KINETIC THEORY OF DUSTY PLASMAS AND EFFECTIVE GRAIN INTERACTIONS

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The basic points of the consistent kinetic theory of dusty plasmas is discussed. The equations for microscopic phase densities of plasma particles and grains are formulated. Using such equations it is possible to derive the kinetic equations, taking into account both elastic and inelastic particle collisions. Obtained equations are used for kinetic description of effective grain-grain potentials.
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1. INTRODUCTION

The interpretation of recent dusty plasma experiments requires more and more sophisticated description of dusty plasmas in terms of microscopic models.

Only this approach makes it possible to formulate the basic points of the rigorous kinetic theory with due regard of grain charging and particle collisions (both elastic and inelastic). Though kinetic descriptions of dusty plasmas have been given in many papers, no such theory has been formulated as yet. Many theoretical studies in this field have been performed using phenomenological generalization of the results known from the kinetic theory of ordinary plasmas. Such treatment provides a possibility to describe the charge and momentum transfer from plasma particles to grains due to inelastic collisions, however, the influence of collective plasma effects (for example, dynamical particle screening) on charging processes are disregarded.

Consistent treatment of these and other effects could be performed in terms of the kinetic theory formulated on the basis of the microscopic description of contact collisions and plasma particle absorption by grains [1, 2].

In some cases, for the sake of simplicity, it is reasonable to describe the collective behavior of grain subsystem using the effective grain-grain interaction potentials. Obviously, such potentials should be calculated taking into account plasma particle absorption by the grain, as well as the effects of nonlinear screening. Usually such potentials are calculated on the basis of numerical solution of the appropriate boundary-value problems, or by means of numerical simulations. However, the description of some interesting phenomena observed in dusty plasma experiments, such as dusty crystal formation, dust acoustic wave propagation, generation of nonlinear dust structure in a plasma requires analytical relations for effective potentials. Such relations can be obtained on the basis of kinetic equations with the effective point sinks [3] which follow from the rigorous equations discussed above.

The purpose of the present paper is to apply the microscopic model of a dusty plasma in order to formulate the basic principles of the kinetic theory and to find explicit relations for the effective grain potentials with regard to plasma particle collisions and the influence of external force fields, if present.

In Sec. 2 we reproduce rigorous equations for the microscopic phase densities (Klimontovich equations) with regard for the electron and ion absorption by grains and contact grain-grain collisions. Then we formulate the appropriate kinetic equations (Sec. 3). Application of these equations to the calculation of the effective grain potentials is given in Sec. 4. Some analysis of the stationary screening is presented in Sec. 5.

2. MICROSCOPIC EQUATIONS FOR DUSTY PLASMAS WITH REGARD FOR ELECTRON AND ION ABSORPTION BY GRAINS

In the case of a dusty plasma consisting of electrons, ions, and monodispersed grains under the assumption that each grain absorbs all encountered electrons and ions the microscopic phase density of plasma particles is given by

$$F_\alpha(X,t)=\sum_{i=1}^{N_\alpha}F_{i\alpha}(X,t)= \\ =\sum_{i=1}^{N_\alpha}\delta\left(X-X_\alpha(t)\right)\theta(t_{i\alpha}-t), \quad (X\equiv\mathbf{rv}), \tag{1}$$

where $t_{i\alpha}$ is the time of i th plasma particle collision with any grain (i.e., the time of i th particle absorption by a grain), $\theta(x)$ is the Heaviside step function, the subscript α labels the plasma particle species.

Combining the derivatives of $F_\alpha(X,t)$ over t, $\mathbf{r}$, and $\mathbf{v}$, $F_\alpha(X,t)$ can be shown to satisfy the equation (see Ref. [2])

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+\frac{e_\alpha}{m_\alpha}\mathbf{E}_M(\mathbf{r},t)\right\}F_\alpha(X,t)= \\ =-\int d\mathcal{X}\;|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|\times \\ \times\delta\left(|\mathbf{r}-\mathbf{r}'|-a\right)F_g(\mathcal{X}',t)F_\alpha(X,t). \tag{2}$$

Here $\mathbf{e}_\mathbf{r}=\mathbf{r}/r$, $\mathbf{E}_M(\mathbf{r},t)$ is the microscopic electric field, and $F_g(\mathcal{X},t)$ is the grain phase density

$$F_g(\mathcal{X},t)=\sum_{i=1}^{N_g}\sum_n\delta\left(\mathcal{X}-\mathcal{X}_i^{(n)}(t)\right)\times \\ \times\theta\left(t_{ig}^{(n+1)}-t\right)\theta\left(t-t_{ig}^{(n)}\right) \tag{3}$$

$\mathcal{X}\equiv(\mathbf{r},\mathbf{v},q)$, $t_{ig}^{(n)}$ is the time of the n th collision of the

i-th grain with any other particle, $X_j^{(n)}(t)$ is the grain trajectory before the $(n+1)$th and after the nth collision, a is the grain radius.

