# NONLINEAR EVOLUTION OF HYPERSONIC WAVES WITH AKHIEZER DAMPING AND THEIR ROLE IN COHERENT BREMSSTRAHLUNG IN CRYSTALS

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The asymptotic and numerical analysis of the nonlinear hypersonic wave propagation is fulfilled. Some peculiarities of the linear propagation of a periodic signal, such as the formation of a periodic shock wave, have been found. Applications to coherent bremsstrahlung of fast electrons are pointed out.

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## 1. INTRODUCTION

The studying of possibilities to increase effective occupation numbers by non-equilibrium phonons arouses great interest. In quantum statistical physics [1] creation and annihilation operators are in concordance with two statistic objects: phonons described by the distribution function satisfying kinetic equation and coherent states with occupation numbers going to infinity along with the volume, so that their ratio remains finite (the thermo-dynamic limit transition in statistical description). It is natural to expect that sound waves with defined wave vector, being solutions of the elasticity equations and representing due to their microscopic nature a statistic coherent state of infinite number of bosons with a certain momentum (Bose-condensate), will take an active part in the electron-phonon interaction in coherent bremsstrahlung (CBS) processes. According to these considerations it could be interesting to investigate the hypersonic (HS) wave (microwave ultrasonic waves in English literature) excitation in crystals, paying a special attention to the problem of HS wave propagation under normal conditions when both dissipation and nonlinearity effects play an essential role.

Microwave ultrasonics is widely used in physics, in particular in solid-state physics [2], mechanics, and medicine [3]. The aim of the proposed analysis of HS wave generation and propagation is to discover a mechanism allowing to obtain quite intensive HS oscillations at target thickness usually used in the CBS ($10^{-2}...10^{-1}$ cm thick) generation.

At a temperature higher than 15 K HS waves [2] undergo strong damping and their propagation length is small and equals $\approx 10^{-3}$ cm at frequency $10^{11}$ Hz. As it is shown below, some of existing damping mechanisms in covalent crystals (silicon, germanium, diamond) do not prevent excitation and propagation of large intensity HS waves.

The large intensity hinders propagation of HS waves on macroscopic distances by virtue of nonlinear effects in generation which strongly damp higher harmonics. However, due to reverse influx of energy from the higher harmonics generated by nonlinear interaction and formation of nonlinear dissipative structures of periodic shock waves, the damping lengths of basic and higher harmonics increase up to the damping length of the first harmonic in the linear theory.

As was assumed [10], large intensity of hypersonic oscillations will be needed to obtain an appreciable effect on the CBS from fast electrons.

## 2. CORRELATIONS AND THE PROBLEM

The theory of thermo-elasticity gives the following equation for longitudinal displacement $u(x,t)$ [2]

$$\ddot{u} - s^2\left(u''_{xx} + \alpha\,\dot{u}''_{xx} - N\,u'_x\,u''_{xx}\right) = a(x,t)\,, \qquad (1)$$

where $a(x,t) = \left(\Gamma/\rho_0\right)\int_0^t dt'\,\partial Q(x,t')/\partial x$ is thermo-acoustic force divided by density $\rho_0$, $\Gamma = \gamma K/C$ is Grüneizen parameter, $\gamma$ is the coefficient of linear expansion, $K$ is the elasticity modulus, $C$ is the heat capacity. $Q(x,t)$ is the loss of energy of concentrated stream of energy. $N$ is the non-dimensional coefficient of non-linearity [3], $s$ is the sound velocity. $\alpha = 2sA/\omega^2$ ($\approx 10^{-15}\,\mathrm{sec}^{-1}$ for a silicon crystal) is the dissipative parameter; $A \propto \omega^2$ is the coefficient of spatial damping proportional to square power of the sound frequency $\omega$. $1/A$ is the length of linear damping of the sound.

### 2.1 MECHANISMS OF DAMPING

For the Akhiezer damping connected with the relaxation of lattice deformation the coefficient of spatial damping equals [2,4]

$$A_{\mathrm{Akh}} = K^2\,\frac{9\gamma^2}{C^2}\,\frac{\kappa\omega^2}{\rho_0 s^5}\,\Phi(T),$$

where $\Phi(T) = T$ for $T > \Theta_D$ and $\Phi(T) = \Theta_D^2/T$ for $T < \Theta_D$, $T$ and $\Theta_D$ are the lattice and Debye temperatures, respectively, $\kappa$ is the coefficient of heat conductivity. Coefficients of heat conductivity due to mechanisms of spatial damping and internal frictions (viscosity damping of longitudinal sound) are defined by formulas [2]:

$$A_{\mathrm{therm}} \approx \omega^2 \kappa\, T\, \gamma^2 \rho_0 \big/ \left(sC^2\right),$$
$$A_{\mathrm{visc}} = \left(4\eta'_1/3 + \eta'_2\right)\omega^2 \big/ \left(2\rho_0 s^3\right),$$

respectively. Here $\eta'_1$ and $\eta'_2$ are viscosity coefficients.

