AZIMUTHAL SURFACE WAVES IN TOROIDAL MAGNETIC TRAPS

I.O. Girka, O.I .Girka, V.O. Girka, I.V. Pavlenko

V.N. Karazin Kharkov National University, Kharkov, Ukraine

Extraordinary polarized electromagnetic surface waves are known to propagate with zero axial wave number ($k_z=0$) along the small azimuth φ nearby the interface of a cold uniform plasma located in cylindrical metal chambers and immersed into axial steady magnetic field B_{0z} [1]. These waves are called as azimuthal surface waves (ASW). ASW are shown to be eigen modes of toroidal plasmas. Their dispersion properties are examined with taking into account numerous peculiarities of plasma in toroidal traps: a magnitude of the external toroidal magnetic field, a thickness of the vacuum layer between plasma and metal chamber, a small radius of plasma, toroidal and ripple variations in steady magnetic field, a deviation of plasma-vacuum and vacuum–metal cross-sections' shapes from a circle, a presence of the poloidal component of the confining magnetic field, a non-uniformity of the density radial profile, etc. ASW with frequencies over the electron cyclotron frequency $|\omega_e|$ are shown to be absorbed within the local resonance region, $\varepsilon_1(r_0)= 0$, where ε_1 is the diagonal element of the plasma dielectric tensor. Possibility is shown of additional heating of a radially inhomogeneous plasma in small toroidal devices with a rippled magnetic field via the absorption of satellite harmonics of the ASW with frequencies below $|\omega_e|$ within the local resonance region, $\varepsilon_1(r_1) = [2\pi c/(\omega L)]^2$, where L is the ripple period.

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1. INTRODUCTION

Interest in studying the properties of ASW stems from the promising outlook for their use in microwave electronics [2–4]. The problem of the SW spectra in a toroidal traps is important for the interpretation of some experimental results in controlled nuclear fusion research, specifically, laboratory data on phenomena in wall plasmas [5]. Surface type electromagnetic waves are often recognized in fusion experiments as one of the most probable reasons of enhanced plasma-wall interaction. These waves can be also responsible for undesirable losses of electromagnetic power launched by antennae.

In a toroidal gaseous plasma [6], the toroidicity manifests itself in the following factors: first, spatial variations of a constant external toroidal magnetic field; second, the radial displacement of magnetic surfaces [7]; and third, the deviation of the shape of the poloidal cross-sections of magnetic surfaces from being circular.

Dispersion properties of electromagnetic waves in toroidal plasma with small aspect ratio are often studied in the framework of the model of plasma cylinder with identified ends. The paper [1] presents the results of studying the dispersion properties of ASW at the boundary of a plasma cylinder, located in a perfectly conducting metal waveguide, in the presence of a vacuum layer between them. The constant magnetic field is oriented along the system axis. In this case Maxwell equations are separated into two independent sets, describing the E- and H-wave. We consider the extraordinary E-wave with the components of E_r, E_φ, H_z.

For cold dense plasma ($\Omega_e>|\omega_e|$) with uniform density, ASW propagate in the frequency ranges:

$$\omega_2<\omega<|\omega_e|,\ |\omega_e|<\omega<\omega_{11}-|\omega_e|,\ \omega_1<\omega<\omega_{11}, \quad (1)$$

where $\omega_{11} =0.5\ |\omega_e| + (\Omega_e^2 + \omega_e^2/4)^{1/2}$; ω_2 and ω_1, and Ω_e are, respectively, the lower and upper hybrid frequencies and the Langmuir frequency.

ASW can propagate in metal cylinder entirely filled by plasma with positive azimuthal wave numbers m in the first range (1) and with negative m in the third range (1).

2. EFFECT OF PLASMA DENSITY NONUNIFORMITY

2.1. DISPERSION PROPERTIES OF LF ASW

We investigate how the dispersion properties of low frequency (LF) ASW from the first and second ranges of (1) are affected by the plasma density non-uniformity [8]. We consider the case of plasma density linear profile:

$$n(r) = \begin{cases} n(r) = (r-a)(dn/dr)|_{r=a} & r \le a \\ n(r) = 0 & a < r \le b \end{cases}. \quad (2)$$

When the plasma cylinder is large ($k_0a>> |\omega_e m|/\omega$, ka), the problem can be solved by perturbation theory. Here $k_0^3 = -d(\delta^{-2})/dr$ at $r = a$, and $\delta \equiv c/\Omega_e$ is skin depth. In the case of linear density profile, the expression form of ASW frequency as a function of the plasma parameters is analogous to what was obtained in the case of a uniform plasma [1] with the substitution $\delta \rightarrow k_0^{-1}$.

