PIC SIMULATION OF NONLINEAR WAKEFIELD EXCITATION BY A TRAIN OF ELECTRON BUNCHES IN A WAVEGUIDE WITH ISOTROPIC PLASMA

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The results of analytical investigations and numerical simulation (in 2.5–dimensional electromagnetic code) of nonlinear wake fields excitation by regular sequence of relativistic electron bunches in isotropic (no external magnetic field) plasma are presented. The dynamics and distribution of electromagnetic field are investigated. The waves predicted in the system by linear theory of wake fields excitation were observed in the numerical experiment.
PACS: 02.60.Cb, 07.05.Tp

1. INTRODUCTION

The results of numerical simulation of nonlinear wake fields excitation by regular sequence of relativistic electron bunches in isotropic (no external magnetic field) plasma are presented. For this purpose the 2.5–dimensional electromagnetic code developed by us [1] and used for simulation of wake field excitation by electron bunches in a dielectric waveguide was modernized. The changes in the code have been made for the purpose of possibility of simulation of electrons and ions of plasma with initial Maxwell distribution by the velocities and initial homogeneous distribution by space. The plasma geometry for simulation has been chosen as the ring. The plasma ring is placed in the cylindrical resonator. Such geometry allows to reduce essentially quantity of macroparticles in the modeled system at compliance of necessary requirements to stability of the scheme and, thus to reduce time of numerical calculations.

Analytical and numerical investigations of a linear stage of wakefields excitation for the investigated geometry of plasma are carried out. It is shown, that for parameters which are used for simulation with help of particle-in-cell (PIC) method, bunches can excite only a plasma wave and the backward surface wave which frequency is close to frequency of a plasma wave. The forward surface wave is not excited.

Carried out numerical simulations by means of PIC method have confirmed results of the linear theory about excitation Langmuir waves which is concentrated in a plasma ring. But along with Langmuir wave not resonant waves which are excited in the vacuum resonator near of cutoff frequency of the resonator are gained. In process of injection of bunches in the resonator the Langmuir waves amplitude increases, the energy stored in the resonator increases also.

2. THE LINEAR THEORY OF WAKE FIELDS EXCITATION IN THE TUBULAR PLASMA WAVEGUIDE

Let's consider an excitation of wake fields by a relativistic electron bunch in a tubular plasma waveguide without of an external magnetic field. Inner radius of tubular plasma is a, outer radius is b, radius of a metal waveguide is R. We will obtain at first the expression for the longitudinal electric field excited by infinitely thin ring relativistic electron bunch of radius r_0 moving in plasma, its charge is equal to Q. Having solved the Maxwell equations with a stationary source which the "rigid" bunch is we will present the derived expression for it in the form of the sum of two parts:

$$E_z = E_z^l + E_z^s \tag{1}$$

The term E_z^l describes excitation of the Langmuir waves with frequency $\omega = \omega_p$:

$$E_z^l = \frac{2Q\omega_p^2}{V_0^2}\frac{1}{F_0(k_p b, k_p a)}\cos(\omega_p \tau)\theta(\omega_p \tau) \times \begin{cases} F_0(k_p r, k_p a)F_0(k_p r_0, k_p b), & a \le r \le r_0 \\ F_0(k_p r, k_p b)F_0(k_p r_0, k_p a), & r_0 \le r \le b \\ 0, \ otherwise \end{cases} \tag{2}$$

The second term in the equation (1), E_z^s describes an excitation of surface waves:

$$E_z^s = -\frac{4Q}{V_0 a}\sum_{s=1,2}\frac{D_2(\omega_s)}{\partial D_2/\partial \omega_s}\Phi_s(r_0)\Phi_s(r)\cos(\omega_s \tau)\theta(\omega_s \tau), \tag{3}$$

$$\Phi_s(r) = \begin{cases} \dfrac{I_0(\kappa_s r)}{I_0(\kappa_s a)}, & r \le a \\ \dfrac{F_0(q_s r, q_s b)}{F_0(q_s a, q_s b)} - \dfrac{\varepsilon_s}{\sqrt{1-\beta_0^2 \varepsilon_s}}\dfrac{F_0(q_s r, q_s a)}{(F_0(q_s a, q_s b))^2} \times \dfrac{F_1(q_s b, q_s b)}{D_2(\omega_s)}, & a \le r \le b \\ \dfrac{\varepsilon_s}{\sqrt{1-\beta_0^2 \varepsilon_s}}\dfrac{F_0(\kappa_s r, \kappa_s R)}{F_0(\kappa_s b, \kappa_s R)}\dfrac{F_1(q_s b, q_s b)}{F_0(q_s a, q_s b)}\dfrac{1}{D_2(\omega_s)}, & otherwise \end{cases}$$