In the temperature interval $\approx 20...250\,K$ the damping is great and damping length is equal to $1/A = 10^{-3}$ cm at $\omega \propto 10^{10}$ Hz [2]. If $T > 600\,K$ the main damping mechanism is connected with the generation of point defects [5] and is not proportional to the sound frequency squared. For crystals of silicon, germanium, diamond, etc. the mechanism of Akhiezer damping is the strongest at normal conditions and in the temperature interval $\approx 300...600\,K$.

### 2.2. BOUNDARY PROBLEM ( $x \ge 0,\ t \ge 0$ )

We shall consider the sound excitation in crystals by concentrated energy flux (for example, by intense laser or ion beam). Thin metal film with the width $d$ is supposed to exist at the boundary $x = 0$ and, for simplicity of solution, its sound wave resistance equals the crystal one. The duration $T_b$ of the pulse incident on film boundary is much greater than the time necessary for sound to pass through film-metal energy interaction region, $T_b >> d/s$; then we have $Q(x,t) = E\delta(x)F(t)$ [6]. $E$ is the full input energy per cm$^2$. We shall consider constantly the solution for periodic temporal dependence, $F(t + 2\pi/\omega) = F(t)$ ($0 \le \omega t$ and $\omega t + 2\pi \le T_b$).

The boundary condition becomes

$$u(x=0,t) = u_{00}\int_0^t F(t')dt',\ \dot{u}(x,t) = u_{00}F(t), \qquad (2)$$

where $u_{00} = GE/\left(\rho s^2\right)$ is an effective displacement under the action of energy $E$. The boundary problem for $x \ge 0,\ t \ge 0$ is solved by asymptotic method under the assumption of smallness of relative deformation, $\varepsilon \propto \varepsilon_{xx} = \partial u/\partial x = -(1/s)\partial U/\partial\Theta$, $\varepsilon << 1$, or of the ratio of the displacement velocity to the sound velocity [7]. Expansion has been supposed $u(x,t) = U(\Theta, X) + O(\varepsilon)$ and only the main term is taken into account. Here $X = \varepsilon x$ is the "slow" coordinate and $\Theta = t - x/s$ is the time variable in the frame of reference moving with the velocity of the sound wave in the linear theory. From Eq. (1) we get the Burgers equation for $U_\Theta$:

$$2U_{X\Theta} - \left(N/s^2\right)U_\Theta U_{\Theta\Theta} = (\alpha/s)U_{\Theta\Theta\Theta}.$$

Now one can introduce new non-dimensional variables $\xi = XA$, $\tau = \omega t$, $\vartheta = \omega\Theta = \tau - \eta$, $\eta = \xi/\sigma$. The known substitution [8] $\partial U/\partial\Theta = U_\Theta(\Theta, X) \equiv V(\vartheta,\xi) = \left(u_{00}/T_b\right)(1/\rho)\left((\partial\phi/\partial\vartheta)/\phi(\vartheta,\xi)\right)$ gives equations of the "thermal conductivity" type in new variables, with inversion of time and spatial variables, and boundary condition from Eq. (2) for $f(\tau) = T_b F(\tau/\omega)$ takes the form:

$$\phi_\xi = \phi_{\vartheta\vartheta},\ \phi(\vartheta, \xi = 0) = \exp\left(\rho f(\vartheta)\right), \qquad (3)$$

where $\rho = u_{00}N/(4\mu s^2\omega T_b)$ is the dimensionless parameter of the intensity; $u_{00} \equiv GE/(\rho_0 s^2)$.