Eq. (3) describes the microscopic state of the grain subsystem which is determined by the microscopic value of the grain charge q along with the grain coordinate and velocity with regard for the sharp changes of the grain charge and velocity.

As it was shown in Ref. [2] the equation for $F_g(X,t)$ has the form

$$
\begin{aligned}
&\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{q}{m_g}\mathbf{E}_M\frac{\partial}{\partial \mathbf{v}}\right\}= \\
&-\sum_{\alpha=e,i}\int dX'\delta(|\mathbf{r}-\mathbf{r}'|-a)\{|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|\times \\
&\times F_g(X,t)-|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_\alpha)\times \\
&\times F_g(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_\alpha q-e_\alpha t)\}F_\alpha(X',t)- \\
&-\int dX'\delta(|\mathbf{r}-\mathbf{r}'|-2a)\,|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|\times \\
&\times\{F_g(X,t)F_g(X',t)-F_g(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_g,q-\delta q;t)\times \\
&\times F_g(\mathbf{r}',\mathbf{v}+\delta\mathbf{v}_g,q';t)\},
\end{aligned}
\tag{4}
$$

where

$$
\begin{aligned}
\delta\mathbf{v}_\alpha &\equiv \delta\mathbf{v}_\alpha(\mathbf{v},\mathbf{v}')=-\frac{m_\alpha}{m_g}(\mathbf{v}-\mathbf{v}'); \\
\delta\mathbf{v}_g &\equiv \delta\mathbf{v}_g(\mathbf{v},\mathbf{v}')=\mathbf{e}_{\mathbf{r}-\mathbf{r}'}\left(\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')\right); \\
\delta q &\equiv \delta q(q,q')=q'-q.
\end{aligned}
\tag{5}
$$

Notice that the integration over X' and X' in Eqs. (2), (4) is performed within the domain $\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')<0$, or $\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_\alpha)<0$, respectively. Eqs. (2), (4) generalize traditional microscopic equation for the case of electron and ion absorption by grains and contact collisions between the grains. In the case of neutral grains Eq. (4) reduces to that derived by Bogolyubov [4].

3. KINETIC EQUATIONS FOR DUSTY PLASMAS

Averaging Eqs. (2), (4) and their combinations it is possible to generalize the Bogolyubov-Born-Green-Kirkwood-Yvon hierarchy to the case of dusty plasma [2]. For example, the first equation of such hierarchy for plasma particles has the following form

$$
\begin{aligned}
&\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right\}f_\alpha(X,t)= \\
&=-\sum_{\alpha'=e,i}^{n_{\alpha;}} n_\alpha\int dX'\hat{V}_{\alpha\alpha'}(X,X')f_{\alpha\alpha'}(X,X';t)- \\
&-n_g\int dX'\,|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|\times \\
&\times\delta\left(|\mathbf{r}-\mathbf{r}'|-a\right)f_{\alpha g}(X,X';t),\quad \alpha=e,i,
\end{aligned}
\tag{6}
$$

where

$$
\hat{V}_{\alpha\alpha'}(X,X')=\frac{e_\alpha e_{\alpha'}}{m_\alpha}\frac{\partial}{\partial\mathbf{r}}\frac{1}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial}{\partial\mathbf{v}};\ (e_g=q)
\tag{7}
$$

$f_{\alpha\alpha'}(X',X';t)$ is the binary distribution function (at $\alpha'=e,i$ $\quad f_{\alpha\alpha'}(X,X';t)=f_{\alpha\alpha'}(X,X;t)\delta(q-e_{\alpha'})$).

