INFLUENCE OF AN EXTERNAL LOW FREQUENCY HELICAL PERTURBATION ON BALLOONING MODES

I.M. Pankratov, A.Ya. Omelchenko

Institute of Plasma Physics,
National Science Center "Kharkov Institute of Physics and Technology",
61108 Kharkov, Ukraine, E-mail: pankratov@kipt.kharkov.ua

Equations for investigation of the influence of external helical magnetic perturbations on the ballooning modes are derived in one-fluid MHD with the plasma response and rotation being taken into account.

PACS: 52.35.Bj, 52.55.Fa

1. INTRODUCTION

ELMs (edge localized modes) are short bursts of particles and energy at the tokamak edge plasma observed in H-mode operations (JET, ASDEX-U, DIII-D) [1]. As results of these bursts the melting, erosion and evaporation of divertor target plates may occur. This problem is also important for ITER [2].

The suppression of ELMs by external helical magnetic perturbations in H-mode of DIII-D was observed [3,4].

ELMs modes are studied using MHD (ballooning and peeling modes) mostly without the external helical magnetic perturbations [5]. Note, the ELMs suppression is not predicted by stochastic layer transport theory taking in account the external magnetic perturbations [4].

Until now, understanding of the underlying physics of ELMs and their suppressions has been far from being complete. In the paper, one-fluid MHD ballooning mode equations are derived with the external helical perturbations. The plasma response and rotation take into account.

2. BASIC EQUATIONS

We start from the one-fluid MHD equations

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p - \nabla \cdot \boldsymbol{\pi}_i + \frac{1}{c}[\mathbf{J}\times\mathbf{B}], \quad \frac{dp}{dt} + \gamma_0 p\,div\mathbf{V} = 0 \qquad (1)$$

the Maxwell's equations

$$rot\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad rot\mathbf{B} = \frac{4\pi}{c}\mathbf{J}, \quad div\mathbf{B} = 0, \qquad (2)$$

$$div\mathbf{J} = 0 \qquad (3)$$

and Ohm's law (σ - conductivity)

$$\mathbf{J} = \sigma\left(\mathbf{E} + \frac{1}{c}[\mathbf{V}\times\mathbf{B}]\right), \qquad (4)$$

where ρ is the plasma mass densities, p is the plasma pressure, $\mathbf{J}$ is the current density, $\boldsymbol{\pi}_i$ is the ion gyroviscosity tensor, respectively.

We consider a current carrying toroidal plasma with nested equilibrium circular magnetic surfaces (ρ_0 is the radius of the magnetic surfaces, ω_0 is the poloidal angle in the cross-section $\zeta = const$, ζ is the toroidal angle). Each magnetic surface is shifted with respect to the magnetic axis (ξ is the shift, R is the radius of the magnetic axis). The equilibrium toroidal contravariant component of the magnetic field, $B_0^{\zeta} = \Phi'/\left(2\pi\sqrt{g}\right)$, is large with respect to the poloidal one, $B_0^{\theta} = \chi'/\left(2\pi\sqrt{g}\right)$, Φ' and χ' are the radial derivatives of toroidal and poloidal fluxes, respectively; $q(a) = \Phi'/\chi'$ is the safety factor, $\mu = 1/q$.

On each magnetic equilibrium surface (see, e.g. [6]) we introduce a straight magnetic field line coordinate system (a, θ, ζ) $\rho_0 = a$, $\omega_0 = \theta + \lambda(a)\sin\theta$

$$\lambda(a) = -\xi'(a) - a/R, \qquad (5)$$

$$\xi'(a) = \frac{1}{aR}\left(\frac{\chi'(a)}{2\pi R}\right)^{-2}\int_0^a\left[16p_0(b) + \left(\frac{\chi'(b)}{2\pi R}\right)^2\right]bdb. \qquad (6)$$

