CHAOTIC DECAYES IN RESONATORS FILLED WITH RARE PLASMA

A.N. Antonov, V.A. Buts, I.K. Kovalchuk, O.F. Kovpik, E.A. Kornilov, V.G. Svichensky, D.V. Tarasov
National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine
E-mail: vbuts@kipt.kharkov.ua

The results of processes theoretical and experimental investigation of the broadband chaotic HF oscillations in the electrodynamics structure with rare plasma are presented. They are based on the nonlinear decay of HF wave into new HF and LF ones. The natural waves which may take part in this interaction are defined. There is a good agreement between theoretical and experimental results.

PACS:52.40. Mj

1. INTRODUCTION

The opportunity of spectra management of HF generators represents significant scientific and practical interest. It is necessary to notice that conventional way for creating such generators with wide noise spectra is very difficult technical problem. We offer to use features of dynamics of the charged particles and electromagnetic fields at enough large intensity for formation of radiation with the desire spectral characteristics. Such approach allows to divide the process of generation of radiation from the process of formation of its spectrum. Mechanisms of widen spectra of excited radiation can be chaotic dynamics of particles, and also chaotic dynamics of nonlinear interaction of waves. The basic attention below we will pay on using of decaying processes with chaotic dynamics. For realization of decay processes it is necessary to have special dispersion of plasma electrodynamics structure. This dispersion should be as decay dispersion. Two cases of occurrence of chaotic dynamics are considered at nonlinear interaction of waves below. In first three waves take part in nonlinear process, in the second cascade of waves participate in disintegration process. Besides dispersion properties of cylindrical waveguide which partially filled by plasma, and results of experimental researches are presented in this work, in which the features of chaotic disintegrations are also investigated and which demonstrate an opportunity of generation of broadband noise signals.

2. NUMERICAL INVESTIGATION OF THREE-WAVE PROCESS OF STOCHASTIC DECAY

The works [1,2] are devoted to research of occurrence of dynamic chaos at nonlinear interaction of three waves in conditions of the modified disintegration. Analytical criterion of occurrence of chaos was fined there, and the results of numerical modeling were represented there too. The numerical results have confirmed analytical criterion of occurrence of chaos. The received realizations can be characterized as irregular. The spectra and the correlation functions, and also main of Lyapunov index were determined for them. At fulfillment of stochastic criterion the spectra were wide, and the correlation functions quickly fell down, the main Lyapunov index had a positive real part. It confirms chaotic character of three-wave process of the modified disintegration. At break down of this criterion (reduction of initial amplitude of a pumping wave) the spectra were narrowed, and the time of correlation was increased. It qualitatively coincides with experimental results which are stated below.

3. CHAOS AT CASCADE WAVES INTERACTION

Above we have observed interacting triplets of waves. However in electrodynamics structure observed by us which represents the cylindrical resonator filled by rare plasma, a great number of triplets can take part in the interacting. The elementary dispersion diagram of interacting waves is presented in Fig.1. It is possible to say, that the scheme of interacting transverse electromagnetic waves with Langmuir waves of plasma is represented in this figure. From this scheme follows, that it is possible the cascade of decaying waves. Really, if initially the high-frequency transverse wave is excited, it can interact with the return transverse high-frequency electromagnetic wave and with the plasma wave. The high-frequency wave excited at such process, in turn, can decay on the same high-frequency and on Langmuir. This process can contain a significant amount of stages (see Fig.2). In order to describe it let's present, as well as earlier, the electric field of the transverse electromagnetic waves in the form of:

$$E = \sum A_n \exp\left[i\left(\omega_0 + n\Delta\omega\right)t - i\left(\vec{k}_0 + n\Delta\vec{k}\right)\right], \qquad (1)$$

where $\omega_0, \vec{k}_0$ – the minimal frequency and wave vector of one of eigen transverse waves of the resonator; $\Delta\omega$ – the distance between eigen transverse waves of the resonator; A_n complex amplitudes of eigen transverse waves.