The hierarchy thus obtained makes it possible to introduce the kinetic equations. In particular, in the approximation of the dominant influence of contact collisions the asymptotics of the binary distribution functions can be written as

$$
f^{(0)}_{\substack{\alpha g\\ |\mathbf{r}-\mathbf{r}'|\gg a}}(X,X't)=f_\alpha(X,t)f_g(X',t)\theta(\vartheta-\vartheta^{\alpha g}_{\min}),\quad \alpha=e,i
$$

where $\vartheta^\alpha_{\min}$ is given by

$$
\sin^2\vartheta^\alpha_{\min}=\frac{a^2_{\alpha g}}{|\mathbf{r}-\mathbf{r}'|^2}\left(1-\frac{2e_\alpha q}{a_\alpha m_\alpha(\mathbf{v}-\mathbf{v}')^2}\right).
\tag{8}
$$

Using these relations for estimates of the main contributions to binary collision terms we can write the kinetic equations as

$$
\begin{aligned}
&\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{e_\alpha}{m_\alpha}\left\langle\mathbf{E}^{\mathrm{eff}}_\alpha\right\rangle\frac{\partial}{\partial \mathbf{v}}\right\}f_\alpha(X,t)= \\
&=-\int d\mathbf{v}'\int dq'\sigma_{\alpha g}(q',\mathbf{v}-\mathbf{v}')\,|\mathbf{v}-\mathbf{v}'|\times \\
&\times f_\alpha(X,t)f_g(\mathbf{r},\mathbf{v}',q',t)+I_\alpha,
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
&\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{q}{m_g}\left\langle\mathbf{E}^{\mathrm{eff}}_g\right\rangle\frac{\partial}{\partial \mathbf{v}}\right\}f_g(X,t)= \\
&=-\sum_{\alpha=e,i}\int d\mathbf{v}'\big[\sigma_{\alpha' g}(q,\mathbf{v}-\mathbf{v}')\,|\mathbf{v}-\mathbf{v}'|\,f_g(X,t)- \\
&-\sigma_{\alpha' g}(q-e_\alpha,\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_{\alpha'})\,|\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_{\alpha'}|\times \\
&\times f_g(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_{\alpha'},q-e_{\alpha'},t)\big]f_{\alpha'}(\mathbf{r},\mathbf{v}',t)- \\
&-\int\frac{d\Omega}{2\pi}\int d\mathbf{v}'\int dq'\,|\mathbf{n}(\mathbf{v}-\mathbf{v}')|\times \\
&\times\big[\sigma_{gg}(q,q',\mathbf{v}-\mathbf{v}')f_g(X,t)f_g(\mathbf{r},\mathbf{v}',q',t)- \\
&-\sigma_{\alpha g}(q-\delta q_g,q';\mathbf{v}-\mathbf{v}')f_g(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_g,q- \\
&-\delta q_g,t)\,f_g(\mathbf{r},\mathbf{v}'+\delta\mathbf{v}_g,q',t)\big]+I_g,
\end{aligned}
\tag{10}
$$

where I_α and I_g are the Coulomb collision terms which can be calculated in terms of the correlation functions of particle density fluctuations (similarly to the case of ordinary plasmas), $\sigma_{\alpha\beta}(q,\mathbf{v})$ is the charging cross-section. In the case of collisionless plasma with no external fields

$$
\begin{aligned}
&\sigma_{\alpha g}(q,v)=\pi a^2_{\alpha g}\left(1-\frac{2e_\alpha q}{m_\alpha v^2 a_\alpha}\right), \\
&a_{\alpha g}=a,\quad \alpha=e,i,\quad a_{gg}=2a.
\end{aligned}
$$

Applications of Eqs. (9), (10) to the calculation of stationary grain distributions are given in [2].

4. EFFECTIVE POTENTIAL OF CHARGED GRAINS (GENERAL RELATIONS)

Now let us apply the obtained equation to the calculations of the effective grain potentials. In the case of single immovable grain $f_g(\mathbf{r}',\mathbf{v}',q')=\delta(\mathbf{r}')\delta(\mathbf{v}')\delta(q'-q)$ Eq. (9) reduces to

