1}{2}Fd_{R,e}\cong\frac{1}{2}\frac{M}{Z}\left(\frac{M}{Z}\frac{V_i^2}{R}+\frac{2mV_{e//}^2}{R}\right)^2\left(\frac{\eta c}{eB}\right)^2, \quad (5)$$

which is delivered through the Lorentz force and field $E_\perp$ to the mass M/Z (and to electron mass, which may be neglected), increasing the kinetic energy $MV_\perp^2/2Z$ of plasma transversal motion. Equating this energy to the right member of Eq. (5) and solving it for the velocity $V_\perp$, we find $V_\perp=\frac{\eta c}{eB}\left(\frac{M}{Z}\frac{V_i^2}{R}+\frac{2mV_{e//}^2}{R}\right)$ that coincides with the expression (3). From this it follows: 1) there is no necessity to know the field $E_\perp$ for determining of transversal displacement of central part of the plasma jet, it is enough to have only the field E_R and the mean radial displacement d_R, which can be easily calculated from expression $d_{R,i}\approx d_{R,e}=\int_0^\tau dt\int_0^t\left(\frac{V_i^2}{R}+\frac{eZ}{M}E_R\right)dt$ and Eq. (2) [12,13]; 2) the identity of expressions for the velocity $V_\perp$ obtained in a different ways is an additional evidence of the full self-consistency of the fields E_R and $E_\perp$ [see Eqs. (1), (2)] with a drift motion of electrons and direct motion of ions.

Note that the same interpretation is applicable to a single particle moving across curvilinear magnetic field. The particle (or its leading center) having a charge q and mass M, when flying into the toroidal magnetic field, experience (along with transversal drift motion) an average radial shift from a circular trajectory (see Fig. 3). This shift is $d_R\approx\rho^2_{//}/R$ (if $\rho_{//}<<R$), where $\rho_{//}=cMV_{//}/qB$ [4]. As this shift is proportional to the centrifugal force $d_R\approx M\left(\frac{c}{qB}\right)^2\frac{MV_{//}^2}{R}$, the particle obtains perpendicular energy $\frac{1}{2}Fd_R=\frac{M}{2}\left(\frac{c}{qB}\right)^2\left(\frac{MV_{//}^2}{R}\right)^2$, which transforms by means of the Lorentz force (see the electron trajectory on Fig. 2) into kinetic energy $MV_\perp^2/2$ of a drift motion. Solving equation $\frac{MV_\perp^2}{2}=\frac{M}{2}\left(\frac{c}{qB}\right)^2\left(\frac{MV_{//}^2}{R}\right)^2$ for the velocity $V_\perp$, we find $V_\perp=cMV_{//}^2/qBR$, i.e. the velocity of centrifugal drift. It is clear that this simple example can be easily extended on a case of arbitrary force.

To estimate a range of validity of the model [12,13] and this analysis too, let us take into consideration that the ion may drift perpendicular to some force F_i, if this force is equalized mainly by the Lorentz force (as it is shown on Fig. 2 for the electron trajectory). This condition is satisfied, when the deflection $d_R\cong\rho^2_{//}/R$ of a single ion is lesser than a charge separation parameter δ in a plasma. This parameter is $\delta=E_R/2\pi en_0$ [12,13], if the plasma density profile is Gaussian. Then, accounting Eq. (2), we can write $\delta=E_R/2\pi en_0=MV_{i//}^2/2\pi Ze^2n_0R>$ $>\rho_{//}{}^2/R$, and find an estimation: $B>c\sqrt{2\pi Mn_0/Z}$, or $B>0.1\sqrt{n_0\ A/Z}$ (in Gauss), where A is a relative atomic weight, and n_0 is the plasma density on the axis of the plasma flux. So, if $B/\sqrt{n_0}<0.1\sqrt{A/Z}$, the ion centrifugal drift in a plasma is impossible, because the centrifugal force is balanced by a polarization force during much lesser time than the ion Larmor period. For a typical vacuum-arc plasma density in filters, $n\sim10^{12}\,cm^{-3}$ [2,3,7,10,11], ions may experience the centrifugal drift, if the magnetic field strength would exceed $5\cdot10^5$ G. At the usual 150–600 G in filtering devices, the drift of ions may take place, if the plasma density would not exceed $10^5\,cm^{-3}$. So, the range of applicability of this consideration includes all practically important cases.