Let us consider, that distance between eigen transverse modes is equal to low-frequency plasma waves: $\Delta\omega = \Omega$. These (Langmuir) waves have equal frequency, but different wave vectors. The electric field of Langmuir waves can be presented in the following form:

$$E_l = \sum B_n \exp\left[i\Omega t - i\left(\vec{k}_{0,l} + n\Delta\vec{k}_l\right)\right]. \qquad (2)$$

The equations which present evolution of complex amplitudes of the transverse and plasma waves, can be presented in the following set of equations:

$$\frac{\partial A_0}{\partial t} = V_{10}A_1B_0^*, \qquad \frac{\partial A_1}{\partial t} = V_{00}A_0B_0 + V_{21}A_2B_1^*,$$

$$\frac{\partial A_2}{\partial t} = V_{11}A_1B_1 + V_{32}A_3B_2^*, \quad \frac{\partial A_n}{\partial t} = V_{n-1,n-1}A_{n-1}B_{n-1}, \qquad (3)$$

$$\frac{\partial B_0}{\partial t} = W_1A_1A_0^*, \quad \frac{\partial B_1}{\partial t} = W_2A_2A_1^*, \quad \frac{\partial B_n}{\partial t} = W_nA_nA_{n-1}^*.$$

While getting (3) we supposed the following satisfied requirement of synchronism $\omega_n - \omega_{n-1} = \Omega$, $\vec{k}_n - \vec{k}_{n-1} = \Delta\vec{k}_l$ (the law of conservation of energy and momentum) for all three waves participating in interacting.

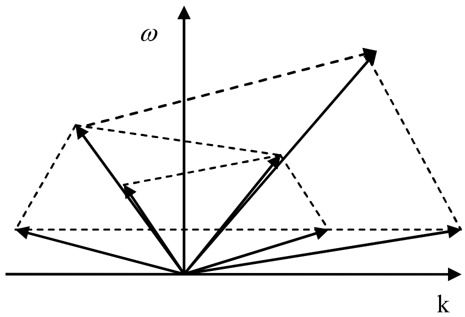


Fig.1. The dispersion diagram of interacting waves

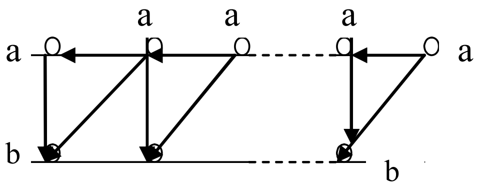


Fig.2. The diagram of decay

From system (3), and from Fig.1 and diagram 2 we can see, that the first equation presents dynamics of the lowest-frequency transverse electromagnetic wave. This wave does not decay on other waves. The last equation for amplitudes of the transverse electromagnetic waves presents dynamics of the highest-frequency transverse electromagnetic wave. This wave decays. The inflow of energy to it occurs only when the phase of the low-frequency wave with which it interreacts changes the sign. For the further analysis of the set of equations (3) it is convenient to transfer to real variables amplitude and the phase:

$$A_k = a_k e^{1\varphi k}, \quad B_k = b_k e^{1\psi k}. \qquad (4)$$

The interaction matrix elements V_{ik} also W are not arbitrary. We shall consider, that all the diagonal elements of this matrix are equal, as well as nondiagonal elements $V_{ii} = V_-$; $V_{ik} = V_+ = W$, $i \neq k$. In order to define the specific value of these elements of a matrix, it is necessary to take into account the law of conservation of energies and momenta. From these laws it follows $V_{ii} = -1$; $V_{ik} = 1 = W$, $i \neq k$. Except the law of conservation of energies and momentums it is possible to get the following integral:

$$\sum a_k^2 = const, \qquad (5)$$

which expresses the law of conservation the number of high-frequency quantums.