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{e_\alpha}{m_\alpha}\mathbf{E}(\mathbf{r},t)\frac{\partial}{\partial \mathbf{v}}+\frac{1}{m_\alpha}\mathbf{F}_\alpha(X,t)\frac{\partial}{\partial \mathbf{v}}\right\}\times \tag{11}$$
$$\times f_\alpha(X,t)=I_\alpha-v\sigma_{\alpha q}(q,v)f_\alpha(X,t)\delta(\mathbf{r}),$$

where $\mathbf{F}_\alpha(X,t)$ is the external force field, if present. In what follows, for the sake of simplicity, instead of the collision term calculated in terms of correlation functions of microscopic fluctuations we shall use the simple version of the model collision integral (simple Bhatnagar–Gross–Krook model) proposed in Ref. [5], namely

$$I_\alpha=-\nu_\alpha\left\{f_\alpha(X,t)-\Phi_\alpha(\mathbf{v})\int d\mathbf{v}f_\alpha(X,t)\right\}. \tag{12}$$

Here ν_α is the collision frequency, $\Phi_\alpha(\mathbf{v})$ is the distribution function generated in course of plasma particle collisions.

In view of the fact that plasma particle absorption considerably suppress the influence of nonlinearity we can suggest that the presence of sinks causes small perturbation of the effective electric field and thus to use the linearized version of Eq. (12).

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_\alpha}\mathbf{F}_\alpha^{\text{ext}}\frac{\partial}{\partial \mathbf{v}}\right\}\delta f_\alpha(X,t)-$$
$$-\frac{e_\alpha}{m_\alpha}\nabla\Phi(\mathbf{r},t)\frac{\partial f_{0\alpha}(\mathbf{v})}{\partial \mathbf{v}}= \tag{13}$$
$$=-v\sigma_\alpha(q(t),v)f_{0\alpha}(\mathbf{v})-$$
$$-\nu_\alpha\left\{\delta f_\alpha(X,t)-\Phi_\alpha(\mathbf{v})\int d\mathbf{v}\delta f_\alpha(X,t)\right\}.$$

The potential $\Phi(\mathbf{r},t)$ is governed by the Poisson equation

$$\Delta\Phi(\mathbf{r},t)=-4\pi q(t)\delta(\mathbf{r})-4\pi\sum_\alpha e_\alpha n_\alpha\int d\mathbf{v}\delta f_\alpha(X,t). \tag{14}$$

The solution of Eq. (13) can be written as

$$\delta f_\alpha(X,t)=\frac{e_\alpha}{m_\alpha}\int_{-\infty}^{t}dt'\int dX'W_\alpha(X,X';t-t')\times$$
$$\times\frac{\partial\delta\Phi(\mathbf{r}',t')}{\partial\mathbf{r}'}\frac{\partial f_{0\alpha}(\mathbf{v}')}{\partial\mathbf{v}'}- \tag{15}$$
$$-\int_{\infty}^{t}dt'\int dX'W_\alpha(X,X';t-t')S_\alpha^{(0)}(\mathbf{v}',t'),$$

where $W_\alpha(X,X';t-t')$ satisfies the equation

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m_\alpha}\mathbf{F}_\alpha^{\text{ext}}\frac{\partial}{\partial \mathbf{v}}\right\}W_\alpha(X,X';\tau)= \tag{16}$$
$$=-\nu_\alpha\{W_\alpha(X,X';\tau)-\Phi_\alpha(\mathbf{v})\int d\mathbf{v}W_\alpha(X,X';\tau)\}$$

with the initial condition

$$W_\alpha(X,X';0)=\delta(X-X'). \tag{17}$$

Substituting Eq. (15) into the Poisson equation (14) one obtains

$$\Phi_{\mathbf{k}\omega}=\frac{4\pi q_\omega}{k^2\varepsilon(\mathbf{k},\omega)}-$$
$$-\frac{4\pi}{k^2\varepsilon(\mathbf{k},\omega)}\sum_\alpha e_\alpha n_{0\sigma}\int d\mathbf{v}\int d\mathbf{v}'W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')S_{\alpha\omega}^{(0)}(\mathbf{v}'), \tag{18}$$

where $\varepsilon(\mathbf{k}\omega)$ is the dielectric response function

$$\varepsilon(\mathbf{k}\omega)=1-i\sum_\alpha\frac{4\pi e_\alpha^2 n_{0\sigma}}{k^2}\int d\mathbf{v}\int d\mathbf{v}'W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')\mathbf{k}\times$$
$$\times\frac{\partial f_{0\sigma}(\mathbf{v}')}{\partial\mathbf{v}'}; \tag{19}$$
$$W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')=\int d\mathbf{R}e^{-i\mathbf{k}\mathbf{R}}\int_0^\infty d\tau e^{i\omega\tau}W_\alpha(X,X',\tau),$$
$$\mathbf{R}=\mathbf{r}-\mathbf{r}'.$$