3. DISCUSSION

As it follows from Eq. (5), on entering at the curvilinear path, the part $W_\perp$ of the initial plasma kinetic energy $W_{//}$ begins to circulate in the cross-section of the plasma flux. This perpendicular energy is approximately equal to $W_\perp\approx(\eta\rho_{//}/R)^2W_{//}$ [see Eq. (5)] and appears as a result of centrifugal acceleration of the ions. In the first stage, the energy $W_\perp$ is absorbed by electrons during of its displacement in the radial polarization field E_R. At the second stage, this energy is passed to the ions through the Lorentz force and positive polarization field $E_\perp$ [see Eq. (1) and Fig. 2)], which accelerate ions in the $\boldsymbol{B}\times\boldsymbol{R}$ direction. As soon as transversal velocity of the ions passes via a maximum, field $E_\perp$ becomes negative [see Eq. (1)]. In this field electrons experience electric drift opposite to $\boldsymbol{R}$, taking away the energy $W_\perp$ from the ions. This last drift returns the ion component (through the field $-E_R$, see Fig. 2) on the circular trajectory. At the moment of return, the ions (and plasma as a whole) fully lose its transversal energy and then a cycle repeats itself again.

Thus, *the transversal movement of the plasma in the* $\boldsymbol{B}\times\boldsymbol{R}$ *direction is only a link in the chain of transformations of the energy* $W_\perp \approx (\eta\rho_{//}/R)^2 W_{//}$.

Since always $W_\perp < W_{//}$, the inequality $\eta\rho_{//}/R << 1$ is a condition of plasma movement along the toroidal magnetic field. In reality, at the point of entry, plasma *always* moves straightforward, on tangent to the magnetic field line, and the question only consist in: how long this motion does continue (see Fig. 3). Evidently, its duration is proportional to $W_\perp/W_{//}$, which grows with the radial displacement d_R or ratio $MV_{//}/qB$. In more accurate approximation, the coefficient at $W_{//}$ in Eq. $W_\perp \approx (\eta\rho_{//}/R)^2 W_{//}$ is modified and asymptotically approaches to one, if the ratio $MV_{//}/qB$ increases. Nevertheless, the approximate condition $\eta\rho_{//}/R < 1$ shows too that vacuum-arc plasma may be guided by essentially weak magnetic field (of a few dozens of Gauss [2]) and, in this case, the plasma loss in curved magnetic filters occurs due to a large characteristic radius of the plasma flux rather [17] than because of its radial displacement.

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Article received 22.09.08.

ПРОИСХОЖДЕНИЕ ПОПЕРЕЧНОГО СМЕЩЕНИЯ ПОТОКА ПЛАЗМЫ, ДВИЖУЩЕГОСЯ ВДОЛЬ КРИВОЛИНЕЙНОГО МАГНИТНОГО ПОЛЯ

А.И. Тимошенко

Показано, что поперечное смещение плазменного потока может быть интерпретировано как результат преобразования (посредством силы Лоренца) энергии, полученной электронами в поле поляризационной силы и в поле сил, связанных с кривизной магнитного поля, в кинетическую энергию поперечного движения всей плазмы. Такая же интерпретация справедлива и для дрейфового движения отдельной частицы.

ПОХОДЖЕННЯ ПОПЕРЕЧНОГО ЗМІЩЕННЯ ПОТОКУ ПЛАЗМИ, ЩО РУХАЄТЬСЯ ВЗДОВЖКРИВОЛІНІЙНОГО МАГНІТНОГО ПОЛЯ

О.І. Тимошенко

Показано, що поперечне зміщення плазмового потоку може бути інтерпретовано як результат перетворення (за допомогою сили Лоренца) енергії, отриманої електронами в полі поляризаційної сили і в полі сил, пов'язаних з кривизною магнітного поля, в кінетичну енергію поперечного руху всієї плазми. Така ж інтерпретація справедлива і для дрейфового руху окремої частинки.