CLOTHED PARTICLE REPRESENTATION IN QUANTUM FIELD THEORY: BOOST GENERATORS OF THE POINCARÉ GROUP

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The method of the unitary clothing transformations is used to construct the generators of the Poincaré group in the instant form of relativistic dynamics in the second and third orders in the coupling constant. It is shown that the respective algebra of generators is fulfilled up to the sixth order. The relativistic invariance of the mass and vertex corrections derived is proven.

PACS: 21.45.+v; 24.10. Jv; 11.80.-m

1. INTRODUCTION. GENERATORS OF THE POINCARÉ GROUP

Poincaré invariance requires that there exists a unitary representation of the Poincaré group defined in a Hilbert space. Corresponding ten generators fulfill the set of commutation relations:

$$[N_i, N_k] = -i\varepsilon_{ikl} J_l \, , \; [J_i, N_k] = i\varepsilon_{ikl} N_l \, , \; [J_i, J_k] = i\varepsilon_{ikl} J_l \, ,$$

$$[H, N_l] = -iP_l \, , \; [P_i, N_l] = -i\delta_{il} H \, , \; [J_i, P_k] = i\varepsilon_{ikl} J_l \, ,$$

$$[P_i, H] = 0 \, , \; [J_i, H] = 0 \, ,$$

$$[P_i, P_k] = 0 \, , \; (i,k,l = 1,2,3) \, . \tag{1}$$

Here H is the Hamiltonian operator, P_i, J_i and N_i are three components of the momentum, angular momentum and boost operators, respectively. In the instant form of relativistic dynamics (after Dirac), four operators H and $\mathbf{N}$ carry interaction.

One-particle eigenstates of H and $\mathbf{N}$ differ from states of their free parts:

$$H(\alpha)\left|\alpha^{\dagger}\Omega\right\rangle \neq H_F(\alpha)\left|\alpha^{\dagger}\Omega\right\rangle , \tag{2}$$

$$\mathbf{N}(\alpha)\left|\alpha^{\dagger}\Omega\right\rangle \neq \mathbf{N}_F(\alpha)\left|\alpha^{\dagger}\Omega\right\rangle . \tag{3}$$

Here α denotes the whole set of creation and destruction operators of the bare particles with bare masses which interact by means of the bare coupling.

Therefore, one may ask a question whether there exists such a set of creation and destruction operators α_c for which the total Hamiltonian $K(\alpha_c)$, the boost operators $\mathbf{B}(\alpha_c)$ and their free counterparts $K_F(\alpha_c)$ and $\mathbf{B}_F(\alpha_c)$ would fulfill the conditions

$$K(\alpha_c)\left|\alpha_c{}^{\dagger}\Omega\right\rangle = K_F(\alpha_c)\left|\alpha_c{}^{\dagger}\Omega\right\rangle , \tag{4}$$

$$\mathbf{B}(\alpha_c)\left|\alpha_c{}^{\dagger}\Omega\right\rangle = \mathbf{B}_F(\alpha_c)\left|\alpha_c{}^{\dagger}\Omega\right\rangle , \tag{5}$$

keeping the algebra?

Greenberg and Schweber [1] assumed that such set of operators α_c existed and was connected with the initial set of operators α via the unitary transformation which kept S-operator intact

$$\alpha_c = e^{-R}\alpha e^{R} \, , \; \alpha = e^{R}\alpha_c e^{-R} \, , \; R^{\dagger} = -R \, . \tag{6}$$

Generator R of the unitary transformation, called “clothing”, was chosen in a way the Hamiltonian operator

$$H(\alpha) = e^{R} H\left(\alpha_c\right) e^{-R} \equiv K\left(\alpha_c\right) = K_F\left(\alpha_c\right) + K_I\left(\alpha_c\right) , \tag{7}$$

satisfied the condition (4).

However, herewith it was not obvious that simultaneously the boost operator could be presented in a similar form

$$\mathbf{N}(\alpha) = e^{R}\mathbf{N}\left(\alpha_c\right)e^{-R} \equiv \mathbf{B}\left(\alpha_c\right) = \mathbf{B}_F\left(\alpha_c\right) + \mathbf{B}_I\left(\alpha_c\right) , \tag{8}$$

so that the condition (5) would be automatically fulfilled.

