MULTIPLET WITH COMPONENTS OF DIFFERENT MASSES

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A principale possibility of the existence of a multiplet including the components with the different masses is indicated. This paper is dedicated to the memory of Anna Yakovlevna Gelyukh (Kalaida).

1. INTRODUCTION

We start with the citation of a very surprising (for us) appraisal of supersymmetry [1-4] given by Yury Abramovich Golfand during the Conference "Supersymmetry-85" at Kharkov State University in 1985. He said [5] that supersymmetry did not justify his hopes to find a generalization of the Poincaré group such that every its representation includes the particles of different masses. Golfand and Likhtman had missed their aim, but had instead found supersymmetry, every representation of which contains the fields of different spins.

So, the problem was raised and requires its solution. In the present paper we give a possible solution of the problem of the multiplet which components have the different masses[1]. We illustrate the solution on the example of the centrally extended $(1+1)$-dimensional Poincaré algebra [7-12].

2. TENSOR EXTENSION OF THE POINCARÉ ALGEBRA

In the paper [10] the tensor extension of the Poincaré algebra in D dimensions

$$[M_{ab}, M_{cd}] = (g_{ad}M_{bc} + g_{bc}M_{ad}) - (c \leftrightarrow d),$$
$$[M_{ab}, P_c] = g_{bc}P_a - g_{ac}P_b,$$
$$[P_a, P_b] = Z_{ab},$$
$$[M_{ab}, Z_{cd}] = (g_{ad}Z_{bc} + g_{bc}Z_{ad}) - (c \leftrightarrow d),$$
$$[P_a, Z_{bc}] = 0,$$
$$[Z_{ab}, Z_{cd}] = 0 \tag{2.1}$$

was introduced and its Casimir operators

$$Z_{a_1 a_2} Z^{a_2 a_3} \cdots Z_{a_{2k-1} a_{2k}} Z^{a_{2k} a_1}, \quad (k = 1, 2, ...); \tag{2.2}$$

$$P^{a_1} Z_{a_1 a_2} Z^{a_2 a_3} \cdots Z_{a_{2k-1} a_{2k}} Z^{a_{2k} a_{2k+1}} P_{a_{2k+1}}$$
$$+ Z^{aa_1} Z_{a_1 a_2} \cdots Z_{a_{2k-1} a_{2k}} Z^{a_{2k} a_{2k+1}} M_{a_{2k+1} a},$$
$$(k = 0, 1, 2, ...); \tag{2.3}$$

$$\varepsilon^{a_1 a_2 ... a_{2k-1} a_{2k}} Z_{a_1 a_2} \cdots Z_{a_{2k-1} a_{2k}}, \quad 2k = D \tag{2.4}$$

were constructed. Here M_{ab} are generators of rotations, P_a are generators of translations, Z_{ab} is a tensor generator and $\varepsilon^{a_1 ... a_{2k}}$, $\varepsilon^{01..2k-1} = 1$ is the totally antisymmetric Levi-Civita tensor in the even

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[1]Concerning another approach to this problem see the paper [6].

symmetric Levi-Civita tensor in the even dimensions $D = 2k$.

Generators of the left shifts with a group element G, acting on the function $f(y)$

$$[T(G)f](y) = f(G^{-1}y), \quad y = (x^a, z^{ab}),$$

have the form

$$P_a = -\left(\partial_{x^a} + \frac{1}{2} x^b \partial_{z^{ab}}\right),$$

$$Z_{ab} = -\partial_{z^{ab}},$$

$$M_{ab} = x_a \partial_{x^b} - x_b \partial_{x^a} + z_a{}^c \partial_{z^{bc}} - z_b{}^c \partial_{z^{ac}} + S_{ab}, \tag{2.5}$$

where coordinates x^a correspond to the translation generators P_a, coordinates z^{ab} correspond to the generators Z_{ab} and S_{ab} is a spin operator. In the expressions (2.5) $\partial_y \equiv \frac{\partial}{\partial y}$.

3. TWO-DIMENSIONAL CASE

In the case of the extended two-dimensional Poincaré algebra the Casimir operators (2.2), (2.3) and (2.4) can be expressed as degrees of the following generating Casimir operators:

$$Z = -\frac{1}{2}\varepsilon^{ab} Z_{ab}, \tag{3.1}$$

$$C = P^a P_a + Z^{ab} M_{ba}, \tag{3.2}$$

where $\varepsilon^{ab} = -\varepsilon^{ba}$, $\varepsilon^{01} = 1$ is the completely antisymmetric two-dimensional Levi-Civita tensor. The relations (2.5) can be represented as

$$P_t \equiv P_0 = -\partial_t - \frac{x}{2}\partial_y, \tag{3.3}$$

$$P_x \equiv P_1 = -\partial_x + \frac{t}{2}\partial_y, \tag{3.4}$$

$$J = \frac{1}{2}\varepsilon^{ab} M_{ab} = -t\partial_x - x\partial_t + S_{01}, \tag{3.5}$$

$$Z = \partial_y, \tag{3.6}$$

where $t \equiv x^0$ is a time, $x \equiv x^1$ is a space coordinate, $y \equiv z^{01}$ is a coordinate corresponding to the central

element Z and the space-time metric tensor has the following nonzero components $g_{11} = -g_{00} = 1$.