In the stationary case $q(t)=q$, and Eq. (18) reduces to

$$\Phi_{\mathbf{k}}=\frac{4\pi q}{k^2\varepsilon(\mathbf{k},0)}-$$
$$-\frac{4\pi}{k^2\varepsilon(\mathbf{k},0)}\sum_\alpha e_\alpha n_{0\sigma}\int d\mathbf{v}\int d\mathbf{v}'W_{\alpha\mathbf{k}}(\mathbf{v},\mathbf{v}')S_\alpha^{(0)}(\mathbf{v}'). \tag{20}$$

Here,

$$W_{\alpha\mathbf{k}}(\mathbf{v},\mathbf{v}')=W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')|_{\omega=0},\quad \varepsilon(\mathbf{k},0)=1+\frac{k_D^2}{k^2},$$
$$k_D^2=\sum_\alpha\frac{4\pi e_\alpha^2 n_{0\sigma}}{T_\alpha}. \tag{21}$$

5. INFLUENCE OF PLASMA PROPERTIES ON THE EFFECTIVE GRAIN POTENTIALS

Let us consider in more details the influence of plasma properties on the specific features of the effective potential of grain which charge is maintained by plasma currents. We start from the case of isotropic plasma with no external field. In this case

$$W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')=\frac{i\delta(\mathbf{v}-\mathbf{v}')}{\omega-\mathbf{kv}+i\nu_\alpha}-$$
$$-\frac{i\nu_\alpha\Phi_\alpha(\mathbf{v})}{(\omega-\mathbf{kv}+i\nu_\alpha)(\omega-\mathbf{kv}'+i\nu_\alpha)}\times \tag{22}$$
$$\times\left[1-i\nu_\alpha\int d\mathbf{v}\frac{\Phi_\alpha(\mathbf{v})}{\omega-\mathbf{kv}+i\nu_\alpha}\right]^{-1}.$$

That leads to the following stationary grain potential

$$\Phi(\mathbf{r})=\frac{qe^{-k_D r}}{r}+$$
$$+i\sum_\alpha 4\pi e_\alpha n_\alpha\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{e^{i\mathbf{kr}}}{k^2+k_D^2}\frac{\int d\mathbf{v}\frac{v\sigma_\alpha(q,v)f_{0\sigma}(v)}{\mathbf{kv}-i\nu_\alpha}}{1+i\nu_\alpha\int d\mathbf{v}\frac{\Phi_\alpha(\mathbf{v})}{\mathbf{kv}-i\nu_\alpha}}. \tag{23}$$

In the collisionless limit $(\nu_\alpha\to 0)$ this relation is especially simplified

$$\Phi(\mathbf{r})=\frac{qe^{-k_D r}}{r}+$$
$$+i\sum_\alpha 4\pi e_\alpha n_{0\sigma}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{e^{i\mathbf{kr}}}{k^2+k_D^2}\int d\mathbf{v}\frac{v\sigma_\alpha(q,v)f_{0\sigma}(v)}{\mathbf{kv}-i0} \tag{24}$$

that gives

$$\Phi(\mathbf{r})=\frac{qe^{-k_D r}}{r}-\frac{\tilde{Q}}{r}g(k_D r), \tag{25}$$

where

$$g(X) = e^{-X} Ei(X) - e^{X} Ei(-X),$$
$$\tilde{Q} = \frac{2\pi}{k_D}\sum_\alpha e_\alpha n_\alpha \int_0^\infty dv\, v\sigma_\alpha(q,v) f_{0\sigma}(v). \tag{26}$$

At $k_D r \gg 1$ Eq. (23) gives the well-known result

$$\Phi(r) \simeq -\frac{2Q}{k_D r^2}.$$

In strongly collisional limit ($\nu_\alpha \gg ks_\alpha$, $s_\alpha = (T_\alpha / m_\alpha)$) Eq. (23) reduces to

$$\Phi(\mathbf{r}) = (q+\tilde{S})\frac{e^{-k_D r}}{r} - \frac{\tilde{S}}{r}, \tag{27}$$

where

$$\tilde{S} = \sum_\alpha \frac{e_\alpha S_\alpha}{D_\alpha};\; S_\alpha = n_\alpha \int d\mathbf{v}' v\sigma_\alpha(q,v) f_{0\alpha}(\mathbf{v});\; D_\alpha = \frac{T_\alpha}{m_\alpha \nu_\alpha},$$

that is in agreement with the results obtained on the basis of the description in the drift-diffusion approximation [6, 7]. Deriving (27) we put $\Phi_\alpha(\mathbf{v}) = f_{0\sigma}(\mathbf{v})$.