### 2.3. SOLUTION. GENERAL EXPRESSIONS

Solution of the problem (3) with phase integral is

$$\phi(\vartheta,\xi) = \left(1/2\sqrt{\pi\xi}\right)\int_{-\infty}^{\infty} d\vartheta' e^{-\Gamma(\vartheta',\vartheta,\xi)}, \qquad (4)$$

$$\Gamma(\vartheta',\vartheta,\xi) = (\vartheta' - \vartheta)^2/(4\xi) - \rho\int_0^{\vartheta'} dt\, f(t).$$

From the normalizing condition $\int_0^{T_b} dtF(t) = 1$ for $F(t) \ge 0$ and definition of $f(\tau) = T_b F(\tau/\omega)$ it follows that the integral

$$\int_0^{\omega T_b} d\tau\, f(\tau) = \omega T_b >> 1$$

and parameter $\rho\,\omega T_b \approx u_{00}/(2\alpha s)$ are large. An asymptotic calculation of integral (4) by standard Laplace method [9] includes the contribution of the vicinity points of the exponent maxima $\vartheta_s(\vartheta,\xi)$:

$$\partial\Gamma/\partial\vartheta' = (\vartheta' - \vartheta)/(2\xi) - \rho f(\vartheta') = 0 \Rightarrow \vartheta' = \vartheta_s(\vartheta,\xi) \qquad (5)$$

The exponent $\Gamma$ has the expansion

$$\Gamma(\vartheta',\vartheta,\xi) = \Gamma_s + (\vartheta' - \vartheta)^2\,\Gamma''_s/2 + ...;$$
$$\Gamma_s = \Gamma(\vartheta_s, \vartheta, \xi); \qquad (6)$$
$$\Gamma''_s = 1/(2\xi) - \rho f'(\vartheta_s) = \gamma''_s/(2\xi),$$

where $\vartheta_s = \vartheta_s(\vartheta,\xi)$, $\gamma''_s = 1 - 2\rho\xi f'(\vartheta_s)$. For the boundary signal in the form $f(\tau) = \sin^2(\tau/2)$ in limits $\rho \to 0$, $\vartheta \to \infty$, when the transient processes are ended, asymptotic result is

$$V(\vartheta,\xi) = \left(u_{00}/(2T_b)\right)\left(1 - e^{-\xi}\cos\vartheta\right).$$

For a **single point of maximum** and conditions $\gamma''_s > 0$, $f'(\vartheta_s) > 0$ the velocity of displacement equals [8]

$$V(\vartheta,\xi) = \frac{u_{00}}{2T_b}\,\frac{f(\vartheta_s)}{1 + h(\vartheta,\xi)e^{\Gamma_s}}.$$

Here $h(\vartheta,\xi) = \sqrt{\left|\gamma''_s\right|}\,\dfrac{\operatorname{erfc}\left(\vartheta/\left(2\sqrt{\xi}\right)\right)}{\operatorname{erfc}\left(-\vartheta_s\sqrt{\left|\gamma''_s\right|/4\xi}\right)}$;

$$\Gamma_s = \rho^2\xi f^2(\vartheta_s) - \rho\int_0^{\vartheta_s} d\tau f(\tau),$$

$$\operatorname{erfc} z = \left(2/\sqrt{\pi}\right)\int_z^{\infty} ds\, e^{-s^2}.$$

The computer simulation of the displacement velocity calculation by the means independent from the asymptotic method of the calculation used for integral (4) was fulfilled. The computer calculations allow investigating of the displacement velocity at values of variable $\xi > 0$ up to $\approx 3{,}03\pi\xi_{char}$ for all $\vartheta$, both negative and positive. Here $\xi_{char} = 1/\left(2\rho\max f'(\vartheta_s)\right)$.

## 3. NONLINEAR HS WAVES. STRUCTURES

If to write the exponential term in the form

$$e^{\Gamma_s}h(\vartheta,\xi) = \exp\Delta(\vartheta,\xi) \equiv \exp\Delta,$$

where $\Delta = \Gamma_s + \ln h(\vartheta,\xi)$, we have expression for the displacement velocity in the form of shock wave

$$V(\vartheta,\xi) = \left(u_{00}/(2T_b)\right) f(\vartheta_s) \left(1 - \mathrm{tg}\,\Delta(\vartheta,\xi)\right). \qquad (7)$$

At small $\xi$ and large and negative $\Delta(\vartheta,\xi)$ the displacement velocity equals

$$V(\vartheta,\xi) = \left(u_{00}/(2T_b)\right) f(\vartheta_s). \qquad (8)$$

For periodic boundary signal the displacement velocity has the form of nonlinear HS wave at frequencies $\omega \geq 2\pi 10^9$. In the interval of distances $0 < \xi < \xi_{\mathrm{char}}$, by iteration from the formula (5) it follows $\vartheta_s = \vartheta + 2\rho\,\xi\, f(\vartheta)$. These formulas allow to find out the effect of steepening of the wave and to investigate its profile. The steepening of the wave is demonstrated by numerical calculations and is presented in Fig. 1.

