

MAGNETIC MONOPOLE AND CP SYMMETRY VIOLATION AT FINITE TEMPERATURE

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An ideal gas of relativistic massive electrons in the background of a Dirac magnetic monopole is considered. We find that in the case of CP symmetry violation this system acquires, in addition to charge, also squared orbital angular momentum, squared spin, and squared total angular momentum. The functional dependence of these quantities on the temperature and the CP-violating vacuum angle is determined. Thermal quadratic fluctuations of conserved quantities are examined, and we analyze, when charge and squared total angular momentum become sharp quantum observables rather than mere expected averages of many quantum measurements.

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1. INTRODUCTION

The interaction of quantized Dirac fermion fields with classical background fields of nontrivial topology can give rise to quantum states with rather unusual eigenvalues [1-3]. In particular, the ground state of a Dirac electron in the background of a Dirac magnetic monopole acquires nonzero electric charge, and this results in the monopole becoming a CP symmetry violating dyon [4-6]. The effect persists when thermal fluctuations of the quantized Dirac electron field are taken into account, and this yields temperature dependence of the induced charge [7-9].

The aim of the present paper is to show that, in addition to charge, also other quantum numbers are induced in the magnetic monopole background both at zero and nonzero temperatures. We find relationships between all these quantum numbers and discuss, which of them become sharp quantum observables rather than quantum averages and also when this happens. At nonzero temperature all quantum numbers are not sharp observables, but, instead, are thermal averages; and, appropriately, the thermal quadratic fluctuations are nonvanishing. If a quadratic fluctuation vanishes at zero temperature, then a corresponding quantum number at zero temperature becomes a sharp observable. We find out, in particular, that induced charge and squared total angular momentum at zero temperature are sharp observables for almost all values of the vacuum angle with the exception of the one corresponding to zero energy of the bound state in the one-particle electron spectrum.

2. OPERATORS OF PHYSICAL OBSERVABLES AND THEIR VACUUM AND THERMAL EXPECTATION VALUES

In the second-quantized theory, the operator of a dynamical variable (physical observable) is given by the integrated commutator of the electron field operators,

$$\hat{O}_Y = \frac{1}{2} \int d^3r \operatorname{tr} \left[\Psi^+(\vec{r}, t), Y \Psi(\vec{r}, t) \right]_-, \quad (1)$$

where Y is the appropriate one-particle operator in the first-quantized theory, and tr denotes the trace over spinor indices; in particular, \hat{O}_H is the operator of energy, where H is the one-particle Hamiltonian, and \hat{O}_{eI} is the operator of charge, where I is the unity matrix in spinor indices, and e is the electron charge. The vacuum expectation value of the observable corresponding to Eq. (1) can be presented as

$$\langle \operatorname{vac} | \hat{O}_Y | \operatorname{vac} \rangle = -\frac{1}{2} \operatorname{Tr} Y \operatorname{sgn}(H), \quad (2)$$

where Tr is the trace of an integro-differential operator in the functional space: $\operatorname{Tr} U = \int d^3\vec{r} \operatorname{tr} \langle \vec{r} | U | \vec{r} \rangle$. The thermal expectation value of the observable is conventionally defined as (see, e.g., Ref. [10])

$$O_Y(T) = \langle \hat{O}_Y \rangle_\beta \equiv \frac{\operatorname{Sp} \hat{O}_Y \exp(-\beta \hat{O}_H)}{\operatorname{Sp} \exp(-\beta \hat{O}_H)}, \quad (3)$$

$$\beta = (k_B T)^{-1},$$

where T is the equilibrium temperature, k_B is the Boltzmann constant, and Sp is the trace or the sum over the expectation values in the Fock state basis in the second-quantized theory. Evidently, the zero-temperature limit of Eq. (3) coincides with Eq. (2):

$$O_Y(0) = \langle \operatorname{vac} | \hat{O}_Y | \operatorname{vac} \rangle. \quad (4)$$

Thus, Eq. (3) can be presented in a way similar to that of Eq. (2), i.e., through the functional trace of operators in the first-quantized theory, see, e.g., Ref. [11],

$$O_Y(T) = -\frac{1}{2} \operatorname{Tr} Y \tanh \left(\frac{1}{2} \beta H \right). \quad (5)$$

The self-adjointness of the one-particle Hamiltonian ensures the conservation of energy in the second-quantized theory, and the corresponding operator is diagonal in creation and destruction operators; the operator of any other conserved observable is diagonal as well.