Assuming that $div\,\mathbf{V}_{\perp} \approx 0$ and perturbation $B^{\zeta} \approx 0$ (see, e.g. [7]) we get for perturbations (*m>>1, nq>>1*)

$$\frac{0.5cB_0^{\zeta}}{\Phi'}\cdot L_{//}\left[\left(-\frac{\partial}{\partial\theta}g_{11} + \frac{\partial}{\partial a}g_{12}\right)B^a + \left(-\frac{\partial}{\partial\theta}g_{12} + \frac{\partial}{\partial a}g_{22}\right)B^{\theta}\right] -$$

$$-\frac{cRB_0^{\zeta}}{ma\mathbf{B}_0^2}\left[\frac{\partial}{\partial a}a\rho_0\omega'\frac{\partial}{\partial a}aV^a - m\rho_0\omega'V^a + m\frac{\partial}{\partial a}[\rho_0(\frac{\partial}{\partial a}a^2V_0^{\theta})V^a - 2m\rho_0V_0^{\theta}\frac{\partial}{\partial a}aV^a\right] - \qquad (7)$$

$$-\frac{2\pi c}{\Phi'}\left[p_0'\frac{\partial}{\partial\theta}\frac{\mathbf{B}_0\mathbf{B}}{\mathbf{B}_0^2} + \frac{1}{(2\pi)^2p_0'\sqrt{g}}\left(\Omega - \frac{1}{c}\mu'\Phi'^2\alpha_0\right)\frac{\partial p}{\partial\theta}\right] + \frac{2\pi c}{\Phi'\sqrt{g}}\frac{\partial\sqrt{g}}{\partial\theta}\cdot\frac{\partial p}{\partial a} + \qquad (9)$$

$$+\frac{2\pi c}{\Phi' p_0'}\frac{\partial p}{\partial\theta}\cdot\left[\mathbf{B}_0\cdot\nabla\left(\frac{v}{p_0'}\right)'\right] + \frac{2\pi c}{\Phi'}\frac{\partial p}{\partial\theta}\cdot(\mathbf{B}_0\cdot\nabla\frac{B_{0a}}{\mathbf{B}_0^2}) + \left(B^a\frac{\partial}{\partial a} + B^{\theta}\frac{\partial}{\partial\theta}\right)\frac{J_t' + \partial v/\partial\theta}{\Phi'} = 0,$$

$$\left[1+\frac{c_s^2}{\omega_i\omega'}\frac{(B_0^\varsigma)^2}{\mathbf{B}_0^2}\dot{L}_{//}^2+\frac{i\gamma_0 V_0^\theta}{\omega_i\sqrt{g}}\frac{\partial\sqrt{g}}{\partial\theta}\right]p=-\frac{c_s^2p_0'}{\omega_i\omega'}\frac{(B_0^\varsigma)^2}{\mathbf{B}_0^2}\dot{L}_{//}\frac{B^a}{B_0^\varsigma}-\left[\frac{c_s^2\rho_0}{\omega_i\omega'}\left(\mu\frac{\partial}{\partial a}(a^2V_0^\theta)+R^2\frac{\partial V_0^\varsigma}{\partial a}\right)\frac{(B_0^\varsigma)^2}{\mathbf{B}_0^2}\dot{L}_{//}+\frac{ip_0'}{\omega_i}\right]V^a, \quad (8)$$

$$\left[i\omega-\frac{1}{\sqrt{g}}\left(V_0^\theta\frac{\partial}{\partial\theta}\sqrt{g}+V_0^\varsigma\sqrt{g}\frac{\partial}{\partial\varsigma}\right)\right]B^a=\frac{c^2}{4\pi\sigma\sqrt{g}}\frac{g_{33}}{\sqrt{g}}\left(\frac{\partial}{\partial a}g_{22}B^\theta-\frac{\partial}{\partial\theta}g_{11}B^a\right)-B_0^\varsigma\dot{L}_{//}V^a \quad . \quad (9)$$

Here

$$\begin{aligned}
\dot{L}_{//}&=\mu(\partial/\partial\theta)+(\partial/\partial\varsigma),\\
\Omega&=cp_0'V''-\Phi''I'+\chi''J_t',\\
J_0^\varsigma&=\left(J_t'+\partial\nu/\partial\theta\right)/2\pi\sqrt{g},\\
J_0^\theta&=I'/2\pi\sqrt{g},\quad \alpha_0=(\mathbf{J}_0\cdot\mathbf{B}_0)/\mathbf{B}_0^2,\\
\chi'(\partial\nu/\partial\theta)&=4\pi^2cp_0'\left((\sqrt{g})_{(0)}-\sqrt{g}\right),\\
\omega_i&=\omega-mV_0^\theta+nV_0^\zeta .
\end{aligned} \quad (10)$$

We included the equilibrium poloidal plasma rotation with the velocity V_0^θ due to existence of an equilibrium radial electric field and the ion diamagnetic drift; and the equilibrium toroidal plasma rotation with a velocity V_0^ζ, ω is the frequency of the external perturbation. In ω' the poloidal rotation is due to only of a radial electric field.