Resonance frequencies of surface waves ω_s are determined from the solution of the next dispersion equation:

$$D(\omega_s) \equiv D_1(\omega_s)\cdot D_2(\omega_s) + \frac{\varepsilon_s^2}{1-\beta_0^2 \varepsilon_s}\frac{F_1(q_s b, q_s b)F_1(q_s a, q_s a)}{F_0(q_s a, q_s b)} = 0, \tag{4}$$

$$D_1(\omega_s) = \gamma_0 \frac{I_1(\kappa_s a)}{I_0(\kappa_s a)} - \frac{\varepsilon_s}{\sqrt{1-\beta_0^2 \varepsilon_s}}\frac{F_1(q_s a, q_s b)}{F_0(q_s a, q_s b)}, \tag{5}$$

$$D_2(\omega_s) = \gamma_0 \frac{F_1(\kappa_s b, \kappa_s R)}{F_0(\kappa_s b, \kappa_s R)} - \frac{\varepsilon_s}{\sqrt{1-\beta_0^2 \varepsilon_s}}\frac{F_1(q_s b, q_s a)}{F_0(q_s b, q_s a)}, \tag{6}$$

where: $F_1(x,y) \equiv I_1(x)K_0(y) + K_1(x)I_0(y)$, $I_{0,1}(x)$, $K_{0,1}(x)$ are modified Bessel function and McDonald functions of zero and first order, $\kappa_s = \omega_s / V_0\gamma_0$, $\beta_0 = V_0 / c$, $\gamma_0 = (1-\beta_0^2)^{-1/2}$, $\varepsilon_s = 1 - \omega_p^2 / \omega_s^2$.

The function $D_1(\omega_s)$ describes surface waves existing at inner boundary of a plasma ring, and function $D_2(\omega_s)$ describes surface waves existing at outer boundary of a plasma ring. If the inner radius of a ring to limit to zero we will come to the dispersion equation [2]:

$$D_2(\omega_s) = 0, \qquad (7)$$

and if the outer radius of a plasma ring is equal to radius of a metal waveguide the dispersion equation (4) is reduced to the equation [2]:

$$D_1(\omega_s) = 0. \qquad (8)$$

The resonant wave (7) has the normal law of a dispersion, the range of frequencies is located in an interval $0 \le \omega \le \omega_p / \sqrt{2}$. The surface wave excited by a bunch (8) has the abnormal law of dispersion, and the range of excited frequencies is located in an interval $\omega_p / \sqrt{2} \le \omega \le \omega_p$. Generally the electron bunch excites two waves. But the surface wave supported by outer boundary of plasma, is excited by a bunch if only its energy exceeds some threshold value which value is defined from the solution of the dispersion equation (4). As has shown the computation of the dispersion equation (4) for parameters of a bunch and the plasma waveguide, used at numerical simulation, the surface wave supported by outer boundary of plasma, is not excited, and frequency of the surface wave supported by inner boundary of plasma, is close to frequency of the Langmuir wave.

Before to proceed to results of numerical simulation of wake fields excitation it is necessary to note, that the Langmuir wave is localized only in a plasma layer, while surface waves in whole volume of a waveguide. This fact can be used at treatment of results of numerical simulation for identification of the waves excited by a chain of electron bunches.

3. RESULTS OF THE NUMERICAL SIMULATION

The 2.5–dimensional electromagnetic code developed and modernized by us was used for simulation of wake field excitation by electron bunches in a dielectric waveguide. For numerical calculations the following parameters has been chosen:

drift chamber radius	R = 4.3 cm,
inner radius of tubular plasma	a = 0.4 cm,
outer radius of plasma	b = 0.5 cm,
chamber length	L = 22.13 cm,
tubular electron beam radiuses coincide with plasma radiuses	
electron beam energy	5 MeV,
beam current	I_b = 0.5 A
repetition period	T_i = 0.372 ns,
bunch duration	0.05833 ns,
plasma density	$8.974 \cdot 10^{10}$ cm^{-3}
plasma temperature	15 eV
number of macroparticles	$\sim 1.2 \cdot 10^6$.

