ELECTRON TEMPERATURE GRADIENT DRIFT INSTABILITY IN THE FINITE BETA EDGE PLASMA OF A FIELD REVERSED MAGNETIC CONFIGURATION

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Electromagnetic drift instabilities in the plasma of a field reversed configuration are considered with no assumption of adiabatic response of ions and/or electrons in the range of perpendicular wave number values from $k_\perp < 1/\rho_{Ti}$ up to $k_\perp \sim 1/\rho_{Te}$ (ρ_{Ti} and ρ_{Te} are ion and electron thermal gyroradiuses). Stabilization by finite plasma length is studied. Stabilising effect of low temperature gradients on electron mode is discussed.

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Drift waves and drift instabilities driven by gradients of plasma density, ion and electron temperatures are the most significant phenomena closely connected with turbulent transport of high temperature plasma in the magnetic confinement devises [1]. From the classical theory of drift instabilities [2], it is known that parallel component of the wave vector $k_\|$ is much less then the perpendicular wave number $k_\perp$ ($k_\| \ll k_\perp$). The range of $k_\|$ in infinite plasma is determined only by the positive growth rate solutions of the dispersion equation. In finite length configuration instability satisfy the following condition:

$$2\pi/k_\| < L, \qquad (1)$$

where L is the length of plasma configuration along magnetic field force lines.

We analyze gradient-driven drift instabilities taking into account of electromagnetic effects and condition (1) for edge finite β plasma of a field reversed configuration (FRC). Ion temperature gradient/electron temperature gradient (ITG/ETG) instability takes into account non-adiabatic responses for ions and electrons for any $k_\perp$. The analysis is carried out in the framework of the local electromagnetic kinetic approach. The local model of low frequency ($\omega \ll \omega_{ci}$, ω_{ci} is the ion cyclotron frequency) drift instabilities is based on the linearized Vlasov equation, quasineutrality condition, and Ampere's law for parallel and perpendicular perturbations of the magnetic field [3–8]. Basic equations are

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{q_j}{m_j}(\mathbf{v}\times\mathbf{B}_0)\cdot\nabla_{\mathbf{v}}\right] f_{1j} = -\frac{q_j}{m_j}(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\nabla f_{0j}, \qquad (2)$$

$$\sum_j q_j \int f_{1j} d^3v = 0, \qquad (3)$$

$$\nabla\times\mathbf{B} = \mu_0 \sum_j q_j \int \mathbf{v} f_{1j} d^3v, \qquad (4)$$

$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t}, \qquad (5)$$

$$\mathbf{B} = \nabla\times\mathbf{A}, \qquad (6)$$

$$f_{0j}(\mathbf{v}, x) = f_{Mj}(\mathbf{v})(1 - \varepsilon_j x), \qquad (7)$$

$$\varepsilon_j = -\frac{1}{f_{0j}}\left.\frac{\partial f_{0j}}{\partial x}\right|_{x=0}. \qquad (8)$$

Here $\mathbf{v}$ is the velocity of the particle (variable of integration); q_j and m_j are the charge and the mass of the particle of kind j ($j = i, e$), respectively; μ_0 is the magnetic permittivity of the vacuum; f_{1j} is perturbation of the velocity distribution function; f_{0j} is unperturbed velocity distribution function; $\mathbf{B}_0$ is unperturbed static magnetic field inside the plasma; $\mathbf{E}$ is the electric field of the wave; $\mathbf{B}$ is the magnetic field of the wave; $\mathbf{k}$ is the wave vector; φ is the scalar potential; $\mathbf{A}$ is vector potential; Coulomb gauge $\nabla\cdot\mathbf{A} = 0$ is used; x is the coordinate along density and temperature gradients; $f_{Mj}(\mathbf{v})$ is the Maxwellian velocity distribution function at $x = 0$.

Using standard integrating procedure one can obtain the following system of equations, containing three independent variables (φ, $A_\perp$, $A_\|$)

$$\sum_j \int \left[\frac{q_j\varphi}{k_B T_j} f_{Mj} + h_j J_0(\Lambda_j)\right] d^3v = 0, \qquad (9)$$

$$k_\perp^2 A_\perp = -\mu_0 \sum_j q_j n_j \int h_j J_1(\Lambda_j) v_\perp d^3v, \qquad (10)$$

$$k_\perp^2 A_\| = -\mu_0 \sum_j q_j n_j \int h_j J_0(\Lambda_j) v_\| d^3v. \qquad (11)$$