We take perturbations in the form (see, e.g. [6])

$$X(a,\theta,\varsigma)=\left[X_{(0)}(a)+X_{(1)}(a,\theta)\right]\exp i(m\theta-n\varsigma-\omega t), \quad (11)$$

$$X_{(1)}(a,\theta)\approx X_s(a)\sin\theta+X_c(a)\cos\theta\ ,\quad \left|X_{(0)}\right|>>\left|X_{(1)}\right|.$$

We consider a quadratic approximation in *1/R*. The known expressions for metric tensor are used [6]. For perturbations $X_{(0)}$, X_s, X_c we have:

$$\begin{aligned}
&F(a)\left[\frac{d}{da}\left(a\frac{d}{da}aB_{(0)}^a\right)-m^2B_{(0)}^a\right]+\frac{8\pi^2aR}{\Phi'}\left[\frac{d}{da}\left(a\rho_0\omega'\frac{d}{da}aV_{(0)}^a\right)-m^2\rho_0\omega'V_{(0)}^a-\right.\\
&\left.-m\rho_0V_0^\theta\frac{d}{da}(aV_{(0)}^a)+m\left(a\rho_0\frac{d}{da}V_E^\theta+\frac{d}{da}(a\rho_0\frac{d}{da}V_0^\theta)\right)V_{(0)}^a\right]+\\
&+p_{(0)}\frac{16im^2\pi^2a^2}{R\Phi'}\left(\mu^2-1+\frac{4\pi^2aR^2p_0'}{\chi'^2}\right)-\frac{dp_s}{da}\frac{8\pi^2ma^2\mu}{\chi'}-8im^2\pi^2p_c\frac{d}{da}\left(\frac{\mu a^2}{\chi'}\right)-\frac{4\pi maR}{c}\frac{d}{da}\left(\frac{J_t'}{\Phi'}\right)\cdot B_{(0)}^a=0,
\end{aligned} \quad (12)$$

$$\begin{aligned}
&iF(a)\left[\frac{d}{da}\left(a^2B_c^\theta\right)-imB_c^a\right]+\mu\left[\frac{d}{da}\left(a^2B_s^\theta\right)-imB_s^a\right]+\left[\mu-\frac{4i\pi aR}{c}\frac{d}{da}\left(\frac{J_t'}{\Phi'}\right)\right]B_c^a-iF(a)B_s^a-\\
&-\frac{8\pi^2aR}{m\Phi'}\left[\frac{d}{da}\left(a\rho_0\omega'\frac{d}{da}aV_c^a\right)-m^2\rho_0\omega'V_c^a-m\rho_0V_0^\theta\frac{d}{da}(aV_c^a)+m\left(a\rho_0\frac{d}{da}V_E^\theta+\frac{d}{da}(a\rho_0\frac{d}{da}V_0^\theta)\right)V_c^a\right]=\\
&=2m\frac{p_c}{p_0'}\frac{8\pi^2a^2p_0'}{R\Phi'}\left(\mu^2-1+\frac{4\pi^2aR^2p_0'}{\chi'^2}\right)+\frac{4\pi}{\mathbf{B}_0^2}\frac{16\pi^2ma^2p_0'}{\Phi'}p_{(0)}+i\frac{dp_{(0)}}{da}\frac{16\pi^2a^2\mu}{\chi'}-16m\pi^2p_{(0)}\frac{d}{da}\left(\frac{\mu a^2}{\chi'}\right)\\
&+\frac{8\pi^2ma^2\mu^2}{\Phi'}\frac{a^3}{R}\left(\xi''+\frac{\xi'}{R}+\frac{1}{R}\right)p_{(0)}+32i\pi^3aR\frac{d}{da}\left(\frac{a^2p_0'}{\chi'\Phi'}\right)\cdot B_{(0)}^a-\frac{8i\pi^2a^2}{c}\frac{d}{da}\left(\frac{J_t'}{\Phi'}\right)\cdot B_{(0)}^a-iF(a)G_1-\mu G_2,
\end{aligned} \quad (13)$$