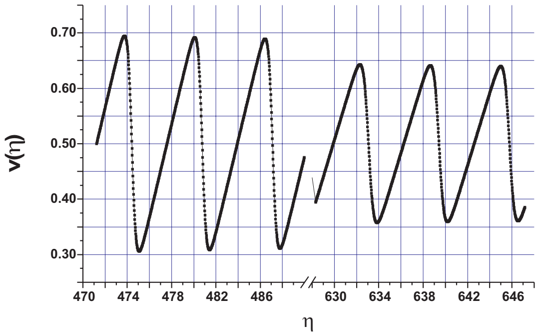


***Fig. 1.*** *The velocity of displacement depending on the dimensionless spatial coordinate* $\eta = 2\pi X/\lambda$*, at* $\rho = 10$*,* $\sigma = 0.001$ *(these parameters may correspond to frequencies* $10^{11}$ *Hz,* $\tau = 6000\pi$*). It is possible to see the steepening of the wave profile*

### 3.1. ENRICHMENT BY HIGH HARMONICS

In the above mentioned interval the wave enrichment by means of higher harmonics happens. For harmonic boundary signal a forward front of the wave in its period is steepening. The back front becomes less steep than in the boundary signal $F(t)$. The computer simulation of these processes is reflected in Fig. 1 and 2.

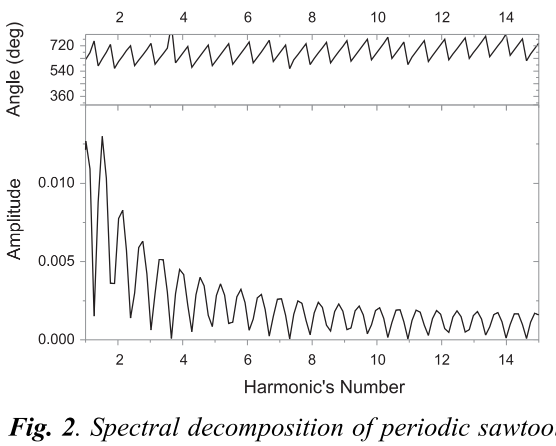


***Fig. 2****. Spectral decomposition of periodic sawtooth shock wave without zero harmonic (parameters are the same of previous figure). Both from this diagram, previous figure and analytical investigations it may be seen the wave enrichment by higher harmonics*

From the solution (8) at $N > 0$, using non-obvious expression (5) for a point of maximum, it is possible to make conclusion: if the boundary velocity is positive, $\dot{u}_0(t) = u_{00}F(t) > 0$, with the propagation deeper into crystal the forward front of the wave undergoes steepening ($dF/dt > 0$) and the back one – tension ($dF/dt < 0$). Actually, if maximum of $F(t)$ is achieved at point $t_m$, for given $X$ the maximum value of $U_\Theta$ is achieved at point $\Theta_m = t_m - Xf(t_m)Nu_{00}/s^2$, i.e. the velocity reaches its maximum value at point $X$ at earlier "moment of time" from the start of the signal, than in the linear theory.

### 3.2. SAWTOOTH PERIODIC SHOCK WAVE

At values $(\vartheta;\vartheta_s) >> 2\sqrt{\pi\xi}$ there are two maximum points in the integral (4). The displacement velocity takes the form

$$V(\vartheta,\xi) = \frac{u_{00}}{T_b}\,\frac{e^{-\Gamma_1} f_1\Big/\sqrt{|\gamma_1''|} + e^{-\Gamma_2} f_2\Big/\sqrt{|\gamma_2''|}}{e^{-\Gamma_1}\Big/\sqrt{|\gamma_1''|} + e^{-\Gamma_2}\Big/\sqrt{|\gamma_2''|}}$$

or another form is

$$V(\vartheta,\xi) = U_\Theta(\Theta, X)\big|_{\Theta=\omega t;\, X=\xi/A}$$
$$= \left(u_{00}/(2T_b)\right)\left(f_1 + f_2 + (f_1 - f_2)\,\mathrm{th}(\Delta\Gamma)\right), \qquad (9)$$

where $f_j = f(\vartheta_j)$, $\Gamma_j = \Gamma(\vartheta_j,\vartheta,\xi)$, $j = 1, 2$. The exponent maximum points; $\vartheta_j = \vartheta_j(\vartheta,\xi)$, are found from Eq. (5) and $\Delta\Gamma = \Gamma_1 - \Gamma_2 - (1/2)\ln\left|\gamma_2''/\gamma_1''\right| \equiv$ $\equiv \Delta\Gamma(\vartheta,\xi)$. Solution (9) has the structure of the sawtooth shock wave of the displacement velocity, shown in fig. 3.