If an observable is conserved, then its thermal quadratic fluctuation,

$$\Delta(T; \hat{O}_Y) = \langle \hat{O}_Y^2 \rangle_\beta - \left(\langle \hat{O}_Y \rangle_\beta \right)^2,$$

takes form

$$\Delta(T; \hat{O}_Y) = \frac{1}{4} \text{Tr } Y^2 \text{sech}^2 \left(\frac{1}{2} \beta H \right). \quad (6)$$

Eqs. (5) and (6) are transformed into integrals over the energy spectrum

$$O_Y(T) = -\frac{1}{2} \int_{-\infty}^{\infty} dE \tau_Y(E) \tanh \left(\frac{1}{2} \beta E \right) \quad (7)$$

and

$$\Delta(T; \hat{O}_Y) = \frac{1}{4} \int_{-\infty}^{\infty} dE \tau_{Y^2}(E) \text{sech}^2 \left(\frac{1}{2} \beta E \right), \quad (8)$$

where

$$\tau_Y(E) = \text{Tr } Y \delta(H - E) \quad (9)$$

and

$$\tau_{Y^2}(E) = \text{Tr } Y^2 \delta(H - E) \quad (10)$$

are the appropriate spectral densities.

3. DIRAC ELECTRON IN THE DIRAC MONOPOLE BACKGROUND

Since the monopole mass is estimated to be much heavier (by two orders) than the electron mass, it is reasonable to adopt a viewpoint according to which the monopole only provides a background where the electron moves. Within this approach we consider quantization of the electron field in the background of a classical monopole field configuration.

3.1. ONE-PARTICLE OPERATORS

A configuration of a pointlike monopole with magnetic charge g at the origin is given by the field strength in the form

$$B(\vec{r}) = g \frac{\vec{r}}{r^3}, \quad \vec{\partial} \cdot \vec{B}(\vec{r}) = 4\pi g \delta^3(\vec{r}). \quad (11)$$

Following Wu and Yang [12], one can divide space into two overlapping regions, $R_a: 0 < \vartheta < \frac{\pi}{2} + \delta$, and

$R_b: \frac{\pi}{2} - \delta < \vartheta < \pi$ (here $0 \leq \vartheta \leq \pi$ stands for the azimuthal angle in spherical coordinates, $x = r \sin \vartheta \cos \phi$, $y = r \sin \vartheta \sin \phi$, $z = r \cos \vartheta$, and $0 < \delta < \frac{\pi}{2}$), and define the patched vector potential:

$$\vec{A}(\vec{r}) d\vec{r} = \begin{cases} g(1 - \cos \vartheta) d\phi, & \vec{r} \in R_a, \\ -g(1 + \cos \vartheta) d\phi, & \vec{r} \in R_b, \end{cases} \quad (12)$$

then $\vec{\partial} \times \vec{A} = \vec{B}$, where \vec{B} is given by Eq. (11). In the overlap

$R_{ab}: \frac{\pi}{2} - \delta < \vartheta < \frac{\pi}{2} + \delta$, the two potentials are related by gauge transformation

$$\vec{A}|_a = \vec{A}|_b + \frac{i}{e} S_{ab} \vec{\partial} S_{ab}^{-1}, \quad (13)$$

with

$$S_{ab} = e^{2ieg\phi}, \quad (14)$$

Therefore, the vector potential serves as a connection on a nontrivial $U(1)$ bundle, and the electron wave function is a section of this bundle, i.e. wave function $\Psi(\vec{r}, t)$ is two-valued with its values in the overlap R_{ab} related by gauge transformation

$$\Psi|_a = S_{ab} \Psi|_b. \quad (15)$$

Generating function S_{ab} (14) is existing (i.e. single-valued) only when

$$eg = \frac{1}{2} n, \quad n \in \mathbb{Z}, \quad (16)$$

which is the celebrated Dirac quantization condition [13] that has already attained its 75-year anniversary.