2. CLOTHED PARTICLES AND BOOST GENERATORS

The problem will be analyzed using the quantum field model in which a spinor fermion (nucleon) field interacts with a neutral meson (pion) field by means of the Yukawa-type threelinear pseudoscalar (PS) coupling. Within the model,

$$H = H(\alpha) = H_F(\alpha) + H_I(\alpha) \, , \tag{9}$$

$$\mathbf{N} = \mathbf{N}(\alpha) = \mathbf{N}_F(\alpha) + \mathbf{N}_I(\alpha) \, . \tag{10}$$

The free parts of H and N have the form

$$H_F = \int d\mathbf{k}\omega_{\mathbf{k}} a^{\dagger}(\mathbf{k})a(\mathbf{k}) + \int d\mathbf{p}E_{\mathbf{p}} \sum_r \left[b^{\dagger}\left(\mathbf{p},r\right) b\left(\mathbf{p},r\right) \right.$$

$$\left. + d^{\dagger}\left(\mathbf{p},r\right) d\left(\mathbf{p},r\right) \right] , \tag{11}$$

$$\mathbf{N}_F(\alpha) = \mathbf{N}_{mes} + \mathbf{N}_{ferm} \, , \tag{12}$$

$$\mathbf{N}_{mes} = \frac{i}{2}\int d\mathbf{k}d\mathbf{k'} \frac{\omega_{\mathbf{k'}}\omega_{\mathbf{k}} + \mathbf{k'}\mathbf{k} + \mu^2}{\sqrt{\omega_{\mathbf{k'}}\omega_{\mathbf{k}}}}$$

$$\times \frac{\partial\delta\left(\mathbf{k}-\mathbf{k'}\right)}{\partial\mathbf{k}} a^{\dagger}(\mathbf{k'})a(\mathbf{k}) \, , \tag{13}$$

$$\mathbf{N}_{ferm} = \frac{i}{2}\int d\mathbf{p}' d\mathbf{p}\sqrt{\frac{m}{E_{\mathbf{p}'}}}\sqrt{\frac{m}{E_{\mathbf{p}}}}E_{\mathbf{p}}\frac{\partial\delta(\mathbf{p}'-\mathbf{p})}{\partial\mathbf{p}}$$
$$\times\sum_{r,r'}\left[u^\dagger(\mathbf{p}',r')u(\mathbf{p},r)b^\dagger(\mathbf{p}',r')b(\mathbf{p},r)\right.$$
$$\left.-v^\dagger(\mathbf{p}',r')v(\mathbf{p},r)d^\dagger(\mathbf{p}',r')d(\mathbf{p},r)\right]. \quad (14)$$

Here $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2+m^2}$, $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2+\mu^2}$, m and μ are nucleon and meson physical masses; $\mathbf{p}$ and $\mathbf{k}$ are nucleon and meson momenta; r is the spin projection index. The creation (destruction) operators of mesons $a^\dagger(\mathbf{k})\left(a(\mathbf{k})\right)$ and the same for fermions $b^\dagger(\mathbf{p},r)\left(b(\mathbf{p},r)\right)$ and antifermions $d^\dagger(\mathbf{p},r)\left(d(\mathbf{p},r)\right)$ satisfy the commutation relations

$$\left[a(\mathbf{k}),a^\dagger(\mathbf{k}')\right] = \delta(\mathbf{k}-\mathbf{k}');$$
$$\left\{b(\mathbf{p},r),b^\dagger(\mathbf{p}',r')\right\} = \delta_{r,r'}\delta(\mathbf{p}-\mathbf{p}');$$
$$\left\{d(\mathbf{p},r),d^\dagger(\mathbf{p}',r')\right\} = \delta_{r,r'}\delta(\mathbf{p}-\mathbf{p}'). \quad (15)$$

The interaction parts of H and $\mathbf{N}$ have the form

$$H_I(\alpha) = V(\alpha) + M_{ren,mes}(\alpha)$$
$$+M_{ren,ferm}(\alpha) + V_{ren}(\alpha); \quad (16)$$
$$\mathbf{N}_I(\alpha) = \mathbf{W}(\alpha) + \mathbf{N}_{ren,mes}(\alpha)$$
$$+\mathbf{N}_{ren,ferm}(\alpha) + \mathbf{W}_{ren}(\alpha). \quad (17)$$

