THE SPONTANEOUS GENERATION OF MAGNETIC FIELDS AT HIGH TEMPERATURE IN SU(2)-GLUODYNAMICS ON A LATTICE

V.I. Demchik[1] *and V.V. Skalozub*[2]

Dniepropetrovsk National University, Dniepropetrovsk, Ukraine;
[1]e-mail: vadimdi@yahoo.com;
[2]e-mail: skalozub@ff.dsu.dp.ua

In a lattice formulation of the SU(2)-gluodynamics, the spontaneous generation of the chromomagnetic field at high temperature is investigated. The procedure to study this phenomenon is developed and Monte Carlo simulations are carried out on the lattices $2x8^3$, $4x8^3$ and $2x16^3$ at various temperatures. The χ^2-analysis of the obtained data set indicates the presence of the spontaneously created magnetic field in the deconfinement phase. A comparison with the results of other approaches is done.

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1. INTRODUCTION

Among interesting problems of modern cosmology the origin of large-scale magnetic fields is intensively attacked nowadays. Various mechanisms of the field generation at different stages of the universe evolution were proposed [1]. Basically they are grounded on the idea of Fermi, Chandrasekhar and Zel'dovich that to have the present day galaxy magnetic fields seed magnetic fields must be present in the early universe. These fields had been frozen in a cosmic plasma and then amplified by some of the mechanisms of the field amplification. One of the ways to produce seed fields is a spontaneous vacuum magnetization at high temperature T [2,3,4,5]. Actually, this is an extension of the Savvidy model for the QCD vacuum [6], proposed already at T=0 and describing the creation of the Abelian chromomagnetic fields due to a vacuum polarization, in case of non-zero temperature. At zero temperature this field configuration is unstable because of the tachyonic mode in the gluon spectrum. At $T \neq 0$, the possibility of having strong temperature-dependent and stable magnetic fields was discovered [4]. The field stabilization is ensured by the temperature and field dependent gluon magnetic mass.

Another related field of interest is the deconfinement phase of QCD. As it was realized recently, this is not the gas of free quarks and gluons, but a complicate interacting system of them. This was discovered at RHIC experiments [7] and observed in either perturbative [4,8] or nonperturbative [9] investigations of the vacuum state with magnetic fields at high temperature. In Refs. [4,8] the spontaneous creation of the chromomagnetic fields of order $gB{\sim}g^4T^2$ was observed in SU(2)- and SU(3)-gluodynamics within the one-loop plus daisy resummation accounted for. In Ref. [9] the fields of the same order were observed in lattice simulations of two-point correlators. In Ref. [10] the response of the vacuum to the influence of strong external fields at different temperatures has been investigated and it was shown that the confinement is restored by increasing the strength of the applied field. These results stimulated the present investigation. We are going to determine the spontaneous creation of magnetic fields in lattice simulations of SU(2)-gluodynamics. In contrast to the problems in the external field, in the case of interest the field strength is a dynamical variable which values at different temperatures have to be determined by means of the minimization of the free energy. This procedure is not a simple one as in continuum because the field strength on a lattice is quantized. To deal with this peculiarity, we consider magnetic fluxes on a lattice as the main objects to be investigated. The fluxes take continuous values, and therefore the minimization of the free energy in presence of magnetic field can be fulfilled in a usual way. These speculations serve as an explanation of the strategy of our calculations.

One of the methods to introduce a magnetic flux on a lattice is to use the twisted boundary conditions (t.b.c.) [11]. In this approach the flux is a continuous quantity. So, in what follows we consider the free energy $F(\varphi)$ with the magnetic flux φ on a lattice in the SU(2)-gluodynamics and calculate its values at different temperatures by means of Monte Carlo (MC) simulations. We will show that the global minimum of $F(\varphi)$ is located at some non-zero value φ_{min} dependent on the temperature. It means the spontaneous creation of the temperature-dependent magnetic fields in the deconfinement phase.