The Dirac Hamiltonian in the background of a static magnetic monopole takes form

$$H = -\gamma^0 \vec{\gamma} \cdot (i\vec{\partial} + e\vec{A}) + \gamma^0 M, \quad (17)$$

where γ^0 and $\vec{\gamma}$ are the Dirac matrices, M is the electron mass, and \vec{A} is given by Eq. (12). The magnetic monopole background is rotationally invariant and three generators of rotations are identified with the components of vector \vec{J} – the operator of total angular momentum in the first-quantized theory,

$$\vec{J} = \vec{\Lambda} + \vec{\Sigma}, \quad (18)$$

where

$$\vec{\Lambda} = -\vec{r} \times (i\vec{\partial} + e\vec{A}) - eg \frac{\vec{r}}{r} \quad (19)$$

is its orbital part, and

$$\vec{\Sigma} = \frac{i}{4} \vec{\gamma} \times \vec{\gamma} \quad (20)$$

is its spin part; note that the last term in Eq.(19) is necessary in order to ensure the correct commutation relations:

$$[J^j, J^k]_- = i\epsilon^{jkl} J^l.$$

3.2. SELF-ADJOINT EXTENSION AND CP SYMMETRY VIOLATION

A solution to the stationary Dirac equation

$$H \langle \vec{r} | E, j, m \rangle = E \langle \vec{r} | E, j, m \rangle \quad (21)$$

for the lowest partial wave with $j = |eg| - \frac{1}{2}$ cannot be chosen to be regular at the location of the monopole. The procedure of the self-adjoint extension is imple-

mented for the corresponding partial Hamiltonian, yielding boundary condition [14, 15]

$$\begin{aligned} & \cos\left(\frac{\Theta}{2} + \frac{\pi}{4}\right) \lim_{r \rightarrow 0} r f(r) \\ & = i \operatorname{sgn}(eg) \sin\left(\frac{\Theta}{2} + \frac{\pi}{4}\right) \lim_{r \rightarrow 0} r g(r), \end{aligned} \quad (22)$$

where $f(r)$ and $g(r)$ are the radial functions of the upper and lower components of the wave function,

$$\langle \bar{r} | E, |eg| - \frac{1}{2}, m \rangle = \begin{pmatrix} f(r) & \eta_m(\vartheta, \varphi) \\ g(r) & \eta_m(\vartheta, \varphi) \end{pmatrix}, \quad (23)$$

$$\eta_m(\vartheta, \varphi) = \begin{pmatrix} -\sqrt{\frac{|eg| - m + 1/2}{2|eg| + 1}} Y_{|eg|, m-1/2}(\vartheta, \varphi) \\ \sqrt{\frac{|eg| + m + 1/2}{2|eg| + 1}} Y_{|eg|, m+1/2}(\vartheta, \varphi) \end{pmatrix}, \quad (24)$$

$Y_{|eg|, m \mp 1/2}(\vartheta, \varphi)$ are the appropriate monopole harmonics [16], $m = -|eg|, -|eg| + 1, \dots, |eg|$, and Θ is the self-adjoint extension parameter that plays the role of the vacuum angle in Witten's approach [4]. It should be emphasized that CP symmetry is violated, unless $\Theta = n\pi$.

In the case of $\cos \Theta < 0$, there exists, in addition to continuum states with energies $E > M$ and $E < -M$, also a bound state with energy $E_{BS} = M \sin \Theta$:

$$\begin{aligned} \langle \bar{r} | E_{BS}, m \rangle & = \begin{pmatrix} i \operatorname{sgn}(eg) \sin\left(\frac{\Theta}{2} + \frac{\pi}{4}\right) & \eta_m(\vartheta, \varphi) \\ \cos\left(\frac{\Theta}{2} + \frac{\pi}{4}\right) & \eta_m(\vartheta, \varphi) \end{pmatrix} \\ & \times \frac{1}{r} \sqrt{-2M \cos \Theta} e^{Mr \cos \Theta}. \end{aligned} \quad (25)$$

4. INDUCED QUANTUM NUMBERS IN THE DIRAC MONOPOLE BACKGROUND

In the standard representation for the Dirac matrices operators $\bar{\Lambda}$ (19) and $\bar{\Sigma}$ (20) are of the block-diagonal form,

$$\Upsilon = \begin{pmatrix} \Omega & 0 \\ 0 & \Omega \end{pmatrix}, \quad (26)$$

and contain no derivatives in r :

$$\Omega h(r) \xi(\vartheta, \varphi) = h(r) \Omega \xi(\vartheta, \varphi). \quad (27)$$