The interaction operators V and $\mathbf{W}$ are written as

$$V(\alpha) = \int d\mathbf{k}\hat{V}^{\mathbf{k}}a(\mathbf{k}) + H.c., \quad (18)$$

$$\hat{V}^{\mathbf{k}} = \int d\mathbf{p}' d\mathbf{p}\sum_{r,r'}\sum_{i,j}F_i^\dagger(\mathbf{p}',r')V_{i,j}^{\mathbf{k}}(\mathbf{p}',r';\mathbf{p},r)F_j(\mathbf{p},r), \quad (19)$$

$$V_{i,j}^{\mathbf{k}}(\mathbf{p}',r';\mathbf{p},r) = \frac{ig}{(2\pi)^{3/2}}\frac{m}{\sqrt{2\omega_{\mathbf{k}}E_{\mathbf{p}'}E_{\mathbf{p}}}}\delta(\mathbf{p}+\mathbf{k}-\mathbf{p}')$$
$$\times\bar{U}_i(\mathbf{p}',r')\gamma_5 U_j(\mathbf{p},r), \quad (20)$$

$$\boldsymbol{W}(\alpha) = \int d\mathbf{k}\hat{\boldsymbol{W}}^{\mathbf{k}}a(\mathbf{k}) + H.c., \quad (21)$$

$$\hat{\boldsymbol{W}}^{\mathbf{k}} = \int d\mathbf{p}' d\mathbf{p}\sum_{r,r'}\sum_{i,j}F_i^\dagger(\mathbf{p}',r')\boldsymbol{W}_{i,j}^{\mathbf{k}}(\mathbf{p}',r';\mathbf{p},r)F_j(\mathbf{p},r), \quad (22)$$

$$\boldsymbol{W}_{i,j}^{\mathbf{k}}(\mathbf{p}',r';\mathbf{p},r) = \frac{-g}{(2\pi)^{3/2}}\frac{m}{\sqrt{2\omega_{\mathbf{k}}E_{\mathbf{p}'}E_{\mathbf{p}}}}\frac{\partial\delta(\mathbf{p}+\boldsymbol{k}-\mathbf{p}')}{\partial\mathbf{k}}$$
$$\times\bar{U}_i(\mathbf{p}',r')\gamma_5 U_j(\mathbf{p},r). \quad (23)$$

Here g – physical coupling constant and the notations are as follows

$$U(\mathbf{p},r) = \begin{pmatrix}U_1(\mathbf{p},r)\\U_2(\mathbf{p},r)\end{pmatrix} = \begin{pmatrix}u(\mathbf{p},r)\\v(-\mathbf{p},r)\end{pmatrix},$$
$$F(\mathbf{p},r) = \begin{pmatrix}F_1(\mathbf{p},r)\\F_2(\mathbf{p},r)\end{pmatrix} = \begin{pmatrix}b(\mathbf{p},r)\\d^\dagger(-\mathbf{p},r)\end{pmatrix}, \quad (24)$$

where $u(\mathbf{p},r)$ and $v(-\mathbf{p},r)$ are spinors satisfying usual Dirac equations with physical masses.

The mass counterterms are presented as

$$M_{ren,mes}(\alpha) = \frac{\mu_0^2-\mu^2}{4}\int\frac{d\mathbf{k}}{\omega_{\mathbf{k}}}\left(a^\dagger(\mathbf{k})a(\mathbf{k})\right.$$
$$\left.+a(\mathbf{k})a(-\mathbf{k}) + H.c.\right), \quad (25)$$

$$M_{ren,ferm}(\alpha) = m(m_0-m)\int\frac{d\mathbf{p}}{E_{\mathbf{p}}}\sum_{r,r'}\sum_{i,j}F_i^\dagger(\mathbf{p},r')$$
$$\times M_{i,j}(\mathbf{p},r';\mathbf{p},r)F_j(\mathbf{p},r), \quad (26)$$

$$\mathbf{N}_{ren,mes}(\alpha) = \frac{i}{4}\left(\mu_0^2-\mu^2\right)\int d\mathbf{k}d\mathbf{k}'\frac{1}{\sqrt{\omega_{\mathbf{k}}\omega_{\mathbf{k}'}}};$$
$$\times\left(a^\dagger(\mathbf{k})a(\mathbf{k}') + a(\mathbf{k})a(-\mathbf{k}') + H.c.\right)\frac{\partial}{\partial\mathbf{k}}\delta(\mathbf{k}-\mathbf{k}'), \quad (27)$$

$$\mathbf{N}_{ren,ferm}(\alpha) = im(m_0-m)\int d\mathbf{p}d\mathbf{p}'\frac{1}{\sqrt{E_{\mathbf{p}}E_{\mathbf{p}}}}\sum_{r,r'}\sum_{i,j}F_i^\dagger(\mathbf{p}',r')$$
$$\times M_{i,j}(\mathbf{p}',r';\mathbf{p},r)F_j(\mathbf{p},r)\frac{\partial}{\partial\mathbf{p}}\delta(\mathbf{p}-\mathbf{p}'), \quad (28)$$

where $M_{i,j}(\mathbf{p}',r';\mathbf{p},r) = \bar{U}_i(\mathbf{p}',r')U_j(\mathbf{p},r)$, m_0 and μ_0 – nucleon and meson bare masses.