Using all these laws, it is finally possible to get the following set of equations for the description of dynamics of the real amplitudes a_i and the real phases φ_i, ψ_i:

$$\begin{aligned}
\dot{a}_n &= -2\delta a_{n-1} b_{n-1} \cos\left(\varphi_{n-1} - \varphi_n + \psi_{n-1}\right) + \\
&\quad + \delta_2 a_{n+1} b_n \cos\left(\varphi_{n+1} - \varphi_n - \psi_n\right), \\
\dot{b}_{n-1} &= a_m a_{m-1} \cos\left(\varphi_m - \varphi_{m-1} - \psi_{m-1}\right), \\
\dot{\varphi}_n &= -2\delta_1 \frac{a_{n-1} b_{n-1}}{a_n} \sin\left(\varphi_{n-1} - \varphi_n + \psi_{n-1}\right) + \\
&\quad + \delta_2 \frac{a_{n-1} b_n}{a_n} \sin\left(\varphi_{n+1} - \varphi_n - \psi_n\right), \\
\dot{\psi}_{m-1} &= \frac{a_m a_{m-1}}{b_{m-1}} \sin\left(\varphi_m - \varphi_{m-1} - \psi_{m-1}\right),
\end{aligned} \qquad (6)$$

where $n = 0, 1, \ldots. N$; $m \equiv 1, \ldots N$;

$$\delta_1 = \begin{cases} 0, n = o \\ 1, \; n \neq 0 \end{cases} \quad \delta_2 = \begin{cases} 0, n = N \\ 1, \; n \neq N \end{cases}.$$

Fig.3 is the characteristic dependence of amplitude of the third wave on the time; the phase plane for the second wave is in Fig.4, the correlation function is in Fig.4, the characteristic spectra is in Fig.5. From time dependences, from the spectrums and the behavior of the correlation function it is seen that the dynamics of interacting waves is chaotic for all considered cases.

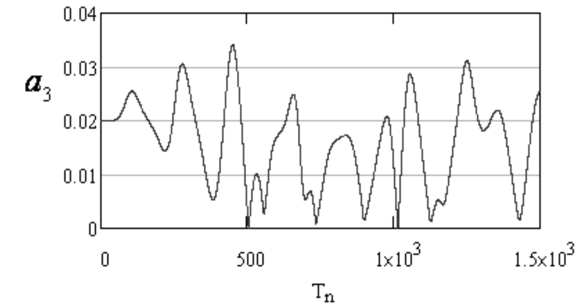


Fig.3. Time evolution of the amplitude of the third wave

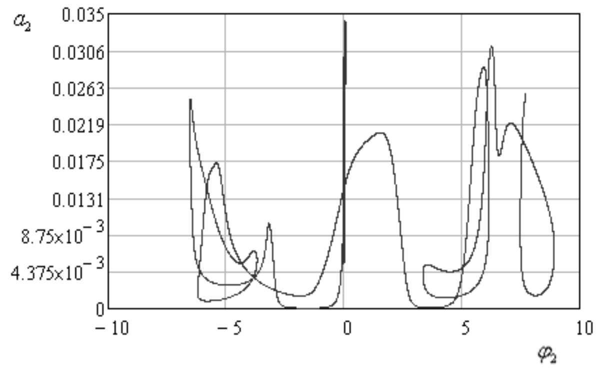


Fig.4. The projection of the phase space of the second wave to the plane $a_2\varphi_2$

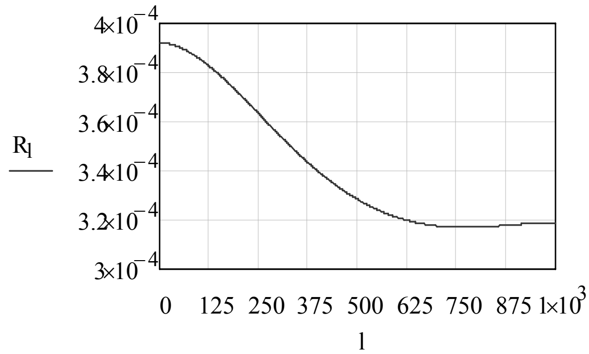


Fig.5. The correlation function for a_3

4. THE DISPERSION EQUATIONS OF THE MAGNETIZED PLASMA IN THE A METAL SHEATH

Let's observe the cylindrical ideally conducting waveguide of radius b partially filled with plasma with plasma frequency ω_p. The plasma is presented as the cylinder of radius a, coaxial with the waveguide. All system is placed in the constant magnetic field guided along the axis of system. There can be both fast and slow waves. The last ones are observed in [3] in details. Some general approaches to studying the magnetized plasma in a metal housing, are studied in [4]. Getting the dispersion equation in a general case is a rather difficult task. Besides such equation will be rather complex. In paper [5] the attention was paid to finding the real solutions in the field of parameters where the transverse wave numbers are complex. In [6] dispersion properties and structure of the fields of axially asymmetrical waves are studied when plasma completely fills a metal sheath.