Here k_B is the Boltzmann constant; T_j is the temperature; $J_0(\Lambda_j)$ and $J_1(\Lambda_j)$ are the Bessel functions; $\Lambda_j = k_\perp v_\perp / \omega_{cj}$; indexes $\perp$ and $\|$ indicate respectively perpendicular and parallel to static magnetic field components of vectors; $v_\perp$ is the perpendicular velocity; $v_\|$ is the parallel velocity; ω_{cj} is the cyclotron frequency;

$$h_j = \frac{\omega + \bar{\omega}_{*j}}{\omega + \omega_{Dj} - k_\| v_\|} \times \left[(\varphi - v_\| A_\|) J_0(\Lambda_j) - v_\perp A_\perp J_1(\Lambda_j)\right] \frac{q_j f_{Mj}}{k_B T_j} \qquad (12)$$

is non-adiabatic portion of the velocity distribution function perturbation; $\omega_{Dj} = \mathbf{k}\cdot\mathbf{V}_{Dj}$ is the magnetic drift frequency; $\mathbf{V}_{Dj}$ is the magnetic drift velocity of the particle; $\overline{\omega}_{*j} = \omega_{*j}\left[1 + \eta_j\left(\frac{m_j v^2}{2k_B T_j} - \frac{3}{2}\right)\right]$;

$\omega_{*j} = k_\perp \frac{k_B T_j}{q_j B_0 L_{nj}}$ is the diamagnetic drift frequency; $\eta_j = L_{nj}/L_{Tj}$; $L_{nj} = -n_j/\nabla_\perp n_j$; $L_{Tj} = -T_j/\nabla_\perp T_j$; n_j is the unperturbed density; $A_\perp = k_\perp B_\parallel$, $B_\parallel$ is the wave magnetic field parallel to the static magnetic field.

Drift frequency can be presented in the form

$$\omega_{Dj} = -\frac{L_{nj}}{L_B}\omega_{*j}\frac{m_j}{k_B T_j}\left(\frac{v_\perp^2}{2} - \alpha_R v_\parallel^2\right), \qquad (13)$$

where $L_B = B_0/\nabla_\perp B_0$, $\alpha_R = L_B/R$, $1/R$ is the particle orbit averaged curvature of the magnetic force line.

The scale of magnetic field gradient is connected with plasma density gradient according the equation

$$\frac{1}{L_B} = \sum_j \frac{(1+\eta_j)\beta_j}{2L_{nj}}, \qquad (14)$$

where $\beta_j = 2\mu_0 n_j k_B T_j / B_0^2$ is local beta parameter calculated by local static magnetic field in the plasma; total local β is

$$\beta = \sum_j \beta_j = \frac{\beta_0}{1-\beta_0}, \qquad (15)$$

where $\beta_0 = \frac{2\mu_0}{B_{0V}^2}\sum_j n_j k_B T_j$; B_{0V} is external (vacuum) magnetic field; $B_0 = B_{0V}\sqrt{1-\beta_0}$.

For low β plasma ($\beta \to 0$) in homogeneous magnetic field effects of magnetic drift and vector potential are negligible, i.e. electrostatic approximation is available. It includes quasineutrality equation and the solution of the Vlasov equation with no perturbed magnetic field. The electrostatic approximation is available if $v_{Tj}A \ll \varphi$, where $v_{Tj} = \sqrt{k_B T_j/m_j}$. From Ampere's law one can estimate $k^2 A \sim \mu_0 \sum_{j=i,e} q_j f_{1j} d^3 v \sim \mu_0 e n_e v_{Te}\frac{e\varphi}{k_B T_e}$ and $v_{Tj}A \sim \frac{\mu_0 e^2 n_e v_{Te} v_{Tj}}{k_\perp^2 k_B T_e}\varphi$. Electrostatic approximation is available at

$$\beta_e = \frac{2\mu_0 n_e k_B T_e}{B_0^2} \ll 2(k_\perp \rho_{Te})^2 \frac{v_{Te}}{v_{Tj}}. \qquad (16)$$

For ions Eq. (16) is satisfied for high β at ETG range of $k_\perp$ and for very low β at ITG range. For $\beta_e \gtrsim 0.1$ this condition is satisfied for electrons and ions at $k_\perp \rho_{Te} > 1$.

So, instability can be considered in framework electrostatic limit with appropriate accuracy at the ETG range.

