Section C. HIGH-ENERGY ELECTRODYNAMICS IN MATTER

POLARIZATION PROPERTIES OF THE VAVILOV-CHERENKOV RADIATION

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We consider polarization properties of the Vavilov-Cherenkov radiation using the relativistic quantum treatment of the one-photon Cherenkov emission. The probability of the one-photon emission by relativistic electron is calculated in the framework of quantum electrodynamics with inclusion of all polarizations (polarized electron 1 emits polarized electron 2 and polarized γ-quantum). Relations between initial and final electron polarizations were found.

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1. INTRODUCTION

More than 70 years were gone since the discovery of the Vavilov-Cherenkov radiation (VCR). This phenomenon was considered in hundreds of works; most of them dealt with classical charge radiation and didn't take into account quantum properties of particles. There are not many works, where a polarization of emitted γ-quantum was investigated [1]. They are Cherenkov's works (1934, 1937), where experimental data for the polarization was from 10…12% till 50%. Also in Mazer's and Zrelov's works it was reported about VCR process of protons with γ-quantum polarization near 100%, and that is in agreement with theory predictions [1]. The distinction between Cherenkov's observations and results obtained later was explained by a lack of good collimated beams of electrons in Cherenkov's experiments.

The quantum theory of VCR first was developed by Ginzburg [2]. In this work were considered VCR features related with spin and recoil of emissive charge. But Ginsburg didn't go beyond the energies of γ-quantum little in comparison with whole energy of electron. Full calculations of electron spin characteristics and γ-quantum polarization for all allowed by the laws of conservation energies of γ-quantum were done in [3]. In particular it was shown, that by ordinary circumstances of experiment the maximum of electromagnetic radiation is located in X-ray part of spectrum. The intensity of radiation in maximum is about 10^4 times more then intensity in ultraviolet part of spectrum. However in this work general relations for intensity with account of all polarizations of electron and photon were written, they were used only in the case, when averaging and summing over polarizations of initial and final electron and photon were made.

The aim of this work is detailed consideration of polarization effects in VCR phenomena with the account of arbitrary electron and photon polarizations and arbitrary recoil of emissive electron.

2. PHOTONS IN MEDIUM

Let's consider Maxwell equations in medium,

$$\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = -\mathrm{rot}\vec{E}, \quad \mathrm{div}\vec{B} = 0,$$
$$\frac{1}{c}\frac{\partial \vec{D}}{\partial t} = \mathrm{rot}\vec{H} - \frac{4\pi}{c}\vec{j}, \quad \mathrm{div}\vec{D} = 4\pi\rho,$$

where $\vec{E}$ and $\vec{H}$ are electrical and magnetic field vectors, $\vec{D} = n^2\vec{E}$ and $\vec{B} = \vec{H}$ are electrical and magnetic induction vectors. Let's determine

$$\hat{\vec{E}} = n\vec{E},\ \hat{\vec{H}} = \vec{H},\ \hat{c} = \frac{c}{n},\ \hat{\vec{j}} = \frac{\vec{j}}{n},\ \hat{\rho} = \frac{\rho}{n}.$$

For these "new" fields (with "hats") we may write "new" Maxwell equations,

$$\begin{aligned} &\frac{1}{c}\frac{\partial \hat{\vec{H}}}{\partial t} = -\mathrm{rot}\,\hat{\vec{E}}, \quad \mathrm{div}\,\hat{\vec{H}} = 0, \\ &\frac{1}{c}\frac{\partial \hat{\vec{E}}}{\partial t} = \mathrm{rot}\,\hat{\vec{H}} - \frac{4\pi}{c}\hat{\vec{j}}, \quad \mathrm{div}\,\hat{\vec{E}} = 4\pi\,\hat{\rho}, \end{aligned} \tag{1}$$

that differ from equations in vacuum only in presence of "hats" under all the letters. It's easy to write (1) in 4-dimensional form, we should only replace usual 4-vector x_μ by 4-vector $\hat{x}_\mu$ with components $\hat{\vec{x}} = \vec{x},\ \hat{x}_4 = i\hat{c}t$:

$$\frac{\partial}{\partial x_\rho}\hat{F}_{\mu\nu} + \frac{\partial}{\partial x_\mu}\hat{F}_{\nu\rho} + \frac{\partial}{\partial x_\nu}\hat{F}_{\rho\mu} = 0,$$
$$\frac{\partial}{\partial x_\nu}\hat{F}_{\mu\nu} = \frac{4\pi}{\hat{c}}\hat{j}_\mu .$$