The vertex counterterms V_{ren} и $\mathbf{W}_{ren}$ are determined by the same formulae (18)-(23) as for V and $\mathbf{W}$ but with g substituted by $\delta g \equiv g_0 - g$ where g_0 is the bare coupling constant.

Total Hamiltonian (9) and boost (10) consist of the g^1 - order operators (18)-(23), called bad, which prevent satisfaction of the conditions (4)-(5). Their removal by the first clothing transformation [2] with the generator

$$R = \int d\mathbf{k}\hat{R}_c^{\mathbf{k}}a_c(\mathbf{k}) - H.c., \quad (29)$$

$$\hat{R}_c^{\mathbf{k}} = \int d\mathbf{p}' d\mathbf{p}\sum_{r,r'}\sum_{i,j}F_{c,i}^\dagger(\mathbf{p}',r')R_{i,j}^{\mathbf{k}}(\mathbf{p}',r';\mathbf{p},r)F_{c,j}(\mathbf{p},r), \quad (30)$$

$$R_{i,j}^{\mathbf{k}}(\mathbf{p}',r';\mathbf{p},r) = \frac{V_{i,j}^{\mathbf{k}}(\mathbf{p}',r';\mathbf{p},r)}{(-1)^{i-1}E_{\mathbf{p}'} - (-1)^{j-1}E_{\mathbf{p}} - \omega_{\mathbf{k}}},$$
$$(i,j=1,2), \quad (31)$$

leads to

$$K(\alpha_c) = H_F(\alpha_c) + M_{ren}(\alpha_c) + V_{ren}(\alpha_c) + \frac{1}{2}[R,V]$$
$$+[R,M_{ren}] + \frac{1}{3}\left[R,[R,V]\right] + \ldots, \quad (32)$$

$$\mathbf{B}(\alpha_c) = \mathbf{N}_F(\alpha_c) + \mathbf{N}_{ren}(\alpha_c) + \mathbf{W}_{ren}(\alpha_c) + \frac{1}{2}[R,\mathbf{W}]$$
$$+[R,\mathbf{N}_{ren}] + \frac{1}{3}\left[R,[R,\mathbf{W}]\right] + \ldots. \quad (33)$$

Thus, $K(\alpha_c)$ and $\mathbf{B}(\alpha_c)$ appear free from bad terms up to second order in g.

3. MASS AND VERTEX RENORMALIZATION IN BOOST GENERATORS

After first clothing, $K(\alpha_c)$ (32) and $\mathbf{B}(\alpha_c)$ (33) contain bad terms of the g^2 and higher orders. In $K(\alpha_c)$, the operators $[R,V]$ and

$M_{ren} = M_{ren,mes} + M_{ren,ferm}$ have parts of the g^2-order, as well as $[R,\mathbf{W}]$ and $\mathbf{N}_{ren} = \mathbf{N}_{ren,mes} + \mathbf{N}_{ren,ferm}$ in $\mathbf{B}(\alpha_c)$.

After normal ordering, $[R,V]$ involves parts bilinear in the meson operators that can be cancelled by the respective counterparts from $M_{ren,mes}$. Then we have meson mass shift of the g^2-order [2]

$$\delta\mu^2 \equiv \mu_0^2 - \mu^2 = \frac{2g^2}{(2\pi)^3}\int\frac{d\mathbf{p}}{E_p}\left\{1+\frac{\mu^4}{4(pk)^2-\mu^4}\right\}, \quad (34)$$

where $p = (E_\mathbf{p},\mathbf{p})$ and $k = (\omega_\mathbf{k},\mathbf{k})$. At the same time, another part of $[R,V]$ partly cancel with $M_{ren,ferm}$ in the same order, giving the fermion mass shift [3]

$$\delta m \equiv m_0 - m = \frac{g^2}{4m(2\pi)^3}\left[I_1(p)+I_2(p)\right],$$

$$I_1(p) = \int\frac{d\mathbf{k}}{\omega_k}pk\left\{\frac{1}{\mu^2-2pk}-\frac{1}{\mu^2+2pk}\right\},$$

$$I_2(p) = \int\frac{d\mathbf{q}}{E_q}\left\{\frac{m^2-pq}{2\left[m^2-pq\right]-\mu^2}+\frac{m^2+pq}{2\left[m^2+pq\right]-\mu^2}\right\}, \quad (35)$$

with $q = (E_q,\boldsymbol{q})$.