First of all we are interested in fast modes which can participate in the processes of nonlinear decay. We shall restrict our consideration to the axially symmetrical oscillations near the upper hybrid resonance which is of interest to specific experimental requirements. The dispersion equation in this case is:

$$\left(y_1-\frac{\kappa\Delta_{00}(a)}{\Delta_{10}(a)}\right)\left(y_2-\frac{\kappa\Delta_{01}(a)}{\Delta_{11}(a)}\right)f_2-\left(y_1-\frac{\kappa\Delta_{01}(a)}{\Delta_{11}(a)}\right)\left(y_2-\frac{\kappa\Delta_{00}(a)}{\Delta_{10}(a)}\right)f_1=$$
$$=\frac{\omega_p^2}{\omega^2}\frac{\kappa\Delta_{00}(a)}{\Delta_{10}(a)}\left[f_1y_1-f_2y_2+\frac{\kappa\Delta_{01}(a)}{\Delta_{11}(a)}(f_2-f_1)\right], \quad (7)$$

where ω – the excited frequency in the system $\kappa^2=k^2-k_z^2$, $k=\omega/c$, k_z – the longitudinal wave number, ω – the oscillation frequency

$y_{1,2}=\left(k_{1,2}J_0(k_{1,2}a)\right)/\left(kJ_0(k_{1,2}a)\right)$

$\Delta_{ik}(r)=N_i(\kappa r)J_k(\kappa b)-J_i(\kappa r)N_k(\kappa b)$ $J_i(x)$, $N_i(x)$ – Bessel and Neumann's functions i-th order accordingly, $k_{1,2}$ – the wave numbers, presenting the transverse structure of the field in plasma which look like

$$k_{1,2}^2=-\left[k_z^2-k^2(1-\omega_p^2/\omega^2)\right]-\frac{\omega_p^2\omega_h^2(k_z^2+k^2)}{2\omega^2\Delta\omega^2}\pm$$
$$\pm\frac{\omega_p^2\omega_h^2\sqrt{4k_z^2k^2\Delta\omega^2+\omega_h^2(k_z^2+k^2)^2}}{2\omega^2\left|\Delta\omega^2\right|}, \quad (8)$$

where $\Delta\omega^2=\omega^2-\omega_e^2-\omega_p^2$, ω_e – electron cyclotron frequency, ω_p – electron plasma frequency

$f_{1,2}=\omega_e\kappa^2\mp sign(\Delta\omega^2)\sqrt{4k_z^2k^2\Delta\omega^2+\omega_e^2(k_z^2+k^2)^2}$.

As it is seen from (8) the squares of the transverse wave numbers $k_{1,2}$ can be both negative and positive. Depending on their signs the dispersion equations will have absolutely different shape and structure of the solutions. The expressions (8) have been simplified and analyzed for the fields of frequencies near upper hybrid resonance $\Delta\omega^2<<\omega^2,\omega_h^2$; $\omega^2 \approx \omega_h^2$. It is also supposed, that $\omega_p^2<<\omega^2$. In the fields $\Delta\omega^2>0$ and $\Delta\omega^2<0$ one of the numbers $k_{1,2}$ is close to vacuum ($k_{1,2}\approx\kappa$) and does not depend on the applied magnetic field. The second one, on the contrary, is defined by the ω_h. For $\Delta\omega^2>0$, at $\Delta\omega^2\to 0$ $k_1\approx\kappa$ $k_2^2\to-\infty$. For $\Delta\omega^2<0$, at $|\Delta\omega^2|\to 0$ $k_1^2\to\infty$, and $k_2\approx\kappa$. As it is seen from the given above relationships, in our case it is possible to put into operation two dimensionless small parameters: $\omega_p^2/\omega^2<<1$ and $\Delta\omega^2/\omega_h^2<<1$. The character of solutions of the dispersion equations essentially depends on the relationship between them. Accomplishing both requirements $\omega_p^2/\omega^2<\left|\Delta\omega^2\right|/\omega_e^2$ the wave numbers are close to vacuum value ($k_{1,2}\approx\kappa$), and solutions of the dispersion equation (7) in this case are close to modes of the empty circular guide. Otherwise the influence of the plasma is essential.