These equations quantization doesn't differ from quantization of electromagnetic field in vacuum. For probability of photon with polarization vector $\hat{\vec{e}}$ radiation by current $\hat{\vec{j}}$ appears the same formula, as for radiation in vacuum:

$$dw = \frac{1}{2\pi^2\hbar\hat{c}}\int \frac{e^{i\hat{k}\hat{x}}\,\hat{\vec{e}}\,\hat{\vec{j}}}{\sqrt{2\omega}}\,d^4\hat{x}^2\,d^3\hat{k}.$$

Presence of all "hats" results only in appearance of n in denominator. Using this simple method we may take into account influence of medium in arbitrary formula.

3. EMISSION OF A PHOTON BY AN ELECTRON IN MEDIUM

In the framework of quantum electrodynamics, the probability of the one-photon process (electron 1 emits electron 2 and γ-quantum) is determined by a matrix element:

$$M^{(1)} = \sqrt{4\pi}(2\pi)^4 \times \frac{e}{n}\frac{\bar{u}_2}{\sqrt{2E_2}}\frac{\gamma_\mu e_\mu}{\sqrt{2\omega}}\frac{u_1}{\sqrt{2E_2}}\delta(p_1 - p_2 - k). \quad (2)$$

Here e is charge of the electron, e_μ is the four-vector of the photon polarizations (we always may get it's fourth component equal zero by correspondent gauge transformation), p_1 and p_2 are initial and final electron four-momenta, k is photon four-momentum, E_1 , E_2 and ω are electron and photon energies respectively, u_1 and u_2 are bispinors, electron wave functions, n is medium refractive index (from this point we put $\hbar = c = 1$),

$$\vec{p}_1 = \vec{p}_2 + \vec{k}, \quad E_1 = E_2 + \omega, \quad k \equiv \left|\vec{k}\right| = n\omega.$$

The probability of the photon with 4-momentum k_μ and polarization e_μ radiation in VCR process with initial and final electron 4-momenta p_1, p_2 and 4-polarizations s_1 , s_2 may be evaluated in such form:

$$dw^{(1)} = \frac{e^2}{2\pi}\frac{|N|^2}{E_1 p_1}dt d\omega d\phi. \quad (3)$$

Formula for $|N|^2$ was obtained in [3]:

$$\begin{aligned}|N|^2 &= -\left(\bar{u}_2\gamma_\mu u_1\right)\left(\bar{u}_1\gamma_\nu u_2\right)e^*_\mu e_\nu \\
&= -(e^*e)\left(m^2 + p_1p_2\right) + (p_2e^*)(p_1e) + (p_1e^*)(p_2e) \\
&+ m\left(\{e^*ep_2s_2\} - \{e^*ep_1s_2\} - \{e^*ep_1s_1\} + \{e^*ep_2s_1\}\right) \\
&- (e^*e)[\left(m^2 + p_2p_1\right)\left(s_2s_1\right) - \left(p_2s_1\right)\left(p_1s_2\right)] \\
&+ \left(m^2 + p_2p_1\right)[(s_2e^*)(s_1e) + (s_1e^*)(s_2e)] \\
&+ \left(s_2s_1\right)[(p_2e^*)(p_1e) + (p_1e^*)(p_2e)] \\
&- \left(p_1s_2\right)[(p_2e^*)(s_1e) + (s_1e^*)(p_2e)] \\
&- \left(p_2s_1\right)[(p_1e^*)(s_2e) + (s_2e^*)(p_1e)].\end{aligned}$$

Braces *{abcd}* mean $\varepsilon_{\mu\nu\rho\sigma}a_\mu b_\nu c_\rho d_\sigma$.