Having fixed the mass corrections in the g^2-order, we immediately verify that similar terms in the operators $[R,\mathbf{W}]$, $\mathbf{N}_{ren,mes}$ and $\mathbf{N}_{ren,ferm}$ cancel.

Operator $V_{ren} + [R,M_{ren}] + \frac{1}{3}[R,[R,V]]$ contains terms of the g^3-order which replicate the operator structure of the interaction operator V and, thus, start the program of vertex renormalization .

Using normal ordering of fermionic operators, several part of the commutator $[R,[R,V]]$ is cancelled with some part of the commutator $[R,M_{ren}]$, providing the meson and nucleon wave function renormalization in the lowest order in g. At the same time, another part of the commutator $[R,[R,V]]$ is cancelled with the part of V_{ren}, determining the charge shift in the g^3-order [4,5]:

$$\delta g = \frac{g^3}{(2\pi)^3 m^2}\left(\int\frac{d\mathbf{q}}{\omega_{\mathbf{q}'}}\frac{p'q'}{\mu^2-2p'q}\frac{m^2+p'p-p'q'}{\mu^2-2pq}\right.$$

$$+\int\frac{d\mathbf{q}}{E_\mathbf{q}}\frac{m^2-p'q}{2\left[m^2+pq\right]-\mu^2}\frac{m^2+p'q-p'k}{\mu^2-2kq}$$

$$\left.+\int\frac{d\mathbf{q}}{E_\mathbf{q}}\frac{m^2+p'q}{2\left[m^2+p'q\right]-\mu^2}\frac{m^2-p'q-p'k}{\mu^2+2kq}\right). \quad (36)$$

Here $E_\mathbf{q} = \sqrt{\mathbf{q}^2+m^2}$; $\omega_{\mathbf{q}'} = \sqrt{\mathbf{q}'^2+\mu^2}$; $p' = (E_{\mathbf{p}'},\mathbf{p}')$; $q' = (\omega_{\mathbf{q}'},\mathbf{q}')$.

Having fixed the vertex correction in the g^3-order, we immediately verify that similar terms in the operators $[R,[R,\mathbf{W}]]$, $\mathbf{W}_{ren}$ and $[R,\mathbf{N}_{ren}]$ cancel.

4. RELATIVISTIC INTERACTIONS AND BOOST GENERATORS

Selecting bad terms of the g^2-order in $K(\alpha_c)$, we further remove them using the second clothing transformation. After that, applying this transformation to $\mathbf{B}(\alpha_c)$, we see that the latter does not contain bad terms of the g^2-order too. Proceeding in such a way, we remove bad terms from $K(\alpha_c)$ and $\mathbf{B}(\alpha_c)$ up to third order in g. The remaining bad terms of higher orders must be removed via successive clothing unitary transformation.

Thus, up to g^4-order, we have

$$K_I(\alpha_c) = K^{(2)}(\alpha_c) + K^{(3)}(\alpha_c) + O(g^4); \quad (37)$$

$$\mathbf{B}_I(\alpha_c) = \mathbf{B}^{(2)}(\alpha_c) + \mathbf{B}^{(3)}(\alpha_c) + O(g^4). \quad (38)$$

Here the operators of physical interactions between physical particles in the second and third orders are as follows

$$K^{(2)}(\alpha_c) = K(NN\to NN) + K(\bar{N}\bar{N}\to\bar{N}\bar{N})$$
$$+K(N\bar{N}\to N\bar{N}) + K(\pi N\to\pi N)$$
$$+K(\pi\bar{N}\to\pi\bar{N}) + K(\pi\pi\leftrightarrow N\bar{N}), \quad (39)$$

$$K^{(3)}(\alpha_c) = K(NN\leftrightarrow\pi NN) + K(\bar{N}\bar{N}\leftrightarrow\pi\bar{N}\bar{N})$$
$$+K(N\bar{N}\leftrightarrow\pi N\bar{N}) + K(N\bar{N}\leftrightarrow\pi\pi\pi)$$
$$+K(\pi N\leftrightarrow\pi\pi N) + K(\pi\bar{N}\leftrightarrow\pi\pi\bar{N}) + \ldots. \quad (40)$$