The dispersion equation (7) can be converted at $\Delta\omega^2<0$ $|\Delta\omega^2|\to 0$. Bessel's functions of zero and the first order from argument $k_1a\to\infty$ in this case can be exchanged by their asymptotic approximation. The equation (7) in this case will look like:

$$\sqrt{\frac{\omega_p^2}{|\Delta\omega^2|}\frac{k_z^2+k^2}{k^2}}ctg\left(\sqrt{\frac{\omega_p^2}{|\Delta\omega^2|}(k_z^2+k^2)}a-\frac{\pi}{4}\right)\times$$
$$\times\left[\frac{k_2}{\kappa}\frac{J_0(k_2a)}{J_1(k_2a)}(k_z^2+k^2)-\left(\frac{\Delta_{01}}{\Delta_{11}}k_z^2+\varepsilon_3\frac{\Delta_{00}}{\Delta_{10}}k^2\right)\right]=$$
$$=\frac{\kappa}{k}\left[\frac{k_2}{\kappa}\frac{J_0(k_2a)}{J_1(k_2a)}\left(\varepsilon_3\frac{\Delta_{00}}{\Delta_{10}}k_3^2+\frac{\Delta_{01}}{\Delta_{11}}k^2\right)-\varepsilon_3\frac{\Delta_{00}\Delta_{01}}{\Delta_{10}\Delta_{11}}(k_z^2+k^2)\right], \quad (9)$$

where $\varepsilon_3=1-\omega_p^2/\omega^2$.

As it is seen, in the case when $\Delta\omega^2<0$ and $|\Delta\omega^2|\to 0$ at fixed k_z the dispersion equation for the fast waves has the infinite number of solutions which are condensed near to the upper hybrid frequency $\omega_h=\sqrt{\omega_e^2+\omega_p^2}$. The dispersion curves are placed in the narrow frequency band between ω_e and ω_h.

The dispersion equation (7) was solved numerically. The character of the dispersion curves is presented in Fig.6; 6a and 7. As it is seen, in the field of frequencies $\omega<\omega_e$ the dispersion of the waveguide partially filled with plasma is similar to the dispersion of the empty cylindrical waveguide. While approaching the electron cyclotron frequency the dispersion curves corresponding the empty waveguide become deformed. They enter the frequency band between electron cyclotron and upper hybrid where they are located parallely to horizontal axis that is shown in Fig.6 in details. Besides in this field of frequencies there is the infinite number of branches which do not go beyond. I.e. their cut-off frequencies are in the field of between ω_e and ω_h.

Thus, it coincides with the made above inference, received due to the analytical analysis, that in the field of the parameters $\omega_p^2/\omega^2>\left|\Delta\omega^2\right|/\omega_e^2$ the dispersion is defined by the properties of plasma. In the field of frequencies $\omega>\omega_h$ the dispersion of the observed waveguide filled with plasma is close to the dispersion of the empty one. The analytical analysis and the numerical calculations show that accomplishing the requirement $\omega_p^2/\omega^2<\Delta\omega^2/\omega_e^2$ we get $k_1\approx k_2\approx\kappa$.