If collisions are present, but $\nu_\alpha \ll ks_\alpha$

$$\Phi_k = \frac{4\pi q}{k^2+k_D^2} - \sum_\alpha \frac{8\pi^3 e_\alpha n_\alpha}{(k^2+k_D^2)}\frac{1}{k}\int_0^\infty dv\, v^2\sigma_\alpha(q,v) f_{0\alpha}(v)\left[1+\frac{\nu_\alpha}{ks_\alpha}(\frac{\pi}{2})^{1/2}\right]. \tag{28}$$

In the coordinate representation one has

$$\Phi(r) = (q+Q)\frac{e^{-k_D r}}{r} - \frac{\tilde{Q}}{r} g(k_D r) - \frac{Q}{r},$$
$$Q \cong \frac{2\pi^2}{k_D}\sum_\alpha \frac{e_\alpha n_\alpha \nu_\alpha}{s_\alpha}\left(\frac{\pi}{2}\right)^{1/2}\int_0^\infty dv v^2 \sigma_\alpha(q,v) f_{0\alpha}(v). \tag{29}$$

Thus, we can conclude that plasma particle collisions generate the Coulomb-like behaviour of the effective potentials at large distances ($r > \lambda_D$).

If the external magnetic field $\mathbf{B}_0 = \mathbf{e}_z B_0$ is present the transition probability is

$$W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') = \tilde{W}_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') + \frac{\nu_\alpha \int d\mathbf{v}' \tilde{W}_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')\Phi_\alpha(\mathbf{v}')\int d\mathbf{v}\Phi_\alpha(\mathbf{v})\tilde{W}_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')}{1-\nu_\alpha \int d\mathbf{v}\int d\mathbf{v}' \tilde{W}_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')\Phi_\alpha(\mathbf{v}')}, \tag{30}$$

where

$$\tilde{W}_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') = W^{(0)}_{\alpha\mathbf{k}\omega+i\nu_\alpha}(\mathbf{v},\mathbf{v}'), \tag{31}$$

and $W^{(0)}_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')$ is the Fourier representation of the transition probability for the collisionless plasma in external magnetic field. With regard to the symmetry properties of the charging cross-sections and unperturbed velocity distributions the quantity

$$W_{\alpha\mathbf{k}\omega}(\mathbf{v}') = \int d\mathbf{v} W_{\alpha\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') \tag{32}$$

in terms of which $\Phi_{\mathbf{k}\omega}$ and $\varepsilon(\mathbf{k}\omega)$ are presented, can be written as

$$W_{\alpha\mathbf{k}\omega}(\mathbf{v}') = \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{k_\perp v'_\perp}{\Omega_\alpha}\right)\frac{i}{\omega - n\Omega_\alpha - k_z v_z t i\nu_\alpha}, \tag{33}$$

where $\Omega_\alpha = e_\alpha B_0 / m_\alpha c$, $J_n(x)$ is the Bessel function. Substituting Eq. (33) into Eq. (20) one obtains

$$\Phi_{\mathbf{k}} = \frac{4\pi q}{k^2 k_D^2} + \frac{4\pi i}{k^2+k_D^2}\times \sum_\alpha e_\alpha n_\alpha \sum_{n=-\infty}^{\infty}\int d\mathbf{v} J_n^2\left(\frac{\mathbf{k}_\perp \mathbf{v}_\perp}{\Omega_\alpha}\right)\frac{v\sigma_\alpha(q,v) f_{0\alpha}(v)}{k_z v_z + n\Omega_\alpha - i\nu_\alpha}\times \frac{1}{1+i\nu_\alpha \sum_n \int d\mathbf{v} J_n^2\left(\frac{k_\perp v_\perp}{\Omega_\alpha}\right)\frac{f_{0\alpha}(\mathbf{v})}{k_z v_z + n\Omega_\alpha - i\nu_\alpha}}. \tag{34}$$

This relation describes the grain potentials in collisional magnetoactive plasma provided that the cross-section $\sigma_\alpha(q,v)$ is known.