3.1. THE CASE OF NEGLIGIBLE ELECTRON RECOIL

Neglecting electron recoil we obtain $E_2 = E_1 \equiv E$ and $\vec{p}_2 = \vec{p}_1 \equiv \vec{p}$. Taking into account this, for matrix element we obtain:

$$M^{(1)}_{i\to f} = \frac{\sqrt{4\pi}e}{nE\sqrt{8\omega}}\bar{u}_2 e_\mu\gamma_\mu u_1 .$$

From Dirac equation:

$$\bar{u}_2\gamma_\mu e_\mu u_1 = -i\frac{p_\mu e_\mu}{m}(\bar{u}_2u_1) = -ip\sin\theta\frac{(\bar{u}_2u_1)}{m}.$$

Here and later θ is angle between initial electron and photon momentum. Vector of photon polarization may lie in the plane of electron and photon momenta or may be perpendicular to this plane. In last case matrix element equals zero.

For matrix element squared we have a formula:

$$\begin{aligned}|M^{(1)}_{i\to f}|^2 &= 2\pi e^2\frac{p^2}{E^2\omega n^2}\sin^2\theta\frac{|\bar{u}_2u_1|^2}{4m^2} \\
&= 2\pi e^2\frac{p^2}{E^2\omega n^2}\sin^2\theta\frac{1+(s_2s_1)}{2}.\end{aligned}$$

And probability of electron emission in time unit:

$$\begin{aligned}dw^{(1)} &= 4\pi^2e^2\frac{p^2}{E^2\omega n^2}\sin^2\theta\delta(E_1 - E_2 - \omega) \times \frac{1+(s_2s_1)}{2}\frac{d^3k}{(2\pi)^3} \\
&= \frac{1}{\pi}e^2\frac{p^2}{E^2\omega n^2}\sin^2\theta\frac{n^3\omega^2}{|\frac{dE_2}{d\theta}|}\frac{1+(s_2s_1)}{2}d\omega d\varphi \\
&= \frac{e^2}{2\pi}v\sin^2\theta\frac{1+(s_2s_1)}{2}d\omega d\varphi.\end{aligned}$$

There are $d^3k = n^3\omega^2 d\omega d\varphi d\cos\theta$, $|\frac{dE_2}{d\theta}| = \frac{p_1k}{E_2}$, $\cos\theta = \frac{1}{vn}$, $E_2 = \sqrt{p_1^2 - 2p_1k\cos\theta + k^2 + m^2}$.

From this result it is easy to obtain standard formulae for VCR: if initial electron isn't polarized and we don't interest in polarization of final electron, s_1 equals zero and result doubles.

3.2. AVERAGING-OUT AND SUMMING OVER ELECTRON POLARIZATIONS

If we don't neglect electron recoil, formulae become a little more complicated. $|N|^2$ summed over final electron polarizations in case of non-polarized initial electron equals

$$\overline{|N|^2} = -2(e^*e)(m^2 + (p_1p_2)) + 2[(p_1e^*)(p_2e) + (p_2e^*)(p_1e)].$$

For vector of photon polarization lying in the plane of emission and perpendicular to it:

$$\begin{aligned}\overline{|N|^2}_{\parallel} &= -2(m^2 + (p_1p_2)) + 4(\vec{p}_1\vec{e})(\vec{p}_2\vec{e}) \\
&= -2(m^2 + (p_1p_2)) + 4p_1^2\sin^2\theta,\end{aligned}$$

$$\overline{|N|^2}_{\perp} = -2(m^2 + (p_1p_2)),$$

and summed over photon polarizations:

$$<\overline{|N|^2}> = 4p_1^2\sin^2\theta + 2\omega^2(n^2 - 1).$$

Finally for integrated on azimuth angle, averaged and summed over all the polarizations probability of photon emission with relativistic quantum corrections,

$$dw^{(1)} = e^2[v\sin^2\theta + 2\tfrac{\omega^2}{E_1 p_1}]dtd\omega$$

$$= e^2\{v[1 - \tfrac{1}{v^2 n^2}(1 + \tfrac{\omega}{2E_1}(n^2-1))^2] + \tfrac{1}{2}\tfrac{\omega^2}{E_1 p_1}(n^2-1)\}dtd\omega.$$