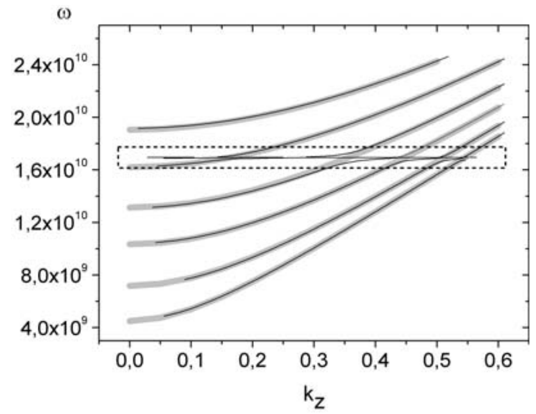
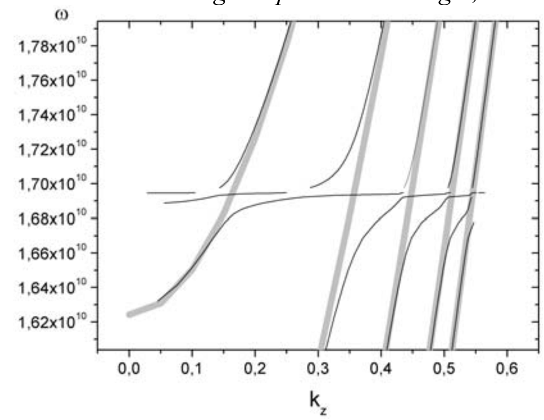


Fig.6. The dispersion of cylindrical waveguide partially filled with plasma. Wide gray curves correspond to dispersion of empty waveguide, and black ones correspond to waveguide filled with plasma. The region pointed by dashed rectangle is presented in Fig.6,a



Puc.6a. The dispersion of cylindrical waveguide partially filled with plasma. This figure corresponds to dashed rectangle in Fig.6

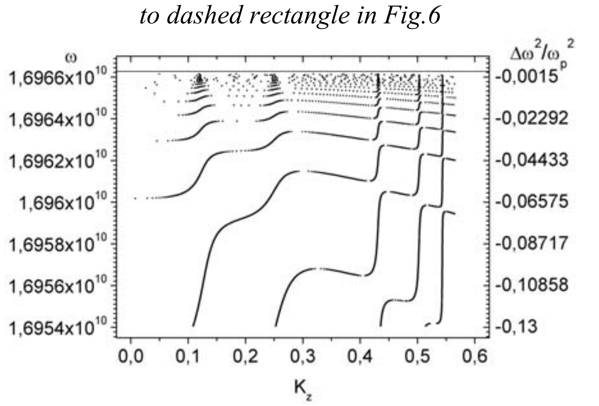


Fig.7. The dispersion of cylindrical waveguide partially filled with magnetoactive plasma near upper hybrid resonance frequency. Horizontal line in upper part of figure corresponds to this frequency

I.e. the structure of the field corresponding this field of frequencies in plasma is close to the structure of the field in vacuum. As it was noted above in the frequency band $\omega_e \le \omega \le \omega_h$ $k_1^2 \to \infty$, and $k_2 \approx \kappa$, that specifies fast oscillating in transverse direction structure of the field in plasma.

In paper [6] the dispersion of the metal waveguide completely filled with the plasma placed in finite magnetic field for axially asymmetrical modes was studied. Its specific features are qualitatively close to the structure observed in the present report.

Thus in metallic cylindrical waveguide partially filled with magnetized plasma in the region of frequencies close to upper hybrid resonance the infinite number of modes exists. We are interested in nonlinear processes of wave decay by the following scheme: HF→HF+LF. One of the natural mode can be used as pump mode, which decays on new HF, localized lower then upper hybrid resonance and other one of the LF plasma modes. Large set of natural oscillations in considered system can be used for realization of cascade decay processes.

5. EXPERIMENTAL RESULTS

The experimental setup is multimode resonator which is laced into longitudinal magnetic field. To excite field in resonator the magnetron generator is used. The resonator is excited at frequency 2.77 GHz. When these experiments were carried out it was revealed that pulse duration of exciting HF oscillations is more longer then duration of magnetron pulse.

When introduced power is 20% more then threshold one, corresponding to decay process beginning the electrons with energy of hundreds of KeV appear. In some microseconds after pulse ending the pulses of HF radiation start appearing. The oscillations with frequencies 1.3; 2.68; 3.75; 5 GHz were registered in them with bandwidth 10 MHz. The temporal interval between pulses may vary in limits 0…30 μs.

The appearance of pulses was random on time and amplitude. When magnetic field value on half resonator length is decreased on 25% the regularity of repeated pulses appearance was increased.