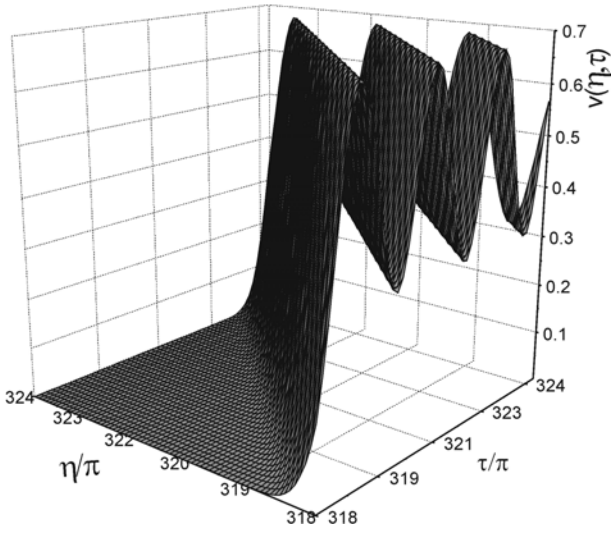


***Fig. 3.*** *Sawtooth shock waves with a sharp steepness depending on intensity of shock wave being propagated along diagonal,* $\tau/\pi = 2t/T$ *and* $\eta/\pi = 2X/\lambda$

## 4. MAIN RESULTS. DISCUSSION

The nonlinear propagation of highly intensive HS waves under dissipative conditions applicable for covalent crystals (silicon, germanium, diamond) was investigated to find the possibilities and conditions for propagation of HS waves in these crystals. Formulated nonlinear elasticity equations with dissipation are reduced to Buergers nonlinear equation with spatial and time inversion of variables expressed in the coordinate system moving with the sound velocity of the linear theory.

The nonlinear wave of large amplitude undergoes temporal evolution in space at some distance $X_{\text{char}} = 1/\rho A$. Forward front of a nonlinear wave with a steep profile separates the area which the wave has not yet reached from the area of a nonlinear solution. The analysis of the obtained solution shows that with the increase of the distance $X$ from the boundary the front line of the wave turns steeper, while trailing one becomes flatter.

The obtained results allow to state, that at $\omega = 2\pi 10^{10}$ Hz, $u_{00} \le 10^{-8}$ cm, $T_b = 10^{-6}$ sec the penetration depth of the nonlinear HS wave achieves several tenth of centimeter.

The asymptotic and the numerical analysis of quadratures have shown the following peculiarities of the nonlinear propagation of periodic signal. The wave was enriched with high harmonics. On the distances a little exceeding $3.1\pi X_{\text{char}}$ the signal is propagated as periodic sawtooth shock wave with weak damping of oscillations.

At extra large times $t >> 3\pi X_{\text{char}}/s$ the spatial behavior qualitatively varies for different boundary impulses. For example, in the solution stated in [8] for periodical in space sources the nonlinear wave damps with time. Its amplitude tends to zero by exponential law.

The steepness of a profile of this front does not depend on the shape of the boundary impulse; also it is defined by the intensity of nonlinear shock wave. Spectral expansion of the shock wave front at the period interval is enriched with higher harmonics for $\rho = 10$ — up to the tenth and higher harmonics.

Thus, investigation of nonlinear propagation of waves in elastic bodies with dissipation shows that the spatial-temporal structure of nonlinear waves is similar to the structure of a shock wave. When time average of the boundary value is not equal to zero, shock wave will propagate deep at such distances at which nonlinear signal will be spread during the time of switching of the boundary signal period (shock waves propagates until boundary signal is switched on).

In accordance with these results the thermo-elastic mechanism of wave excitation is of the essential interest.

## 5. APPLICATIONS AND DEVELOPMENT

Earlier the investigations of this kind were made in order to study the beam interaction with solids and magneto-sonic wave penetration in conducting media [7]. In paper [7] nonlinear sound modulated magnetic field and generated magneto-acoustic waves which penetrated in conducting medium.