3.3. POLARIZATION EFFECTS FOR ELECTRONS WITH DEFINITE HELICITY

Earlier we didn't concretize explicit form of 4-vectors s_1 and s_2. In this section we will consider initial and final electrons with definite helicity, and it means that electron spin projections on the motion direction are equal $\lambda_1/2$ and $\lambda_2/2$, i.e.

$$\vec{s}_1 = \lambda_1 \frac{E_1}{m}\frac{\vec{p}_1}{p_1},\ s_{10} = \lambda_1 \frac{p_1}{m};$$

$$\vec{s}_2 = \lambda_2 \frac{E_2}{m}\frac{\vec{p}_2}{p_2},\ s_{20} = \lambda_2 \frac{p_2}{m}.$$

Photon polarization is arbitrary and described by Stokes parameters $\vec{\xi}$; ξ_1, ξ_2 describe linear and ξ_3 circular polarization [4].

Matrix element squared in this case

$$|M|^2 = A(\lambda_1,\lambda_2) + B(\lambda_1,\lambda_2)\xi_3 + C(\lambda_1,\lambda_2)\xi_2,$$

where

$$A = \frac{1}{4}[4(E_1E_2 - m^2) + k^2 - \frac{(E_1^2 - E_2^2)^2}{k^2}](1 + \frac{\lambda_1\lambda_2}{p_1p_2}(E_1E_2 - m^2)) - \frac{\lambda_1\lambda_2}{p_1p_2}m^2(E_1 - E_2)^2;$$

$$B = \frac{1}{4}[2(p_1^2 + p_2^2) - k^2 - \frac{(E_1^2 - E_2^2)^2}{k^2}](1 + \frac{\lambda_1\lambda_2}{p_1p_2}(E_1E_2 - m^2));$$

$$C = \frac{1}{2}(k - \frac{(E_1 - E_2)^2}{k})(E_1 + E_2)(\frac{\lambda_1}{p_1}E_1 + \frac{\lambda_2}{p_2}E_2) - m^2k(\frac{\lambda_1}{p_1} + \frac{\lambda_2}{p_2}).$$

Stox parameters may be found in this way:

$$\xi_3 = \frac{B}{A},\ \xi_2 = \frac{C}{A}.$$

In case of completely polarized initial and final electrons $|\lambda_1| = |\lambda_2| = 1$ and $\xi_3^2 + \xi_2^2 = 1$, i.e. photon, as it should be, also completely polarized.

Let's discuss come special cases. In experiment, when final electron polarization isn't registered, the observed phenomena are defined by matrix element squared and summed on final electron polarizations:

$$\overline{|M|^2} = \overline{A} + \overline{B}\xi_3 + \overline{C(\lambda_1)}\xi_2;$$

$$\overline{A} = \frac{1}{2}[4(E_1E_2 - m^2) + k^2 - \frac{(E_1^2 - E_2^2)^2}{k^2}];$$

$$\overline{B} = \frac{1}{2}[2(p_1^2 + p_2^2) - k^2 - \frac{(E_1^2 - E_2^2)^2}{k^2}];$$

$$\overline{C} = [(k - \frac{(E_1 - E_2)^2}{k})(E_1 + E_2)E_1 - 2m^2k]\frac{\lambda_1}{p_1}.$$