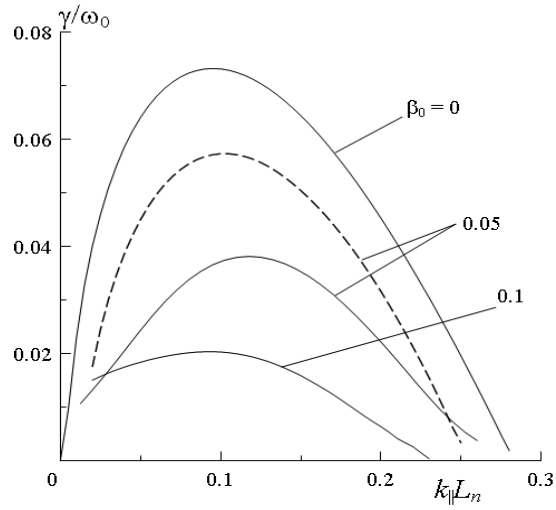

Fig. 1. Growth rates at moderate β_0: ——— – electromagnetic solution, – – – – – electrostatic approximation. $k_\perp\rho_{Ti} = 1$, $T_e/T_i = 1$, $\eta_i = \eta_e = 2$

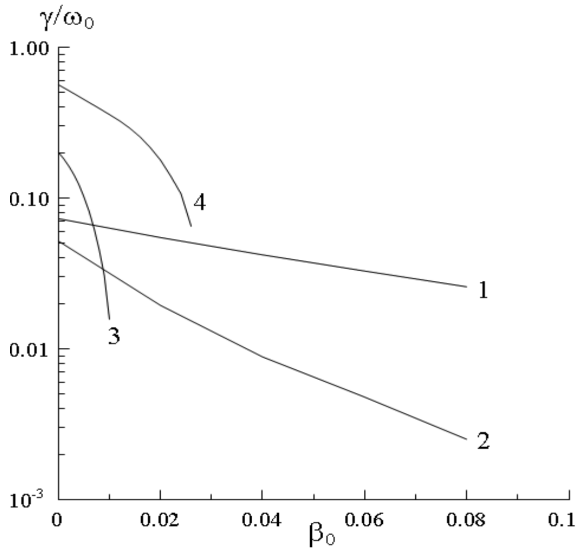

Fig. 2. Growth rates vs β_0 (moderate β_0): 1 – $k_\perp\rho_{Ti} = 1$, $k_\parallel L_n = 0.1$; 2 – $k_\perp\rho_{Ti} = 5$, $k_\parallel L_n = 0.07$; 3 – $k_\perp\rho_{Ti} = 10$, $k_\parallel L_n = 0.025$; 4 – $k_\perp\rho_{Ti} = 15$, $k_\parallel L_n = 0.05$. $T_e/T_i = 1$, $\eta_i = \eta_e = 2$

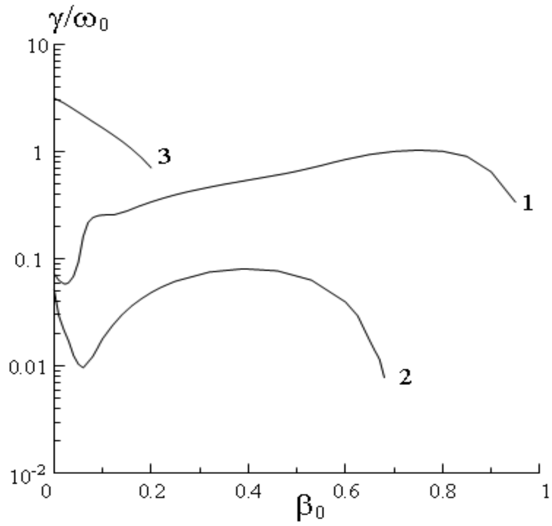

Fig. 3. Growth rates vs β_0 (high β_0): 1 – $k_\perp\rho_{Ti} = 1$, $k_\parallel L_n = 0.1$; 2 – $k_\perp\rho_{Ti} = 5$, $k_\parallel L_n = 0.07$; 3 – $k_\perp\rho_{Ti} = 43$ ($k_\perp\rho_{Te} = 1$), $k_\parallel L_n = 0.1$. $T_e/T_i = 1$, $\eta_i = \eta_e = 2$

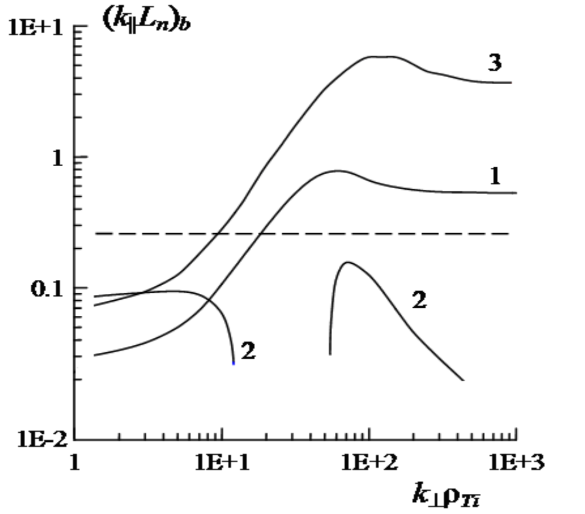

Fig. 4. Upper boundary of $k_{\|}L_n$ (solid lines) and approximate down boundary (dashed lines) of instability in FRC vs $k_{\perp}\rho_{Ti}$: 1 – $\eta_e = 2$, $\eta_i = 0.1$, $T_e/T_i = 0.5$; 2 – $\eta_e = 1$, $\eta_I = 0.1$, $T_e/T_i = 0.5$; 3 – $\eta_e = 2$, $\eta_i = 0.1$, $T_e/T_i = 0.1$