In the Fig. 1 the amplitude of longitudinal E_z (*a*) and transverse E_r (*b*) electric fields is represented. It is seen that growth of field amplitude with time is observed. At the initial time in spectrum $E_z(t)$ the frequency of injection f_i = 2.69 GHz is dominating, then by time moment t = 10 ns the amplitude of multiple harmonics of a field is observed prevails.

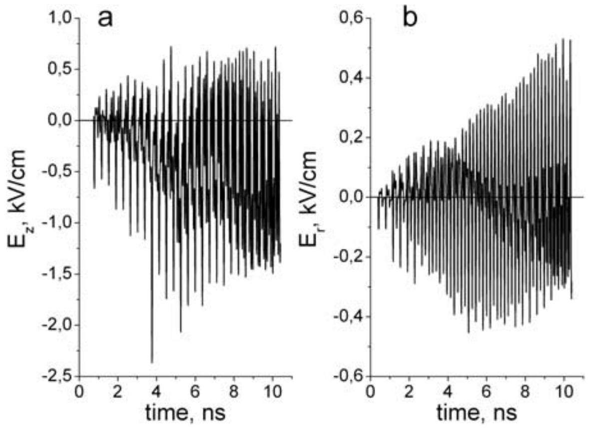

Fig. 1. Amplitude of fields E_z and E_r versus time t: a — E_z at the input end of resonator, at its axis r = 0: b — E_r in the middle part of the resonator at r = R

Fig. 2 shows spectrum of longitudinal E_z (light grey curve) and transverse E_r (black curve) components of an electromagnetic field. The repetition rate of bunches f_i = 2.69G Hz is shown by a vertical line. In the fields spectrum the essential contribution of multiple harmonics 2 f_i and 4 f_i is observed.

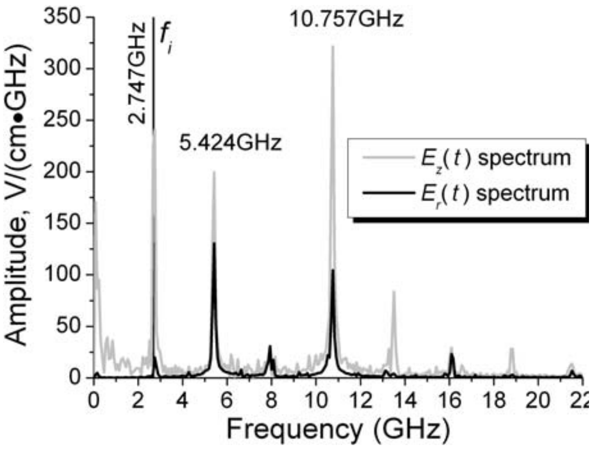

Fig. 2. Spectrum of longitudinal E_z (a dark blue curve) and transverse E_r (a red curve) components of an electromagnetic field

In the Fig. 3 the energy of an electromagnetic field stored in resonator, Joule loss of particles energy and changes of kinetic energy of particles versus time t is shown.

The transverse distribution of longitudinal electric field (black curve) and transverse electric field (light grey curve) at z = 6.653 cm and t = 2.27 ns are drawn in the Fig. 4. It is seen, that the amplitude of a field has the greatest value in the area occupied with plasma. Thus we can say that the electron bunches excite the Langmuir wave substantially; the amplitude of surface wave is smaller than the amplitude of bulk Langmuir wave.

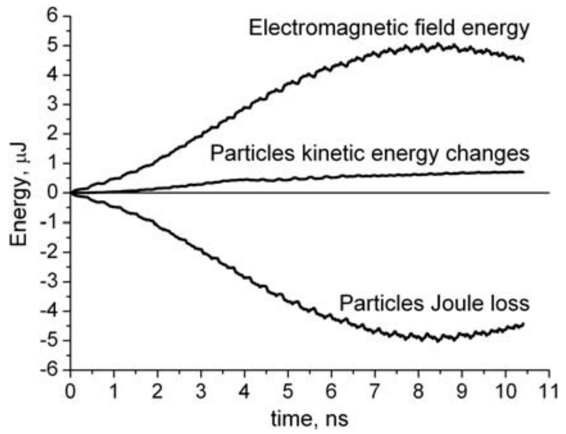

Fig. 3. Energy of electromagnetic field stored in resonator (*W*), *Joule loss of particles energy* (*P*) *and changes of kinetic energy of particles* (*K + R*) *versus time t*