Similarly, the respective operators in $\mathbf{B}_I(\alpha_c)$ have the form

$$\mathbf{B}^{(2)}(\alpha_c) = \mathbf{B}(NN\to NN) + \mathbf{B}(\bar{N}\bar{N}\to\bar{N}\bar{N})$$
$$+\mathbf{B}(N\bar{N}\to N\bar{N}) + \mathbf{B}(\pi N\to\pi N)$$
$$+\mathbf{B}(\pi\bar{N}\to\pi\bar{N}) + \mathbf{B}(\pi\pi\leftrightarrow N\bar{N}); \quad (41)$$

$$\mathbf{B}^{(3)}(\alpha_c) = \mathbf{B}(NN\leftrightarrow\pi NN) + \mathbf{B}(\bar{N}\bar{N}\leftrightarrow\pi\bar{N}\bar{N})$$
$$+\mathbf{B}(N\bar{N}\leftrightarrow\pi N\bar{N}) + \mathbf{B}(N\bar{N}\leftrightarrow\pi\pi\pi)$$
$$+\mathbf{B}(\pi N\leftrightarrow\pi\pi N) + \mathbf{B}(\pi\bar{N}\leftrightarrow\pi\pi\bar{N}) + \ldots. \quad (42)$$

In Eqs. (39) – (42), the transparent notations N, $\bar{N}$ and π are used to distinguish clothed (physical) nucleon, antinucleon and pion, respectively.

For brevity, in what follows we omit the spin indices.

The nucleon-nucleon interaction operator is equal to

$$K(NN\to NN) = \frac{1}{2}\int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}'_1 d\mathbf{p}'_2 d\mathbf{k}$$
$$\times R_{11}^{\mathbf{k}}(\mathbf{p}'_1;\mathbf{p}_1)\cdot V_{11}^{-\mathbf{k}}(\mathbf{p}'_2;\mathbf{p}_2)$$
$$\times F_1^\dagger(\mathbf{p}'_1)F_1^\dagger(\mathbf{p}'_2)F_1(\mathbf{p}_1)F_1(\mathbf{p}_2) + H.c. \quad (43)$$

The antinucleon-antinucleon interaction operator has the form

$$K\left(\bar{N}\bar{N}\rightarrow\bar{N}\bar{N}\right)=\frac{1}{2}\int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_1' d\mathbf{p}_2' d\mathbf{k}$$
$$\times R_{22}^{\mathbf{k}}\left(\mathbf{p}_1';\mathbf{p}_1\right)\cdot V_{22}^{-\mathbf{k}}\left(\mathbf{p}_2';\mathbf{p}_2\right)$$
$$\times F_2\left(\mathbf{p}_1'\right)F_2\left(\mathbf{p}_2'\right)F_2^{\dagger}\left(\mathbf{p}_1\right)F_2^{\dagger}\left(\mathbf{p}_2\right)+H.c. \tag{44}$$

The nucleon-antinucleon interaction operator can be presented as

$$K\left(N\bar{N}\rightarrow N\bar{N}\right)=\frac{1}{2}\int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_1' d\mathbf{p}_2' d\mathbf{k}$$
$$\times\left\{R_{11}^{\mathbf{k}}\left(\mathbf{p}_1';\mathbf{p}_1\right)\cdot V_{22}^{-\mathbf{k}}\left(\mathbf{p}_2;\mathbf{p}_2'\right)-R_{12}^{\mathbf{k}}\left(\mathbf{p}_1';\mathbf{p}_2'\right)\cdot V_{21}^{-\mathbf{k}}\left(\mathbf{p}_2;\mathbf{p}_1\right)\right.$$
$$\left.+R_{22}^{\mathbf{k}}\left(\mathbf{p}_2;\mathbf{p}_2'\right)\cdot V_{11}^{-\mathbf{k}}\left(\mathbf{p}_1';\mathbf{p}_1\right)-R_{21}^{\mathbf{k}}\left(\mathbf{p}_2;\mathbf{p}_1\right)\cdot V_{12}^{-\mathbf{k}}\left(\mathbf{p}_1';\mathbf{p}_2'\right)\right\}$$
$$\times F_1^{\dagger}\left(\mathbf{p}_1'\right)F_2\left(\mathbf{p}_2'\right)F_1\left(\mathbf{p}_1\right)F_2^{\dagger}\left(\mathbf{p}_2\right)+H.c. \tag{45}$$