2.2. ABSORPTION OF LF ASW

ASW with frequencies over $|\omega_e|$ are absorbed in the local resonance region $r=r_0$. ASW damping rates caused by electron collisions and the presence of a resonance point $r=r_0$ are derived, compared and analyzed [8].

3. LF ASW IN A RIPPLED MAGNETIC FIELD

3.1. DISPERSION PROPERTIES OF LF ASW

Toroidal plasma is often confined by a rippled steady magnetic field $B_0 = B_{0z}e_z + B_{0r}e_r$:

$$B_{0r}=B_{00}(\varepsilon'_m/k_m)\sin(k_m z),\ B_{0z}=B_{00}[1+\varepsilon_m(r)\cos(k_m z)], \quad (3)$$

where $\varepsilon_m'\equiv d\varepsilon_m/dr$ and $k_m=2\pi/L$. In particular, in tokamaks the ripple stems from the discreteness of the toroidal magnetic field coils. It is anticipated that the confining magnetic field of a Helias modular stellarator will be dominated by a so-called "mirror" nonuniformity [9]. In this sense, our study relates to the Helias configuration.

The solution to Maxwell equations for the components of the wave field is found [10] in the form of wave packet. The weak coupling between the E- and H-modes is due to the ripple in the constant magnetic field and due to the nonzero axial number of the satellite harmonics $(\partial/\partial z \neq 0)$. Symmetry of spatial distribution of the satellite harmonics is studied.

To second order in ε_m, the dispersion relation is derived:

$$D^{(0)} + D^{(2)} = 0, \quad (4)$$

where $D^{(2)}$ is a second-order quantity. The solution to (4) is found in the form $\omega = \omega^{(0)} + \Delta\omega$, where the correction $\Delta\omega = -D^{(2)}(\partial D^{(0)}/\partial\omega)^{-1}$ is a second- order quantity.

3.2. PLASMA HEATING VIA THE ABSORPTION OF SATELLITE HARMONICS OF ASW

Possibility of additional heating of a radially inhomogeneous plasma with a rippled magnetic field (3) via the absorption of satellite harmonics of ASW with frequencies $\omega < |\omega_e|$ in the local resonance region $r = r_1$ is studied [11]. The electromagnetic power absorbed within the resonance region is calculated and analyzed.

4. ASW IN A MAGNETIC FIELD WITH A WEAK POLOIDAL COMPONENT

4.1. LF ASW

External poloidal magnetic field, $|B_{0\varphi}| << |B_{0z}|$, can be caused in the toroidal traps by, e.g., an axial electric current. In this case, spatial distribution of the LF ASW field is obtained with accuracy up to terms of first order of smallness in $B_{0\varphi}$ [12]. The corrections to the eigen frequency of both ordinary and extraordinary ASW caused by $B_{0\varphi}$ in general case is proportional to the square of its value.

Ordinary and extraordinary ASW interact linearly in a resonance manner if the vacuum layer is sufficiently wide and if the external axial magnetic field differs from zero. In this case the correction to the eigen frequency of the ASW, caused by $B_{0\varphi}$, is linear in its value.

4.2. HF ASW

We investigated also [13] the coupling of ordinary bulk and extraordinary surface waves caused by $B_{0\varphi}$ in HF range (third range of (1)). This phenomenon can be observed in wide plasma waveguides for the waves propagating with negative values of azimuthal mode number if utilized steady axial magnetic field is not small.

The effect of $B_{0\varphi}$ on the spatial distribution of the ASW fields is determined with taking into account the items of the first order of smallness.

Far from the conditions of the waves' linear resonant interaction, the correction to eigen frequency of ASW, caused by $B_{0\varphi}$, is proportional to the square of $B_{0\varphi}$. Nearby the points, in which the dispersion curves of ASW and ordinary bulk waves cross, the correction $\Delta\omega_x \propto B_{0\varphi}$. $B_{0\varphi}$ exerts the strongest influence on the ASW dispersion properties if, for the definite sets of values of the plasma parameters an odd number of quarters of forced azimuthal bulk wave's wavelength compose the plasma radius.

5. EFFECT OF TOROIDAL VARIATION IN AXIAL MAGNETIC FIELD ON THE LF ASW

Toroidal variation in axial magnetic field,

$$B_{0\zeta} = B_0 / \left[1 - (r/R)\cos\varphi \right], \quad (5)$$

is shown [14] to cause the small shift $\Delta\omega$ to the eigen frequency ω_0 of LF ASW, $\Delta\omega \sim \varepsilon_t^2 \omega_0$. Here $\varepsilon_t = a/R << 1$, R is plasma torus large radius. The symmetry of the problem allows us to represent the sought-for solution to Maxwell equation in the form of wave packet. In the second approximation, the boundary condition can be represented in the form similar to (4). The toroidal geometry of the waveguide gives rise to only the second-order correction to the eigen frequency of the ASW.