It can be shown that, when conditions (26) and (27) are satisfied, the contribution of partial waves with $j > |eg| - \frac{1}{2}$ to spectral density (9) is even in energy, and, thus, these waves do not contribute to thermal expectation value (7). The contribution of the lowest partial wave to spectral density (9) is calculated to be

$$\begin{aligned} \tau_Y(E) & = \sum_{m=-|eg|}^{|eg|} \frac{2\pi}{0} \int_0^\pi d\phi \int_0^\pi d\vartheta \sin \vartheta \eta_m^\dagger \Omega \eta_m \\ & \times \left[\theta(-\cos \Theta) \delta(E - M \sin \Theta) - \frac{1}{4} \delta(E - M) \right. \\ & \quad \left. - \frac{1}{4} \delta(E + M) + \frac{\cos \Theta}{2\pi} \frac{\operatorname{sgn}(E)}{E - M \sin \Theta} \right. \\ & \quad \left. \times \frac{M}{\sqrt{E^2 - M^2}} \theta(E^2 - M^2) \right]. \end{aligned} \quad (28)$$

Using the last relation, we get the following expression for the thermal expectation value (7) of the observable which corresponds to an operator satisfying conditions (26) and (27):

$$\begin{aligned} O_Y(T) & = -\frac{1}{2} \sum_{m=-|eg|}^{|eg|} \frac{2\pi}{0} \int_0^\pi d\phi \int_0^\pi d\vartheta \sin \vartheta \eta_m^\dagger \Omega \eta_m \times \\ & \times \left[\theta(-\cos \Theta) \tanh\left(\frac{1}{2} \beta M \sin \Theta\right) \right. \\ & \quad \left. + \frac{\sin 2\Theta}{2\pi} \int_1^\infty \frac{dw}{\sqrt{w^2 - 1}} \frac{\tanh\left(\frac{1}{2} \beta M w\right)}{w^2 - \sin^2 \Theta} \right]. \end{aligned} \quad (29)$$

The integration over angular variables with summation over m can be performed using the orthonormality of η_m 's. In particular, one gets immediately

$$O_{\bar{\Lambda}}(T) = O_{\bar{\Sigma}}(T) = O_{\bar{J}}(T) = 0, \quad (30)$$

and, thus, rotational symmetry is not spontaneously broken. In the case of $Y = eI$, one gets induced charge [7-9]

$$\begin{aligned} O_{eI}(T) & = -e |eg| \left[\theta(-\cos \Theta) \tanh\left(\frac{1}{2} \beta M \sin \Theta\right) \right. \\ & \quad \left. + \frac{\sin 2\Theta}{2\pi} \int_1^\infty \frac{dw}{\sqrt{w^2 - 1}} \frac{\tanh\left(\frac{1}{2} \beta M w\right)}{w^2 - \sin^2 \Theta} \right]; \end{aligned} \quad (31)$$

note that the last expression at $|eg| = \frac{1}{2}$ coincides with the expression for charge which is induced in 2+1-dimensional space-time at finite temperature by a point-like magnetic vortex with flux $\pi \bmod 2\pi$ [17].

All other nonvanishing quantum numbers are related to Eq. (31): squared orbital angular momentum

$$O_{\Lambda^2}(T) = |eg| (|eg| + 1) e^{-1} O_{eI}(T), \quad (32)$$

squared spin

$$O_{\Sigma^2}(T) = \frac{3}{4} e^{-1} O_{eI}(T), \quad (33)$$

and squared total angular momentum

$$O_{J^2}(T) = \left[(eg)^2 - \frac{1}{4} \right] e^{-1} O_{eI}(T). \quad (34)$$

Note that charge tends to finite value at zero temperature

$$O_{eI}(0) = -2e |eg| \frac{1}{\pi} \arctan\left(\tan \frac{\Theta}{2}\right), \quad (35)$$

and vanishes in the high-temperature limit as inverse temperature

$$O_{eI}(T \rightarrow \infty) = -\frac{e}{4} |eg| \beta M \sin \Theta. \quad (36)$$

Thus, one can conclude that at CP-violating values of the vacuum angle (i.e. at $\Theta \neq n\pi$) both charge and the squares of orbital angular momentum, spin, and total angular momentum are induced at finite (zero and non-zero) temperatures.

5. QUANTUM EIGENVALUES OR QUANTUM EXPECTED AVERAGE VALUES

The squares of orbital angular momentum and spin are nonconserved observables, so their values both at zero and nonzero temperatures should be regarded as expected averages of many quantum measurements. The conserved observables are charge and squared total angular momentum; note that the latter vanishes in the case of the minimal monopole strength $|eg| = 1/2$.