In the case of collisionless plasma Eq. (36) is simplified to

$$\Phi_{\mathrm{k}} = \frac{4\pi q}{k^2+k_D^2} - \frac{4\pi^2}{k^2+k_D^2}\times \sum_\alpha e_\alpha n_\alpha \sum_{n=-\infty}^{\infty}\int d\mathbf{v} J_n^2\left(\frac{k_\perp v_\perp}{\Omega_\alpha}\right) v\sigma_\alpha(q,v)\delta\left(k_z v_z + n\Omega_\alpha\right). \tag{35}$$

.

In the case of strongly magnetized plasma ($B_0 \to \infty$) we can put

$$f_{0\sigma}(\mathbf{v}) = \delta(\mathbf{v}_\perp)\left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{1/2}\exp\left(-\frac{m_\alpha v_z^2}{2T_\alpha}\right) \tag{36}$$

and thus

$$\Phi(r) = \frac{qe^{-k_D r}}{r} - \Phi_0 K_0(k_D r). \tag{37}$$

Here, $K_0(k_D r)$ is the modified Bessel function,

$$\Phi_0 = \sum_\alpha e_\alpha n_\alpha \pi a^2 \int dv_z \left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{1/2}\times \exp\left(-\frac{m_\alpha v_z^2}{2T_\alpha}\right)\theta\left(v_z^2 - \frac{2e_\alpha q}{m_\alpha a}\right). \tag{38}$$

The stationary value of q in this case satisfies the equation

$$\sum_\alpha e_\alpha n_\alpha \int_{-\infty}^{\infty} dv_z\, |v_z| \left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{1/2}\exp\left(-\frac{m_\alpha v_z^2}{2T_\alpha}\right)\times \theta\left(v_z^2 - \frac{2e_\alpha q}{m_\alpha a}\right) = 0. \tag{39}$$

Thus, in the case of strongly magnetized collisionless plasma the effective potential is generated by the charged string which appears due to one-dimensional charging currents. Eq. (33) also shows that in the case $|z| \gg s_\alpha / \nu_\alpha$, $R_\perp > s_\alpha / |\Omega_\alpha|$ the relation of the type of Eq. (28) exists, but the effective charge $\tilde{S}$ is dependent on the angle between $\mathbf{r}$ and $\mathbf{B}_0$.

6. SUMMARY AND CONCLUSIONS

Microscopic equations with regard to electron and ion absorption by grains are formulated and the kinetic equations for dusty plasma is presented.

Kinetic description of the effective grain potentials on the basis of the derived equation with regard to plasma particle collisions in terms of BGK-collision integral makes it possible to recover the results known from the numerical solutions of the drift-diffusion and collisionless kinetic equations.

With the appropriate choice of the grain charge, the obtained relations reproduce with good accuracy the numerical solutions of the nonlinear boundary-value problems.

Obtained general relations can be effectively used for the description of the effective grain potentials in the presence of external magnetic field.

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КИНЕТИЧЕСКАЯ ТЕОРИЯ ПЫЛЕВОЙ ПЛАЗМЫ И ЭФФЕКТИВНОЕ ВЗАИМОДЕЙСТВИЕ ПЫЛИНОК

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Обсуждаются основные положения последовательной кинетической теории пылевой плазмы. Приведены уравнения для микроскопических фазовых плотностей для плазменных частиц и пылинок. Показано, что, используя такие уравнения, можно сформулировать кинетические уравнения для пылевой плазмы, учитывающие упругие и неупругие столкновения частиц и поглощение электронов и ионов пылинками. Выполнены кинетические расчеты эффективных потенциалов взаимодействия пылинок в плазме.

КІНЕТИЧНА ТЕОРІЯ ЗАПОРОШЕНОЇ ПЛАЗМИ ТА ЕФЕКТИВНА ВЗАЄМОДІЯ ПОРОШИНОК

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Обговорюються основні положення послідовної кінетичної теорії запорошеної плазми. Наведено рівняння для мікроскопічних фазових густин для плазмових частинок і порошинок. Показано, що, використовуючи такі рівняння, можна сформулювати кінетичні рівняння для запорошеної плазми з урахуванням пружних і непружних зіткнень частинок та поглинання електронів та іонів порошинками. Розглянуто кінетичний опис ефективних потенціалів взаємодії порошинок у плазмі.