These formulae show, that linear polarization of photon doesn't depend on initial electron polarization and it is defined by expression (there symbols from [3] are used, $\beta_0 = \frac{p_1}{E_1}$, $\tilde{\varepsilon} = \frac{\omega}{E_1}$):

$$\xi_3 = \frac{\overline{B}}{\overline{A}} = 1 - \frac{\frac{n^2-1}{2\beta_0^2}\tilde{\varepsilon}^2}{1 - \frac{1}{n^2\beta_0^2} - \frac{n^2-1}{n^2\beta_0^2}\tilde{\varepsilon} + \frac{n^4-1}{4n^2\beta_0^2}\tilde{\varepsilon}^2}. \quad (4)$$

Circular polarization may appear only in the case of polarized initial electron:

$$\xi_2 = \frac{\overline{C}}{\overline{A}} = \frac{1 - \frac{1}{n^2\beta_0^2} - \frac{n^2-1}{2n^2\beta_0^2}\tilde{\varepsilon}}{1 - \frac{1}{n^2\beta_0^2} - \frac{n^2-1}{n^2\beta_0^2}\tilde{\varepsilon} + \frac{n^4-1}{4n^2\beta_0^2}\tilde{\varepsilon}^2}\frac{n\tilde{\varepsilon}}{\beta_0}\lambda_1. \quad (5)$$

In case with no polarized initial electron ($\lambda_1 = 0$) and unknown final electron polarization we obtain result, identical to one from [3]:

$$\overline{|M|^2} = \frac{1}{2}[4(E_1E_2 - m^2) + k^2 - \frac{(E_1^2 - E_2^2)^2}{k^2}] = 2E_1^2\beta_0^2(1 - \frac{1}{n^2\beta_0^2} - \frac{n^2-1}{n^2\beta_0^2}\tilde{\varepsilon} + \frac{n^4-1}{4n^2\beta_0^2}\tilde{\varepsilon}^2). \quad (6)$$

If a completely polarized electron stops ($|\lambda_1| = 1$), then $\xi_3 = 0$ and $\xi_2 = \lambda_1$, i.e. photon is completely polarized and its' helicity has the same direction as initial electron (this is the law of angular momentum conservation: $\pm 1/2 = \mp 1/2 \pm 1$, spin turns over.

In high energy limit, when electron mass is neglected, polarization phenomena also become simpler:

$$A = \frac{1}{4}[4E_1E_2 + k^2 - \frac{(E_1^2 - E_2^2)^2}{k^2}](1 + \lambda_1\lambda_2);$$

$$B = \frac{1}{4}[2(p_1^2 + p_2^2) - k^2 - \frac{(E_1^2 - E_2^2)^2}{k^2}](1 + \lambda_1\lambda_2);$$

$$C = \frac{1}{2}(k - \frac{(E_1 - E_2)^2}{k})(E_1 + E_2)(\lambda_1 + \lambda_2).$$

So, in ultrarelativistic limit electron helicity conserves.

4. RELATION BETWEEN VECTORS OF POLARISATION OF FINAL AND INITIAL ELECTRONS IN CASE OF COMPLETELY POLARIZED PHOTON

General formula for the case of linear polarization is

$$(s_2)_\mu = (s_1)_\mu + \frac{(ks_1)[2(p_1e)e_\mu - k_\mu] - 2(es_1)[(p_1k)e_\mu + (p_1e)k_\mu]}{(p_1k) + 2(p_1e)^2}. \quad (7)$$

One can examine that $s_2^2 = s_1^2$.

It is evident if $s_1^2 = 1$. In this case completely polarized electron after radiation of completely polarized photon stays completely polarized. However if $s_1^2 < 1$, the conservation of polarization extent after emission of completely linear polarized photon is not trivial. It follows from simplicity of matrix element, distinctive for VCR.