2. MAGNETIC FIELDS ON A LATTICE

In perturbation theory, the value of the macroscopic (classical) magnetic field generated inside a system is determined by the minimization of the free energy functional. The interaction with the classical field is introduced by splitting the gauge field potential in two parts: $A_\mu = \overline{A_\mu} + A^R_\mu$, where A^R_μ describes a radiation field and $\overline{A_\mu} = (0,0,Hx^1,0)$ corresponds to the constant magnetic field directed along the third axis. However, on a lattice, the direct detection of the spontaneously generated field strength by straightforward analysis of the configurations, which are produced in the MC simulations, seems to be problematic. Therefore, it is reasonable to follow the approach used in the continuum field theory.

First, let us write down the free energy density,

$$F(\varphi) = -\log \frac{Z(\varphi)}{Z(0)}; \qquad (1)$$

$$Z(\varphi) = \int [DU(\varphi)] \exp\{-S(U(\varphi))\}. \qquad (2)$$

Here, Z(φ) and Z(0) are the partition functions at finite and zero magnetic fluxes, respectively; the link variable U is the lattice analogue of the potential A_μ. The free energy density relates to the effective action as follows,

$$F(\varphi) = \overline{S}(\varphi) - \overline{S}(0), \qquad (3)$$

where $\overline{S}(\varphi)$ and $\overline{S}(0)$ are the effective lattice actions with and without magnetic field, correspondingly.

To detect the spontaneous creation of the field it is necessary to show that the free energy density has the global minimum at a non-zero magnetic flux, $\varphi_{\min} \neq 0$.

In what follows, we use the hypercubic lattice $L_t \times L_s^3$ ($L_t < L_s$) with the hypertorus geometry; L_t and L_s are the temporal and the spatial sizes of the lattice, respectively. In the limit of $L_s \to \infty$ the temporal size L_t is related to physical temperature. The one-plaquette action of the SU(2) lattice gauge theory can be written:

$$S_W = \beta \sum_x \sum_{\mu > \nu} [1 - \tfrac{1}{2} Tr U_{\mu\nu}(x)]; \qquad (4)$$

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x + a\hat{\mu}) U_\mu^+(x + a\hat{\nu}) U_\nu^+(x), \qquad (5)$$

where $\beta = 4/g^2$ is the lattice coupling constant, g is the bare coupling, $U_\mu(x)$ is the link variable located on the link leaving the lattice site x in the μ direction, $U_{\mu\nu}(x)$ is the ordered product of the link variables.

The effective action $\overline{S}$ in (3) is the Wilson action averaged over the Boltzmann configurations, produced in the MC simulations.

The lattice variable $U_\mu(x)$ can be decomposed in terms of the unity, I, and Pauli, σ_j, matrices in the colour space,

$$U_\mu(x) = I U_\mu^0(x) + i\sigma_j U_\mu^j(x) = \begin{pmatrix} U_\mu^0(x) + iU_\mu^3(x) & U_\mu^2(x) + iU_\mu^1(x) \\ -U_\mu^2(x) + iU_\mu^1(x) & U_\mu^0(x) - iU_\mu^3(x) \end{pmatrix}. \qquad (6)$$

The four components $U_\mu^j(x)$ are subjected to the normalization condition $\sum_j U_\mu^j(x) U_\mu^j(x) = 1$. Hence, only three components are independent.

Since the spontaneously generated field is to be the Abelian one, the Abelian parametrization of the lattice variables is used to introduce the magnetic field,

$$U_\mu(x) = \begin{pmatrix} \cos\phi_\mu e^{i\theta_\mu(x)} & \sin\phi_\mu e^{i\chi_\mu(x)} \\ -\sin\phi_\mu e^{-i\chi_\mu(x)} & \cos\phi_\mu e^{-i\theta_\mu(x)} \end{pmatrix}, \qquad (7)$$

where the angular variables are changed in the following ranges $\theta, \chi \in [-\pi; +\pi)$, $\phi \in [0; \pi/2)$.