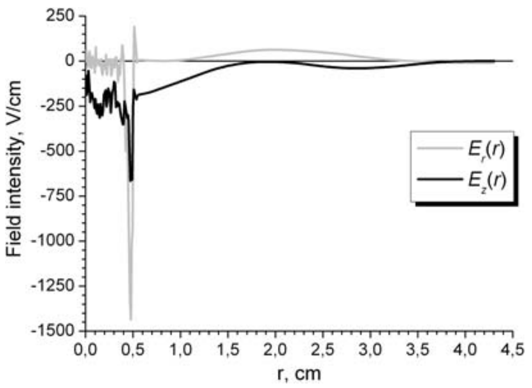

Fig. 4. The transverse distribution of longitudinal electric field (*black curve*) *and transverse electric field* (*light grey curve*) *at z = 6.653 cm and t = 2.27 ns*

4. CONCLUSIONS

Analytical and numerical studies of wake fields excitation in tubular plasma waveguide by relativistic sequence of electronic bunches is carried out.

Analytical researches have shown, in general case the electron bunch excites in an isotropic plasma waveguide a space charge wave (the Langmuir wave) and two surface waves, one of them is supported inner boundary, and the other is supported by outer boundary of tubular plasma. The obtained analytical expressions for excited wake fields allow to estimate amplitudes of each of waves and compare them among themselves.

The 2.5 numerical code is developed for a simulation of wake fields excitation by relativistic electron bunches in the isotropic plasma resonator with cold ions and with the Maxwell distribution in the velocities of plasma electrons. Numerical simulation for parameters of bunches and density of plasma which are expected in planned in NSC KIPT experiments is carried out. It is shown the excited field, basically, is localized in plasma region. It allows making a conclusion about primary excitation of the Langmuir wave. At first the amplitude of wake field grows with increase in quantity of injected into resonator bunches, and then goes on saturation. The total of bunches taking the contribution to growth of amplitude of a field is not large. Restriction is connected as with transverse spread of plasma that leads to decreasing of plasma density in the interaction region and to infringement of synchronism conditions and so with transverse spread on momentums of electronic bunches. In a spectrum of an excited field in an initial stage of bunch injection the plasma frequency, which is equal to the period of injection of bunches, prevails, at next time it is enriched by multiple harmonics of a plasma wave.

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Article received 30.09.08.

PIC МОДЕЛИРОВАНИЕ НЕЛИНЕЙНОГО ВОЗБУЖДЕНИЯ КИЛЬВАТЕРНОГО ПОЛЯ ПОСЛЕДОВАТЕЛЬНОСТЬЮ ЭЛЕКТРОННЫХ СГУСТКОВ В ВОЛНОВОДЕ С ИЗОТРОПНОЙ ПЛАЗМОЙ

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Представлены результаты численного моделирования возбуждения нелинейных кильватерных волн регулярной последовательностью релятивистских электронных сгустков в изотропной (в отсутствие внешнего магнитного поля) плазме. Исследована динамика и распределение электромагнитного поля. Проведенное численное моделирования с помощью метода крупных частиц с параметрами сгустков и плазмы в планируемых в ННЦ ХФТИ экспериментах, подтвердило результаты линейной теории о возбуждении ленгмюровской волны, которая сосредоточена в плазменном кольце.

PIC МОДЕЛЮВАННЯ НЕЛІНІЙНОГО ЗБУДЖЕННЯ КІЛЬВАТЕРНОГО ПОЛЯ ПОСЛІДОВНІСТЮ ЕЛЕКТРОННИХ ЗГУСТКІВ У ХВИЛЕВОДІ З ІЗОТРОПНОЮ ПЛАЗМОЮ

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Представлено результати чисельного моделювання збудження нелінійних кільватерних хвиль регулярною послідовністю релятивістських електронних згустків в ізотропній (тобто, у відсутності зовнішнього магнітного поля) плазмі. Досліджено динаміку й розподіл електромагнітного поля. Проведене чисельне моделювання за допомогою методу великих часток з параметрами згустків і плазми у планованих в ННЦ ХФТІ експериментах, підтвердило результати лінійної теорії про збудження ленгмюровської хвилі, що зосереджена в плазмовому кільці.