$$\begin{aligned}
&iF(a)\left[\frac{d}{da}\left(a^2B_s^\theta\right)-imB_s^a\right]-\mu\left[\frac{d}{da}\left(a^2B_c^\theta\right)-imB_c^a\right]+\left[\mu-\frac{4i\pi aR}{c}\frac{d}{da}\left(\frac{J_t'}{\Phi'}\right)\right]B_s^a-iF(a)B_c^a-\\
&-\frac{8\pi^2aR}{m\Phi'}\left[\frac{d}{da}\left(a\rho_0\omega'\frac{d}{da}aV_s^a\right)-m^2\rho_0\omega'V_s^a-m\rho_0V_0^\theta\frac{d}{da}(aV_s^a)+m\left(a\rho_0\frac{d}{da}V_E^\theta+\frac{d}{da}(a\rho_0\frac{d}{da}V_0^\theta)\right)V_s^a\right]=\\
&=2m\frac{p_s}{p_0'}\frac{8\pi^2a^2p_0'}{R\Phi'}\left(\mu^2-1+\frac{4\pi^2aR^2p_0'}{\chi'^2}\right)-32i\pi^3aR\left(\frac{a^2p_0'}{\chi'\Phi'}\right)\cdot B_{(0)}^\theta-iF(a)G_2+\mu G_1,
\end{aligned} \quad (14)$$

$$\left[\frac{c_s^2}{\omega_i\omega'}\frac{F^2(a)}{R^2}-1\right]p_{(0)}=\frac{c_s^2p_0'}{\omega_i\omega'}\frac{F(a)}{R^2}\frac{B_{(0)}^a}{B_0^\varsigma}+i\left[\frac{c_s^2\rho_0}{\omega_i\omega'}\frac{F(a)}{R^2}\left(\mu\frac{\partial}{\partial a}(a^2V_0^\theta)+R^2\frac{\partial V_0^\varsigma}{\partial a}\right)+\frac{p_0'}{\omega_i}\right]V_{(0)}^a+i\frac{\gamma_0aV_0^\theta}{\omega_iR}p_s\,, \quad (15)$$

$$\left[1-\frac{c_s^2}{\omega_i\omega'}\frac{(\mu^2+F^2(a))}{R^2}\right]p_c+2i\frac{c_s^2}{\omega_i\omega'}\frac{\mu F(a)}{R^2}p_s=-\frac{c_s^2p_0'}{\omega_i\omega'}\frac{(\mu B_s^a+iF(a)B_c^a)}{R^2B_0^\varsigma}-$$
$$-\frac{c_s^2\rho_0}{\omega_i\omega'}\left(\mu\frac{\partial}{\partial a}(a^2V_0^\theta)+R^2\frac{\partial V_0^\varsigma}{\partial a}\right)\frac{(\mu V_s^a+iF(a)V_c^a)}{R^2}+\frac{ip_0'}{\omega_i}V_c^a\,, \quad (16)$$

$$\left[1-\frac{c_s^2}{\omega_i\omega'}\frac{(\mu^2+F^2(a))}{R^2}\right]p_s-2i\frac{c_s^2}{\omega_i\omega'}\frac{\mu F(a)}{R^2}p_c=-\frac{c_s^2p_0'}{\omega_i\omega'}\frac{(-\mu B_c^a+iF(a)B_s^a)}{R^2B_0^\varsigma}-$$
$$-\frac{c_s^2\rho_0}{\omega_i\omega'}\left(\mu\frac{\partial}{\partial a}(a^2V_0^\theta)+R^2\frac{\partial V_0^\varsigma}{\partial a}\right)\frac{(\mu V_c^a+iF(a)V_s^a)}{R^2}-\frac{ip_0'}{\omega_i}V_s^a-\frac{2i\gamma_0aV_0^\theta}{\omega_iR}p_{(0)}-\frac{2c_s^2p_0'}{\omega_i\omega'}\frac{\mu a^2}{R^2}B_{(0)}^a\,, \quad (17)$$

$$\omega_iB_{(0)}^a=-F(a)\frac{\Phi'(a)}{2\pi aR}V_{(0)}^a+\frac{ic^2m}{4\pi\sigma a^2}\left[\frac{d}{da}(a^2B_{(0)}^\theta)-imB_{(0)}^a\right], \quad (18)$$