The Abelian part of the lattice variables is represented by the diagonal components of the matrix and the condensate Abelian magnetic field influences the field $\theta_\mu(x)$, only.

The second important task is to incorporate the magnetic flux in this formalism. The most natural way was proposed by 't Hooft [11]. In his approach, the constant homogeneous external flux φ in the third spatial direction can be introduced by applying the following t.b.c.:

$$\begin{aligned} &U_\mu(L_t, x_1, x_2, x_3) = U_\mu(0, x_1, x_2, x_3); \\ &U_\mu(x_0, L_s, x_2, x_3) = U_\mu(x_0, 0, x_2, x_3); \\ &U_\mu(x_0, x_1, L_s, x_3) = e^{i\varphi} U_\mu(x_0, x_1, 0, x_3); \\ &U_\mu(x_0, x_1, x_2, L_s) = U_\mu(x_0, x_1, x_2, 0). \end{aligned} \qquad (8)$$

It could be seen, the edge links in all directions are identified as usual periodic boundary conditions except for the links in the second spatial direction, for which the additional phase φ is added (Fig. 1). In the continuum limit, such t.b.c. settle the magnetic field with the potential $A_\mu(x) = (0, 0, Hx^1, 0)$. The magnetic flux φ is measured in angular units, $\varphi \in [0, 2\pi)$.

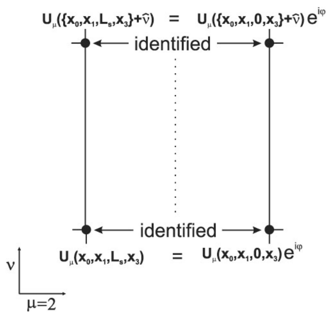

Fig. 1. *The plaquette presentation of the twisted boundary conditions*

The lattice variables (in the Abelian parametrization) in the presence of the magnetic flux φ are

$$U_\mu(x) = \begin{pmatrix} \cos\phi_\mu e^{i(\theta_\mu(x) + \varphi_\mu(x))} & \sin\phi_\mu e^{i\chi_\mu(x)} \\ -\sin\phi_\mu e^{-i\chi_\mu(x)} & \cos\phi_\mu e^{-i(\theta_\mu(x) + \varphi_\mu(x))} \end{pmatrix}, \qquad (9)$$

where $\varphi_\mu(x) = \varphi$ for the edge links at $x = (x_0, x_1, L_s, x_3)$ with $\mu = 2$ and $\varphi_\mu(x) = 0$ for other links.

The total flux through the plane spanned by the plaquettes p, which affects the edge links at $x = (x_0, x_1, L_s, x_3)$ with $\mu = 2$, is

$$g\Phi = \sum_{p \in plane} (\theta_p + \varphi), \qquad (10)$$

$$\theta_p = \theta_\mu(x) + \theta_\nu(x + a\hat{\nu}) - \theta_\mu(x + a\hat{\nu}) - \theta_\nu(x). \qquad (11)$$

Eq. (10) is the lattice analogue of the flux in the continuum $\Phi_c = \int_S d^2\sigma_{\mu\nu} F_{\mu\nu}$. In this approach the variable φ describes a flux through the whole lattice plane, not just through an elementary plaquette.

The t.b.c. for the components (9),

$$\begin{aligned} &U_\mu^0(x) = \cos(\theta_\mu(x) + \varphi_\mu(x)) \cos\phi_\mu(x); \\ &U_\mu^1(x) = \sin\phi_\mu(x) \sin\chi_\mu(x); \\ &U_\mu^2(x) = \sin\phi_\mu(x) \sin\chi_\mu(x); \\ &U_\mu^3(x) = \sin(\theta_\mu(x) + \varphi_\mu(x)) \cos\phi_\mu(x), \end{aligned} \qquad (12)$$

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$$U_\mu^0(x) = U_\mu^0(x)\cos\varphi - U_\mu^3(x)\sin\varphi\,, \qquad (13)$$

$$U_\mu^3(x) = U_\mu^0(x)\sin\varphi + U_\mu^3(x)\cos\varphi \qquad (14)$$

for the edge links at x =(x_0, x_1, L_s, x_3) with μ=2.