Relation (7) is much more complicated in case of completely elliptical polarized photon (in this case 4-vector of photon polarization is complex):

$$(s_2)_\mu = \{-m\{e^*ek\mu\} - \tfrac{1}{m}\{e^*ekp_1\}(p_2)_\mu + +(s_1)_\mu$$
$$\times[(p_1k) + 2(p_1e^*)(p_1e)] + (ks_1)[(p_1e^*)e_\mu + (p_1e)e^*_\mu$$
$$- k_\mu] + (ks_1)[(p_1e^*)e_\mu + (p_1e)e^*_\mu - k_\mu] - (p_1k)$$
$$\times[(e^*s_1)e_\mu + (es_1)e^*_\mu] - [(p_1e^*)(s_1e) + (p_1e)(s_1e^*)]k_\mu\}$$
$$\times[(p_1k) + 2(p_1e^*)(p_1e) - m\{e^*eks_1\}]^{-1}.$$

We see that new effect appears: nonpolarized electron may become polarized. Indeed, in case $s_1 = 0$, we obtain

$$(s_2)_\mu = \frac{-m\{e^*ek\mu\} - \frac{1}{m}\{e^*ekp_1\}(p_2)_\mu}{(p_1k) + 2(p_1e^*)(p_1e)}. \quad (8)$$

Cumbersome evaluations give us comparatively simple formula that relates polarization extent of final and initial electron:

$$1 - s_2^2 = (1 - s_1^2) \times \frac{|e^2(p_1k) + 2(p_1e)(p_1e)|^2}{|(p_1k) + 2(p_1e^*)(p_1e) - m\{e^*eks_1\}|^2}. \quad (9)$$

Assuming $s_1 = 0$ we have

$$s_2^2 = 1 - \frac{|e^2(p_1k) + 2(p_1e)(p_1e)|^2}{|(e^*e)(p_1k) + 2(p_1e^*)(p_1e)|^2}. \quad (10)$$

In (10) $(e^*e) = 1$ is put for symmetry of expressions in denominator. For linear polarized photon 4-vector e is real, $e^* = e$ and $s_2^2 = 0$. In case of circular polarization $e^2 = 0$, and final electron turns out to be polarized, for $(p_1e) = 0$ to be completely polarized.

5. CONCLUSIONS

In this article we considered all possible correlations between polarizations of initial and final electron and emitted photon in VCR. For experimental observation of these mentioned polarizations and correlations is rather difficult, even in case of classical VCR [1]. Questions of experimental studying of VCR polarization quantum effects were left beyond this work.

REFERENCES

1. V.P. Zrelov. *Vavilov-Cherenkov radiation and its application in high energy physics.* Part I. M.: “Atomizdat”, 1968, 276 p. (in Russian).
2. V.L. Ginzburg. Quantum theory of light radiation by electron uniformly moving in medium //*ZhETF.* 1940, v. 10, p. 589-600, (in Russian)
3. G.N. Afanasiev, M.V. Lyubchenko, Yu.P. Stepanovsky. Fine structure of the Vavilov–Cherenkov radiation //*Proc. Roy. Soc. A.* 2006, v. 462, p. 689-699.
4. A.I. Akhiezer, V.B. Berestecky. *Quantum electrodynamics, 3-d ed.* M.: “Nauka”, 1969, 624 p. (in Russian).

ПОЛЯРИЗАЦИОННЫЕ СВОЙСТВА ИЗЛУЧЕНИЯ ВАВИЛОВА-ЧЕРЕНКОВА

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Рассмотрены поляризационные свойства излучения Вавилова-Черенкова. В рамках квантовой электродинамики с учётом всех поляризаций рассчитана вероятность однофотонного процесса. Найдено соотношение между поляризациями начального и конечного электрона.

ПОЛЯРИЗАЦІЙНІ ВЛАСТИВОСТІ ВИПРОМІНЮВАННЯ ВАВІЛОВА-ЧЕРЕНКОВА

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Розглянуті поляризаційні властивості випромінювання Вавілова-Черенкова. У межах квантової електродинаміки з урахуванням всіх поляризацій розрахована ймовірність однофотонного процесу. Знайдене співвідношення між поляризаціями початкового і кінцевого електрона.