$$F(a)\frac{\Phi'(a)}{2\pi aR}V_c^a-\frac{i\mu\Phi'(a)}{2\pi aR}V_s^a=-\omega_i\left(\frac{2a}{R}B_{(0)}^a+B_c^a\right)+$$
$$+\frac{c^2m}{4\pi\sigma a^2}\left[-2\frac{d}{da}(a^2\xi'B_{(0)}^\theta)+2\frac{a^2}{R}\frac{d}{da}(aB_{(0)}^\theta)+\left(\frac{d}{da}(a^2B_c^\theta)-imB_c^a\right)\right], \quad (19)$$

$$F(a)\frac{\Phi'(a)}{2\pi aR}V_s^a+\frac{i\mu\Phi'(a)}{2\pi aR}V_c^a=-\omega_iB_s^a+\frac{c^2m}{4\pi\sigma a^2}\left(\frac{d}{da}(a^2B_s^\theta)-imB_s^a\right), \quad (20)$$

$$G_1=2\frac{d}{da}\left(\lambda a^2B_{(0)}^\theta\right)-2m\xi'B_{(0)}^a-\left(a^2\lambda'-a\xi'\right)B_{(0)}^\theta\,,$$
$$G_2=\frac{d}{da}\left((a^2\lambda'-a\xi')B_{(0)}^a\right)-im(a^2\lambda'-a\xi')B_{(0)}^a+2\xi'B_{(0)}^a$$

.The value of $F(a)=m\mu(a)-n$ is equal to zero inside the plasma, when $q(a_{res})=m/n$.

3. DISCUSSIONS AND CONCLUSIONS

The derived equations allow to study the control of ballooning modes in tokamak because the expression $p+\mathbf{B}_0\mathbf{B}/4\pi\approx 0$. External helical magnetic perturbations will change pressure perturbation.

Expected result may be used to control the plasma stability for experiments in tokamaks JET, DIII-D, TEXTOR and future ITER operation.

REFERENCES

1. M. Becoulet, G. Huysmans, P. Thomas et al. Edge localized modes control: experiment and theory // *Journal of Nuclear Materials.* 2005, v. 337-339, N 3, p. 677-683.
2. P-H. Rebut. From JET to the reactor // *Plasma Phys. Control. Fusion.* 2006, v.48, N12B, p. B1-B14.
3. T.E. Evans, R.A. Moyer, P.R. Thomas et al. Suppression of large edge localized modes in high confinement DIII-D plasmas with a stochastic magnetic boundary//*Phys.Rev.Letters.* 2004, v. 92, N 23, p. 235003.
4. T. E. Evans, R.A. Moyer, K.H. Burrell et al. Edge stability and transport control with resonant magnetic perturbations in collisionless tokamak plasmas // *Nature Physics.* 2006, v.2, N 6, p.419-423.
5. P.B. Snyder, H.R. Wilson, J.R. Ferron et al. ELMs and constrains on the H-mode pedestal: peeling-ballooning stability calculation and comparison with experiment // *Nuclear Fusion.* 2004, v.44, N 2, p.320-328.
6. A.B. Mikhailovskii. *Instabilities of plasma in magnetic traps.* Moscow: "Atomizdat", 1978 (in Russian).
7. O.P. Pogutse, E.I. Yurchenko // *Reviews of Plasma Physics /* Moscow: "Energoizdat", 1982, v.11, p. 56-117 (in Russian).

Article received 8.10.08.

ВЛИЯНИЕ ВНЕШНЕГО НИЗКОЧАСТОТНОГО ВИНТОВОГО ВОЗМУЩЕНИЯ НА БАЛЛОННЫЕ МОДЫ

И.М. Панкратов, А.Я. Омельченко

В рамках одножидкостной МГД получены уравнения для изучения влияния внешнего низкочастотного винтового возмущения на баллонные моды с учетом отклика плазмы и ее вращения.

ВПЛИВ ЗОВНІШНЬОГО НИЗЬКОЧАСТОТНОГО ГВИНТОВОГО ЗБУРЕННЯ НА БАЛОННІ МОДИ

І.М. Панкратов, А.Я. Омельченко

У рамках однорідинної МГД отримані рівняння для вивчення впливу зовнішнього низькочастотного гвинтового збурення на балонні моди з урахуванням відгуку плазми та її обертання.