The relations (13) and (14) have been implemented into the kernel of the MC procedure in order to produce the configurations with the magnetic flux φ. In this case the flux φ is accounted for in obtaining a Boltzmann ensemble at each MC iteration.

3. MC SIMULATIONS AND DATA FITS

The MC simulations are carried out by means of the heat bath method.

The spontaneous generation of magnetic field is the effect of order ~g^4 [4]. The results of MC simulations show the comparably large dispersion. So, the large amount of the MC data is collected and the standard χ^2-method for the analysis of data is applied to determine the effect. We consider the results of the MC simulations as observed "experimental data".

The effective action depends smoothly on the flux φ in the region φ~0. Therefore, the free energy density can be fitted by the quadratic function of the flux φ,

$$F(\varphi) = F_{\min} + b(\varphi - \varphi_{\min})^2\,. \qquad (15)$$

This choice is motivated also by the results obtained already in continuum field theory [13] where it was determined that free energy has a global minimum at $\varphi \neq 0$. The parametrization (15) is the most reasonable in this case. It is based on the effective action accounting for the one-loop plus daisy diagrams [13],

$$F(H) = \frac{H^2}{2} + \frac{11}{48}\frac{g^2}{\pi^2}H^2\log H^2\frac{T^2}{\mu^2} - \qquad (16)$$

$$-\frac{1}{3}\frac{(gH)^{3/2}T}{\pi} - \frac{1}{12}Tr[\Pi_{00}(0)]^{3/2}\,,$$

having g^2 and $(g^2)^{3/2}$ orders in coupling constant. Here, H is field strength (flux φ~H), T is the temperature, μ is the normalization point, $\Pi_{00}(0)$ is the zero-zero component of the gluon polarazation operator calculated in the external field at the finite temperature and taken at zero momentum. The value of β=3, which was used, corresponds to a deep perturbation regime. So, a comparison with perturbation results is reasonable. The systematic errors in fitting function (15) could come from not taking into account the high-order diagrams in (16). However, as it is well-known [15], the lack of an expansion parameter at finite temperature starts from the three-loop diagram contributions that is of g^6 order and could not remove an effect derived in g^2 and g^3 orders. As the finite-size effects are concerned, in the present investigation we just made calculations for two lattices $2x8^3$ and $2x16^3$ and have derived the same results for the φ_{min} (as it will be seen below). A more detailed investigation of this issue requires much more computer resources, which were limited.

There are 3 unknown parameters, F_{min}, b and φ_{min} in Eq.(16). The parameter φ_{min} denotes the minimum position of the free energy, whereas the F_{min} and b are the free energy density at the minimum and the curvature of the free energy function, correspondingly.

The value φ_{min} is obtained as the result of the minimization of the $\chi^2(F_{\min}, b, \varphi_{\min})$-function

$$\chi^2 = \sum_i \frac{(F_{\min} + b(\varphi_i - \varphi_{\min})^2 - F(\varphi_i))^2}{D(F(\varphi_i))}\,, \qquad (17)$$

where φ_i is the array of the set fluxes and $D(F(\varphi_i))$ is the data dispersion. It can be obtained by collecting the data into the bins (as a function of flux),

$$D(F(\varphi_i)) = \sum_{i \in bin} \frac{(F(\varphi_i) - \hat{F}_{bin})^2}{n_{bin} - 1}\,, \qquad (18)$$

where n_{bin} is the number of points in the considered bin, $\hat{F}_{bin}$ is the mean value of free energy density in the considered bin. As it is determined in the data analysis, the dispersion is independent of the magnetic flux values φ. The deviation of φ_{min} from zero indicates the presence of spontaneously generated field.