We analyze thermal correlations between conserved and nonconserved observables and thermal quadratic fluctuations of conserved observables, and find out that these quantities at nonzero temperature are given by the ideal gas expressions, and, thus, are Θ -independent and proportional to the powers of spatial volume. For example, we list here the expressions for the quadratic fluctuations of charge,

$$\Delta(T; \hat{O}_{eI}) = \frac{e^2}{4\pi^2} \frac{V}{\beta^3} \int_{\beta^2 M^2}^{\infty} ds \frac{(s - \beta^2 M^2)^{1/2}}{\cosh^2\left(\frac{1}{2}\sqrt{s}\right)}, \quad (37)$$

and squared total angular momentum,

$$\Delta(T; \hat{O}_{J^2}) = \frac{2}{35\pi^2} \left(\frac{3}{4\pi}\right)^{4/3} \frac{V^{7/3}}{\beta^7} \int_{\beta^2 M^2}^{\infty} ds \frac{(s - \beta^2 M^2)^{5/2}}{\cosh^2\left(\frac{1}{2}\sqrt{s}\right)}. \quad (38)$$

Note that in the high-temperature limit Eqs. (37) and (38) increase as T^3 and T^7 . Thus, the values of charge and squared total angular momentum at nonzero temperature should be regarded as expected averages of many quantum measurements, since the corresponding thermal quadratic fluctuations are nonvanishing.

However, interaction with the monopole background reveals itself at zero temperature, yielding a Θ -dependence of a specific type, which is due to a possibility of appearance of a bound state with zero energy in the one-particle electron spectrum, i.e. at $\cos\Theta < 0$ and $\sin\Theta = 0$, see Eq. (25):

$$\Delta(0; \hat{O}_{eI}) = \begin{cases} 0, & \Theta \neq \pi \bmod 2\pi \\ \frac{e^2}{2} |eg|, & \Theta = \pi \bmod 2\pi \end{cases} \quad (39)$$

and

$$\Delta(0; \hat{O}_{J^2}) = \begin{cases} 0, & \Theta \neq \pi \bmod 2\pi \\ \frac{1}{2} |eg| \left[(eg)^2 - \frac{1}{4} \right]^2, & \Theta = \pi \bmod 2\pi \end{cases}. \quad (40)$$

This fact has immediate consequences when we turn to a question: whether the values of charge and squared total angular momentum at zero temperature are observed in a single quantum measurement, or whether they are to be regarded as expected averages of many such measurements. As it follows from Eqs. (39) and (40), charge and squared total angular momentum are sharp observables (quantum-mechanical eigenvalues), unless $\Theta = \pi \bmod 2\pi$. Thus, CP-conserving values of the vacuum angle, $\Theta = n\pi$, differ significantly. In the case of $\Theta = 2n\pi$, zero charge and zero squared total angular momentum are observed in a single quantum measurement. In the case of $\Theta = (2n+1)\pi$, only zero squared total angular momentum at $|eg| = 1/2$ is a sharp quantum observable, while zero charge at $eg \neq 0$ and zero squared total angular momentum at $|eg| > 1/2$ are expected average values of many quantum measurements.

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МАГНИТНЫЙ МОНОПОЛЬ И НАРУШЕНИЕ СР-СИММЕТРИИ ПРИ КОНЕЧНОЙ ТЕМПЕРАТУРЕ

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Рассматривается идеальный газ релятивистских массивных электронов в присутствии магнитного монополя Дирака. Обнаружено, что при нарушении СР-симметрии в этой системе наряду с зарядом возникают также квадрат орбитального углового момента, квадрат спина и квадрат полного углового момента. Определена функциональная зависимость этих величин от температуры и вакуумного угла, нарушающего СР-симметрию. Исследованы температурные квадратичные флуктуации сохраняющихся величин, и установлено, когда заряд и квадрат полного углового момента становятся точными квантовыми наблюдаемыми, а не просто ожидаемыми средними по многим квантовым измерениям.

МАГНІТНИЙ МОНОПОЛЬ ТА ПОРУШЕННЯ СР-СИМЕТРІЇ ПРИ СКІНЧЕНІЙ ТЕМПЕРАТУРІ

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Розглядається ідеальний газ релятивістських масивних електронів у присутності магнітного монополя Дірака. Знайдено, що при порушенні СР-симетрії в цій системі поряд із зарядом виникають також квадрат орбітального кутового моменту, квадрат спіну та квадрат повного кутового моменту. Визначена функціональна залежність цих величин від температури та вакуумного кута, що порушує СР-симетрію. Досліджені температурні квадратичні флуктуації величин, що зберігаються, і з'ясовано, коли заряд та квадрат повного кутового моменту стають точними квантовими спостережуваними, а не просто очікуваними середніми по багатьом квантовим вимірюванням.