In Fig. 1, the comparison with electromagnetic and electrostatic solutions are presented for moderate values of β_0 ($\beta_0 < 0.1$). As a scale of the real frequency ω_R and growth rate γ we use $\omega_0 = k_B T_i /(eBL_n \rho_{Ti})$. In Figs. 2 and 3, results of electromagnetic calculations for modes with fixed $k_{\perp}$ are shown. Values of $k_{\|}$ for presented modes are close to maximum of the growth rate at fixed $k_{\perp}$.

To calculate instability parameters we use typical plasma conditions in FRC experiments [9-13]: $\eta_i \approx 0.1$, $\eta_e \approx 1–2$, $T_e/T_i \approx 0.5$ (for typical regimes), $T_e/T_i \approx 0.1$ (hot ions and cold electrons). Examples of the results of calculated $(k_{\|}L_n)_b$ are presented in Fig. 4. For not very elongated FRCs $2\pi L_n/L \sim 0.3$, i.e. for finite length stabilized modes $(k_{\|}L_n)_b < 0.3$. The dashed line in Fig. 4 corresponds to this approximate condition of stabilization.

CONCLUSIONS

Our calculations have shown that under FRC experiment conditions typical ITG instability ($k_{\perp} \lesssim 1/\rho_{Ti}$) appears to be hardly restricted by finite size of FRC devises, but in the range of ETG instability ($k_{\perp} \gtrsim 1/\rho_{Te}$) instability can exist. Parameters of such an instability are seems to be close to ETG mode, but ion effects for FRC experiment conditions are significant at $k_{\perp}\rho_{Te} < 1$. Maximum of growth rate is located at $k_{\perp}\rho_{Te} \sim 1$.

Calculated values and real frequencies for the ETG range agree well with data measured by Carlson on TRX-2 device [14].

Special calculations are shown that to decrease growth rate of ETG instability (and ETG driven turbulent transport) one can decrease η_e. To sustain of low-η_e configuration the heating of electrons in the plasma edge can be used.

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ЭЛЕКТРОННАЯ ТЕМПЕРАТУРНО-ГРАДИЕНТНАЯ ДРЕЙФОВАЯ НЕУСТОЙЧИВОСТЬ В КРАЕВОЙ ПЛАЗМЕ ОБРАЩЕННОЙ МАГНИТНОЙ КОНФИГУРАЦИИ С КОНЕЧНЫМ БЕТА

В.И. Хвесюк, А.Ю. Чирков

Электромагнитные дрейфовые неустойчивости впервые рассматриваются для плазмы обращенной магнитной конфигурации без использования приближения адиабатического отклика ионов и/или электронов в диапазоне значений перпендикулярного волнового числа от $k_{\perp} < 1/\rho_{Ti}$ до $k_{\perp} \sim 1/\rho_{Te}$ (ρ_{Ti} и ρ_{Te} – ионный и электронный тепловые гирорадиусы). Исследуется стабилизация конечной длинной плазмы. Обсуждается стабилизирующий эффект низких градиентов температуры на электронную моду.

ЕЛЕКТРОННА ТЕМПЕРАТУРНО-ГРАДІЄНТНА ДРЕЙФОВА НЕСТІЙКІСТЬ У КРАЙОВІЙ ПЛАЗМІ ЗВЕРНЕНОЇ МАГНІТНОЇ КОНФІГУРАЦІЇ З КІНЦЕВИМ БЕТА

В.І. Хвесюк, О.Ю. Чирков

Електромагнітні дрейфові нестійкості вперше розглядаються для плазми звернемої магнітної конфігурації без використання наближення адіабатичного відгуку іонів і/або електронів у діапазоні значень перпендикулярного хвильового числа від $k_{\perp} < 1/\rho_{Ti}$ до $k_{\perp} \sim 1/\rho_{Te}$ (ρ_{Ti} і ρ_{Te} – іонний і електронний теплові гірорадіуси). Досліджується стабілізація кінцевою довжиною плазми. Обговорюється стабілізуючий ефект низьких градієнтів температури на електронну моду.