6. EFFECT OF THE CROSS-SECTION SHAPE NONCIRCULARITY ON THE LF ASW

6.1. EFFECT OF THE SHAPE OF THE CHAMBER CROSS-SECTION

Effect of the deviation from a circle of cross-section shapes for vacuum–metal [15] surface and plasma-vacuum [16, 17] surface on the LF ASW dispersion properties is examined. We thoroughly analyze the particular case, in which the small radius changes with the azimuthal angle φ according to the law $R_2 = b(1 + h_N \sin(N\varphi))$ and, then, generalize the results obtained to the case of an arbitrary shape of the cross section. The case $N = 1$ corresponds to the Shafranov shift. The case $N = 2$ describes a D-shaped cross section of tokamak chamber. The case $N = 3$ corresponds to the "triangular" cross section of stellarator…

The dispersion relation is obtained in the form similar to that of (4), where $D^{(2)}$ is the small value of the second order in h_n. Consequently the correction to the eigen frequency caused by the noncircularity of the chamber cross section is the second-order value.

6.2. EFFECT OF THE SHAPE OF THE CROSS-SECTION OF THE OUTER MAGNETIC SURFACE

Since the most part of the energy of ASW is concentrated nearby the plasma-vacuum interface then dispersion properties of ASW are more sensitive to the noncircularity of this interface. We consider a plasma column with an arbitrary cross section of the radius $R_1(\varphi)$:

$$R_1 = a \cdot \left[1 + \sum_{n=1}^{\infty} h_n \sin(n\varphi - \varphi_n) \right], \quad (6)$$

where a is the mean radius of the plasma column and h_n are the small parameters. Because of the periodic spatial nonuniformity of the plasma–vacuum interface, an ASW propagates as a wave packet containing the fundamental harmonic $\propto exp(im\varphi - i\omega t)$, and an infinite number of satellite harmonics $\propto exp[i(m \pm jN)\varphi - i\omega t]$ ($j = 1, 2, 3,\ldots$). Amplitudes of satellite harmonics are found up to the second order of smallness. The second-order correction to the eigen frequency caused by the noncircularity of the plasma-vacuum cross section is derived in the explicit form in some limiting cases.

This noncircularity affects the eigen frequency and spectral contents of the packet the most strongly in the case if the angular period of the wave perturbations is twice the ripple period of the plasma-vacuum interface. In this resonant case [17] the second-order correction to the eigen frequency is approximately inversely proportional to B_{0z}.

CONCLUSIONS

Extraordinary polarized electromagnetic surface waves are shown to propagate with zero toroidal wave number along the small azimuth nearby the interface of toroidal plasma. Dispersion properties of ASW are examined with taking into account numerous peculiarities of toroidal plasma. Special attention is given to the ASW absorption in plasma with nonuniform density profile.

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АЗИМУТАЛЬНЫЕ ПОВЕРХНОСТНЫЕ ВОЛНЫ В ТОРОИДАЛЬНЫХ МАГНИТНЫХ ЛОВУШКАХ

И.А. Гирка, А.И. Гирка, В.А. Гирка, И.В. Павленко

Показано, что необыкновенно поляризованные электромагнитные волны могут распространяться с нулевым тороидальным волновым числом вдоль малого азимута вблизи поверхности плазмы в тороидальной металлической камере с магнитным полем. Их дисперсионные свойства исследованы с учетом многочисленных особенностей плазмы тороидальных ловушек. Особое внимание уделено поглощению этих волн в плазме с радиально- неоднородным профилем плотности.

АЗИМУТАЛЬНІ ПОВЕРХНЕВІ ХВИЛІ В ТОРОЇДНИХ МАГНІТНИХ ПАСТКАХ

І.О. Гірка, О.І. Гірка, В.О. Гірка, І.В. Павленко

Показано, що незвичайно поляризовані електромагнітні хвилі можуть поширюватися з нульовим тороїдним хвильовим числом уздовж малого азимута поблизу поверхні плазми у тороїдній металевій камері з магнітним полем. Їхні дисперсійні властивості досліджено з урахуванням численних особливостей плазми тороїдних пасток. Особливу увагу приділено поглинанню цих хвиль у плазмі з радіально-неоднорідним профілем густини.