The extended Poincaré algebra (2.1) in this case can be rewritten in the following form (see also [9]):

$$[P_a, J] = \varepsilon_a{}^b P_b,$$

$$[P_a, P_b] = \varepsilon_{ab} Z,$$

$$[P_a, Z] = 0, \quad [J, Z] = 0 \tag{3.7}$$

and for the Casimir operator (3.2) we have the expression

$$C = P^a P_a - 2ZJ. \tag{3.8}$$

For simplicity let us consider the spin-less case $S_{01} = 0$. Then with the help of the relations (3.3)–(3.6) we obtain a mass square operator

$$M^2 \equiv \partial_{xx} - \partial_{tt} = P_x{}^2 - P_t{}^2 - JZ - \frac{t^2 - x^2}{4} Z^2, \tag{3.9}$$

where the notations $\partial_{xx} = \frac{\partial^2}{\partial x^2}$ and $\partial_{tt} = \frac{\partial^2}{\partial t^2}$ are used.

4. NEW COORDINATES

By a transition from t, x and y to the new coordinates

$$x_\pm = \frac{t \pm x}{2},$$

$$y_- = y - \frac{t^2 - x^2}{4}, \tag{4.1}$$

we obtain the following expressions for the generators:

$$P_- = \partial_{x_-}, \tag{4.2}$$

$$P_+ = 2x_- \partial_{y_-} - \partial_{x_+}, \tag{4.3}$$

$$J = x_- \partial_{x_-} - x_+ \partial_{x_+}, \tag{4.4}$$

$$Z = \partial_{y_-}, \tag{4.5}$$

where

$$P_\pm = P_x \pm P_t.$$

These generators satisfy the following commutation relations:

$$[P_+, P_-] = -2Z,$$

$$[J, P_\pm] = \pm P_\pm,$$

$$[J, Z] = 0,$$

$$[P_\pm, Z] = 0. \tag{4.6}$$

We see that $P_\pm$ are step-type operators.

The Casimir operator (3.8) in the new coordinates takes the form

$$C = P_+ P_- + Z - 2ZJ \tag{4.7}$$

and the mass square operator is

$$M^2 = P_+ P_- + Z - ZJ - x_+ x_- Z^2. \tag{4.8}$$

5. MULTIPLET

As a complete set of the commuting operators we choose the Casimir operators Z, C and rotation operator J. Let us assume that there exists such a state $\Psi_{z,j}(x_+, x_-, y_-)$ that

$$P_- \Psi_{z,j}(x_+, x_-, y_-) = 0, \tag{5.1}$$

$$Z \Psi_{z,j}(x_+, x_-, y_-) = z \Psi_{z,j}(x_+, x_-, y_-), \tag{5.2}$$

$$J \Psi_{z,j}(x_+, x_-, y_-) = j \Psi_{z,j}(x_+, x_-, y_-). \tag{5.3}$$

The equations (4.2) and (5.1) mean that $\Psi_{z,j}(x_+, x_-, y_-)$ is independent on the coordinate x_-. Then, as a consequence of the relations (4.4), (4.5), (5.2) and (5.3), we come to the following expression for the state $\Psi_{z,j}(x_+, y_-)$:

$$\Psi_{z,j}(x_+, y_-) = a x_+{}^{-j} e^{zy_-}, \tag{5.4}$$

where a is some constant.

For the states

$$P_+{}^k \Psi_{z,j}(x_+, y_-), \quad (k = 0, 1, 2, ...) \tag{5.5}$$

we obtain

$$J P_+{}^k \Psi_{z,j}(x_+, y_-) = \mathscr{J} P_+{}^k \Psi_{z,j}(x_+, y_-)$$
$$= (j + k) P_+{}^k \Psi_{z,j}(x_+, y_-), \tag{5.6}$$

$$M^2 P_+{}^k \Psi_{z,j}(x_+, y_-) = \mathscr{M}^2 P_+{}^k \Psi_{z,j}(x_+, y_-)$$
$$= [(k + 1 - j)z - x_+ x_- z^2] P_+{}^k \Psi_{z,j}(x_+, y_-), \tag{5.7}$$

$$C P_+{}^k \Psi_{z,j}(x_+, y_-)$$
$$= z(1 - 2j) P_+{}^k \Psi_{z,j}(x_+, y_-), \tag{5.8}$$

from which we have

$$\mathscr{J} = j + k, \tag{5.9}$$

$$\mathscr{M}^2 = (k + 1 - j)z - x_+ x_- z^2. \tag{5.10}$$

The states (5.5) are the components of the multiplet.

By excluding k from the relations (5.9) and (5.10), we come to the Regge type trajectory

$$\mathscr{J} = \alpha(0) + \alpha' \mathscr{M}^2 \tag{5.11}$$

with parameters

$$\alpha(0) = 2j - 1 + x_+ x_- z, \tag{5.12}$$

$$\alpha' = \frac{1}{z}. \tag{5.13}$$

6. CONCLUSION

Thus, on the example of the centrally extended (1+1)-dimensional Poincaré algebra we solved the problem of the multiplet which contains the components with the different masses.

It would be interesting to construct the models based on such a multiplet.

Note that, as can be easily seen from the commutation relation

$$[\partial_{x^a} + A_a, \partial_{x^b} + A_b] = F_{ab},$$

where A_a is an electromagnetic field and F_{ab} is its strength tensor, the above mentioned extended $D = 2$ Poincaré algebra (3.7) is arisen in fact when an "electron" in the two-dimensional space-time is moving in the constant homogeneous electric field.

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МУЛЬТИПЛЕТ С КОМПОНЕНТАМИ РАЗЛИЧНЫХ МАСС

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Указана принципиальная возможность существования мультиплета, содержащего компоненты с различными массами. Эта статья посвящена памяти Анны Яковлевны Гелюх (Калайда).

МУЛЬТИПЛЕТ З КОМПОНЕНТАМИ РІЗНИХ МАС

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Вказано на принципову можливість існування мультиплета, що містить компоненти з різними масами. Ця стаття присвячено пам'яті Ганни Яковлевни Гелюх (Калайда).