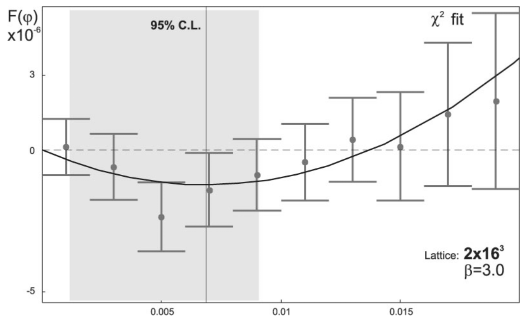

Fig. 2. *χ^2-fit of the free energy density on lattice 2×16^3 for β=3.0 (grey region describe the φ_{min}= $0.0069^{+0.0022}_{-0.0057}$ at the 95% C.L.)*

The values of the generated fluxes φ_{min} for different lattices (at the 95% C.L.)

φ_{min}	$2x8^3$	$2x16^3$	$4x8^3$
β=3	$0.019^{+0.013}_{-0.012}$	$0.0069^{+0.0022}_{-0.0057}$	$0.005^{+0.005}_{-0.003}$
β=5	$0.020^{+0.011}_{-0.010}$	-	-

The fit results are given in Table 1. As one can see, φ_{min} demonstrates the 2σ-deviation from zero. The dependence of φ_{min} on the temperature is also in accordance with the results known in perturbation theory: the increase in temperature results in the increasing of the field strength [4].

The fit for the lattice $2x16^3$ at β=3.0 is shown in Fig. 2. The maximum-likelihood estimate of F(φ) by the whole data set is shown as the solid curve. In addition, all φ values are divided into 10 bins. The mean values and the 95% confidence intervals are presented as points for each bin. The first 7 bins contain about 600-2000 points per bin. The large number of points in the bins allow to find the free energy F with the accuracy which substantively exceeds the dispersion, $\sqrt{D(F(\varphi_i))} \sim 10^{-4}$. It makes possible to detect the effect of interest. As it is also seen, the maximum-likelihood estimate of F(φ) is in a good accordance with the bins pointed, because the solid line is located in the 95% confidence intervals of all bins.

The 95% confidence level (C.L.) area of the parameters b and φ_{min} is represented in Fig. 3. The black cross marks the position of the maximum-likelihood values of b and φ_{min}. It can be seen that the flux is positive determined. The 95% C.L. area becomes more symmetric with the center at the F_{min}, b and φ_{min} when the statistics is increasing. This also confirms the results of the fitting.

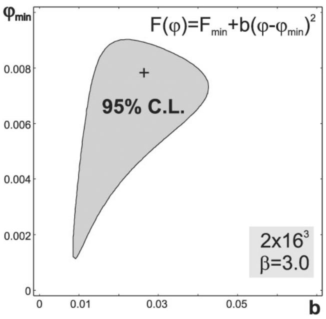

Fig. 3. *The 95% C.L. area for the parameters φ_{min} and b, determining the free energy density dependence on the flux φ_{min} on lattice 2×16^3 for β=3.0*

4. DISCUSSION

The main conclusion from the results obtained is that the spontaneously created temperature-dependent chromomagnetic field is present in the deconfinement phase of QCD. This supports the results derived already in the continuum quantum field theory [4,12] and in lattice calculations [9].

Let us first discuss the stability of the magnetic field at high temperature. It was observed in Refs. [4,12] that the stabilization happens due to the gluon magnetic mass calculated from the one-loop polarization operator in the field at temperature. This mass has the order $m^2_{magn} \sim g^2 (gH)^{1/2} T \sim g^4 T^2$ as it should be because the chromomagnetic field is of order $(gH)^{1/2} \sim g^2 T$ [4]. The stabilization is a non-trivial fact that, in principle, could be changed when the higher order Feynman diagrams to be accounted for. Now we see that the stabilization of the field really takes place.