The pion-nucleon interaction operator can be given as

$$K\left(\pi N\rightarrow\pi N\right)$$
$$=\frac{1}{2}\int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{k}_1 d\mathbf{k}_2\left[R^{\mathbf{k}_1},V^{-\mathbf{k}_2}\right]_{11}\left(\mathbf{p}_2;\mathbf{p}_1\right)$$
$$\times F_1^{\dagger}\left(\mathbf{p}_2\right)a^{\dagger}\left(\mathbf{k}_2\right)F_1\left(\mathbf{p}_1\right)a\left(\mathbf{k}_1\right)+H.c., \tag{46}$$

where

$$\left[R^{\mathbf{k}_1},V^{\mathbf{k}_2}\right]_{ij}\left(\mathbf{p}';\mathbf{p}\right)=\int d\mathbf{q}\left[R_{im}^{\mathbf{k}_1}\left(\mathbf{p}';\mathbf{q}\right)V_{mj}^{\mathbf{k}_2}\left(\mathbf{q};\mathbf{p}\right)\right.$$
$$\left.-V_{im}^{\mathbf{k}_2}\left(\mathbf{p}';\mathbf{q}\right)R_{mj}^{\mathbf{k}_1}\left(\mathbf{q};\mathbf{p}\right)\right]. \tag{47}$$

The pion-antinucleon interaction operator has the form

$$K\left(\pi\bar{N}\rightarrow\pi\bar{N}\right)=\frac{1}{2}\int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{k}_1 d\mathbf{k}_2\left[R^{\mathbf{k}_1},V^{-\mathbf{k}_2}\right]_{22}\left(\mathbf{p}_2;\mathbf{p}_1\right)$$
$$\times F_2\left(\mathbf{p}_2\right)a^{\dagger}\left(\mathbf{k}_2\right)F_2^{\dagger}\left(\mathbf{p}_1\right)a\left(\mathbf{k}_1\right)+H.c. \tag{48}$$

The interaction operator for nucleon-antinucleon pair production (annihilation) by pair of pions is equal to

$$K\left(\pi\pi\leftrightarrow N\bar{N}\right)=\frac{1}{2}\int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{k}_1 d\mathbf{k}_2\left[R^{\mathbf{k}_1},V^{\mathbf{k}_2}\right]_{12}\left(\mathbf{p}_2;\mathbf{p}_1\right)$$
$$\times F_1^{\dagger}\left(\mathbf{p}_2\right)F_2\left(\mathbf{p}_1\right)a\left(\mathbf{k}_1\right)a\left(\mathbf{k}_2\right)+H.c.. \tag{49}$$

The interaction operators $\mathbf{B}^{(2)}\left(\alpha_c\right)$ are determined by the same formulae (43)-(49) as for $K^{(2)}\left(\alpha_c\right)$ but with $V(\alpha_c)$ substituted by $\mathbf{W}(\alpha_c)$.

The operators of the third order in g have similar forms but more difficult structure. For example, the operator of the pion production on the pair of nucleons has the form [6]