Within STCU project № 285 it was made an attempt to analyze the optimal conditions in order to increase the CBS emission of fast electrons. This attempt belongs to V.F. Boldyshev and Yu.P. Peresunko [10], Fig. 4. For Si crystal the displacement amplitude of about $0.125 \cdot 10^{-8}$ cm should be achieved at frequency $\omega \propto 2\pi 10^{10}$ Hz.

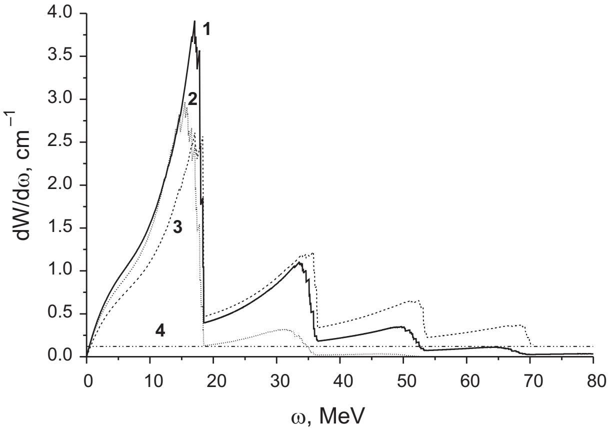


***Fig. 4.*** *Spectral radiation intensity of the electron with energy 1 GeV, moving in the Si crystal along the axis (0 0 1) in the field of transverse HS wave (HSW) with the frequency* $\omega = 2\pi 10^{10} s^{-1}$*; HSW propagates in the same direction. Solid curve (1) corresponds to the HSW amplitude* $a_s = 0.05a$*, dashed line (2) −* $a_s = 0.025a$*, dotted line (3) −* $a_s = 0.10a$*, chain line (4) − to the amorphous level.* $a = 5.43 \cdot 10^{-8}$cm *is the crystal lattice parameter*

Fig. 4 displays (with shown parameters) the spectral intensity $dW/d\omega$ of the electron radiation calculated with the obtained formulae. Also diagrams are given here for various values of the HS wave amplitude. It is evident from Fig. 4 that the spectral radiation intensity has distinct maxima at the energies of $l \cdot 18.98$ MeV, $l = 1,2,...$. It is interesting to consider how $dW/d\omega$ depends on amplitude of the hypersonic wave $a_s$. The obtained results make it possible to find an optimal value of $a_s$ for the effect under consideration. This value equals $a_s^{\text{eff}} \propto 0.05\,a$ (remind for comparison that the mean-square thermal displacement for Si at room temperature is equal to $0.014\,a$). We can see from the given picture that the CBS contribution exceeds the amorphous layer 30-35 times in the presence of the HS wave with the optimal amplitude at the region of the first maximum. It should be noted that analogous values for radiation intensity can be found for the case of the HS wave with the longitudinal polarization

The analysis of influences on radiation of fast electrons in crystals and the analysis of nonlinear HS waves with actual structure as well as actual physical requirements of HS wave excitation of various intensities should be a subject of the further investigations (see, e.g. [11]).

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## НЕЛИНЕЙНАЯ ЭВОЛЮЦИЯ ГИПЕРЗВУКОВЫХ ВОЛН С АХИЕЗЕРОВСКИМ ЗАТУХАНИЕМ И ИХ РОЛЬ В КОГЕРЕНТНОМ ТОРМОЗНОМ ИЗЛУЧЕНИИ В КРИСТАЛЛАХ

***А.А. Водяницкий***

Представлены асимптотический и численный анализы распространения нелинейных гиперзвуковых волн. Исследовано формирование периодических нелинейной и ударной волн для периодического граничного сигнала. Указаны приложения к когерентному тормозному излучению быстрых электронов.

## НЕЛІНІЙНА ЕВОЛЮЦІЯ ГІПЕРЗВУКОВИХ ХВИЛЬ З АХІЕЗЕРОВСЬКИМ ЗАГАСАННЯМ ТА ЇХ РОЛЬ В КОГЕРЕНТНОМУ ГАЛЬМІВНОМУ ВИПРОМІНЮВАННІ В КРИСТАЛАХ

***О.А. Водяницький***

Представлені асимптотичний і чисельний аналізи розповсюдження нелінійних гіперзвукових хвиль. Досліджено формування періодичних нелінійної та ударної хвиль для періодичного граничного сигналу. Вказано на застосування до когерентного гальмівного випромінювання швидких електронів.