Our approach based on the joining of calculation of the free energy functional and the consequent statistical analysis of its minimum positions at various temperatures and flux values. This overcomes the difficulties peculiar to the description of the field on a lattice. Here we mean that the field strength on a lattice is quantized and therefore a non-trivial tuning of the coupling constant, temperature and field strength values has to be done in order to determine the spontaneously created magnetic field.

We also would like to note that in the present paper the flux dependence on temperature remains not investigated in details. This is because of the small lattice size considered. That restricts the number of points permissible to study. However, at this stage we have determined the effect of interest as a whole. Even at the small lattice, one needs to take into consideration thousands points of free energy (that corresponds to an analysis of 5-10 millions MC configurations for different lattices) to determine the flux value φ_{min} at the 95% C.L. In case of larger lattices this number and corresponding computer resources should be increased considerably. This problem is left for the future.

It is interesting to compare our results with that of in Ref. [10] where the response of the vacuum on the external field was investigated. These authors have observed in lattice simulations for the SU(2)- and SU(3)-gluodynamics that the external field is completely screened by the vacuum at low temperatures, as it should be in the confinement phase. With the temperature increase, the field penetrates into the vacuum and, moreover, increase in temperature results in existing more strong external fields in the vacuum. On the other hand, increase in the applied external field strength leads to the decreasing of the deconfinement temperature. These interesting properties are closely related to the studies in the present work. Actually, we have also investigated the vacuum properties as an external field problem when the field is described in terms of fluxes. This was the first step of the calculations. The next step was the statistical analysis of the minimum position of free energy, in order to determine the spontaneous creation of the field. In fact, at the first step we reproduced the results of Refs. [10] (in terms of fluxes).

Note that the present investigations also correspond to the case of the early universe. They support our previous results on the magnetic field generation in the standard model [13] and in the minimal supersymmetric standard model [14]. As it was discussed by Pollock [5], the field generated by this mechanism at the Planck era might serve as a seed field to produce the present day magnetic fields in galaxies.

We would like to conclude with the note that the deconfinement phase of gauge theories is a very interesting object to study. The temperature dependent magnetic fields living in this state influence various processes that should be taken into consideration to have an adequate concept about them.

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СПОНТАННАЯ ГЕНЕРАЦИЯ МАГНИТНЫХ ПОЛЕЙ ПРИ ВЫСОКОЙ ТЕМПЕРАТУРЕ В SU(2)-ГЛЮОДИНАМИКЕ НА РЕШЕТКЕ

В.И. Демчик, В.В. Скалозуб

Исследована спонтанная генерация хромомагнитного поля при высокой температуре в решеточной формулировке SU(2)-глюодинамики. Разработана процедура для исследования этого эффекта на решетке. Проведено моделирование методом Монте Карло на решетках $2x8^3$, $4x8^3$ и $2x16^3$ при различных температурах. χ^2-анализ полученных данных указывает на существование в фазе деконфайнмента спонтанно рожденного магнитного поля. Проведено сравнение с результатами, полученными в других приближениях.

СПОНТАННА ГЕНЕРАЦІЯ МАГНІТНИХ ПОЛІВ ПРИ ВИСОКІЙ ТЕМПЕРАТУРІ В SU(2)-ГЛЮОДИНАМІЦІ НА ГРАТЦІ

В.І. Демчик, В.В. Скалозуб

Досліджено спонтанну генерацію хромомагнітного поля при високій температурі в гратковій формулюровці SU(2)-глюодинаміки. Розроблено процедуру для дослідження цього ефекта на гратці. Проведено моделювання методом Монте Карло на гратках $2x8^3$, $4x8^3$ та $2x16^3$ при різних температурах. χ^2-аналіз отриманих даних вказує на наявність у фазі деконфайнменту магнітного поля, що народжується спонтанно. Проведено порівняння з результатами, отриманими у інших наближеннях.