$$K\left(NN\rightarrow\pi NN\right)=\frac{1}{3}\int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_1' d\mathbf{p}_2' d\mathbf{k}_1 d\mathbf{k}_2$$
$$\times\left\{R_{11}^{\mathbf{k}_1}\left(\mathbf{p}_1';\mathbf{p}_1\right)\cdot\left[R^{\mathbf{k}_1},V^{\mathbf{k}_2}\right]_{11}^{\dagger}\left(\mathbf{p}_2';\mathbf{p}_2\right)\right.$$
$$+2R_{11}^{\mathbf{k}_1}\left(\mathbf{p}_1';\mathbf{p}_1\right)\cdot\left[R^{\mathbf{k}_2},V^{\mathbf{k}_1}\right]_{11}^{\dagger}\left(\mathbf{p}_2';\mathbf{p}_2\right)$$
$$+\left[R^{\mathbf{k}_1},V^{-\mathbf{k}_2}\right]_{11}\left(\mathbf{p}_1';\mathbf{p}_1\right)\cdot R_{11}^{\mathbf{k}_1^{\dagger}}\left(\mathbf{p}_2';\mathbf{p}_2\right)$$
$$+2\left[R^{\mathbf{k}_2},V^{-\mathbf{k}_1}\right]_{11}\left(\mathbf{p}_1';\mathbf{p}_1\right)\cdot R_{11}^{\mathbf{k}_1^{\dagger}}\left(\mathbf{p}_2';\mathbf{p}_2\right)$$
$$+V_{11}^{\mathbf{k}_1}\left(\mathbf{p}_1';\mathbf{p}_1\right)\cdot\left[R^{\mathbf{k}_2},R^{\mathbf{k}_1}\right]_{11}^{\dagger}\left(\mathbf{p}_2';\mathbf{p}_2\right)$$
$$\left.+\left[R^{\mathbf{k}_2},R^{\mathbf{k}_1^{\dagger}}\right]_{11}\left(\mathbf{p}_1';\mathbf{p}_1\right)\cdot V_{11}^{-\mathbf{k}_1}\left(\mathbf{p}_2';\mathbf{p}_2\right)\right\}$$
$$\times F_1^{\dagger}\left(\mathbf{p}_1'\right)F_1^{\dagger}\left(\mathbf{p}_2'\right)F_1\left(\mathbf{p}_1\right)F_1\left(\mathbf{p}_2\right)a^{\dagger}\left(\mathbf{k}_2\right). \tag{50}$$

5. WHETHER THE POINCARÉ ALGEBRA IS FULFILLED?

In order to answer this question it is necessary to evaluate explicitly the set of all commutation relations (1) in the respective orders. It is important to emphasize that the corresponding verification is not difficult though quite tedious. Thus, we just give only some necessary explanations. The commutation relations which do not contain K and $\boldsymbol{B}$ are automatically fulfilled. The rotational and translational invariance of the interaction operator V leads to the fact that the commutation relations $\left[P_i,H\right]=0$, $\left[J_i,H\right]=0$ and $\left[J_i,N_k\right]=i\varepsilon_{ikl}J_l$ are also satisfied. The relation $\left[P_i,N_l\right]=-i\delta_{il}H$ is proven by partial integration and the way of verifying $\left[H,N_l\right]=-iP_l$ and $\left[N_i,N_k\right]=-i\varepsilon_{ikl}J_l$ up to the sixth order is similar [5].

6. CONCLUSION

Using the method of the unitary clothing transformation, we have constructed the generators of the Poincaré group in the instant form of relativistic dynamics in the second and third orders in the coupling constant. In the model of the three-linear Yukawa type pseudoscalar interaction between nucleon and meson fields we have shown how the generators of the Poincaré group acquire one and the same sparse structure in the Fock space of particle states.

The explicit form for the boost generators enables us to check directly the properties of the bare and clothed operators and states under the Lorentz transformations. Contrary to the bare vacuum and bare one-particle states, the clothed vacuum and the clothed one-particle states appear invariant under these transformations. The new states being expressed in terms of the bare ones have a very difficult structure which witnesses that the clouds of virtual particles are included in the clothed operators and states.

It is important to emphasize that the algebra of generators of the Poincaré group is fulfilled up to the six order in the coupling constant. As a byproduct of our clothing procedure, the mass and vertex renormalization in the second and third orders in the coupling constant respectively is performed in a relativistic manner.

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ПРЕДСТАВЛЕНИЕ ОДЕТЫХ ЧАСТИЦ В КВАНТОВОЙ ТЕОРИИ ПОЛЯ: ГЕНЕРАТОРЫ БУСТОВ ГРУППЫ ПУАНКАРЕ

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С помощью метода унитарных одевающих преобразований построены генераторы группы Пуанкаре в мгновенной форме релятивистской динамики во втором и третьем порядках по константе взаимодействия. Показано, что алгебра генераторов группы удовлетворяется до шестого порядка. Обоснована релятивистская инвариантность рассчитанных поправок массам частиц и константе взаимодействия.

ЗОБРАЖЕННЯ ОДЯГНЕНИХ ЧАСТИНОК В КВАНТОВІЙ ТЕОРІЇ ПОЛЯ: ГЕНЕРАТОРИ БУСТІВ ГРУПИ ПУАНКАРЕ

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За допомогою методу унітарних одягаючих перетворень побудовані генератори групи Пуанкаре в миттєвій формі релятивістської динаміки в другому і третьому порядках за константою взаємодії. Показано, що алгебра генераторів групи задовольняється до шостого порядку. Обґрунтовано релятивістську інваріантність розрахованих поправок до мас частинок і константі взаємодії.