When repeated pulses were appeared the pulses of plasma luminescence and X-ray radiation appeared too, conditioned by the slowing-down of accelerated electrons with tens – hundreds keV on neutral molecules of gas.

The spectrum of low frequency oscillations in cavity was defined experimentally versus introducing power. The signal oscillograms from resonator are presented in Fig.8-10 for three values of introduced power 17, 58 and 167 kW. It is seen that when introduced power is increased the kind of registered signal in resonator becomes less regular. In Fig.11 the experimental dependence of bandwidth versus introduced power is presented. Growth of bandwidth of spectrum and kind of oscillograms point out that when amplitude of pumping wave increases the processes in resonator, partially filled by magnetoactive plasma became chaotic. Analogous dynamics was observed at numerical simulation of process of three wave decay.

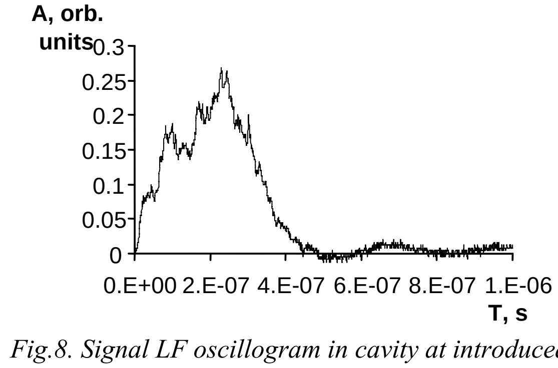


Fig.8. Signal LF oscillogram in cavity at introduced power 17 kW

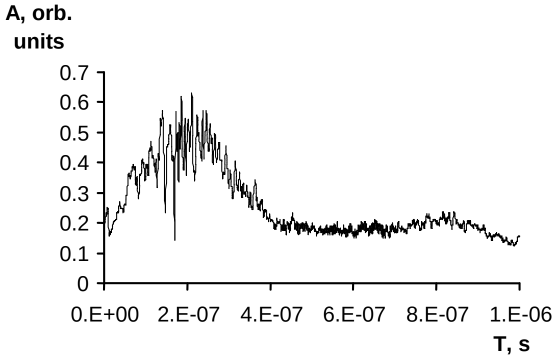


Fig.9. Signal LF oscillogram in cavity at introduced power 58 kW

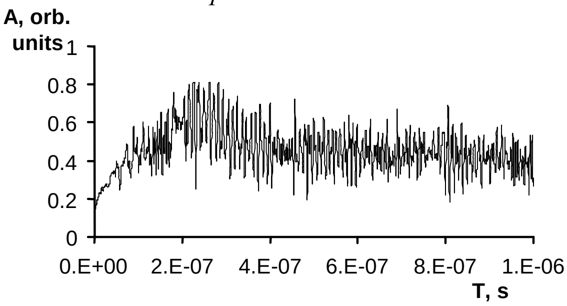


Fig.10. Signal LF oscillogram in cavity at introduced power 167 kW

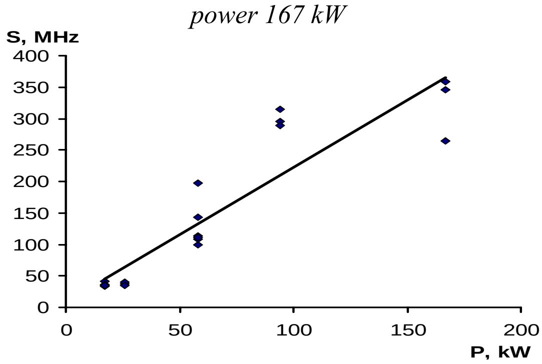


Fig.11. Bandwidth of spectrum versus introducing power

CONCLUSIONS

In this article the results of theoretical and experimental investigations on exciting of wideband noise oscillations in cylindrical resonator partially filled by plasma are presented. This process is basing on the modified decay of high frequency wave in this system into new high frequency and low frequency ones. Briefly results of numerical simulation of such decay have been stated. The possibility of the chaos development in many wave cascade regime has been shown. The dispersion properties of natural axial symmetric modes of cylindrical resonator partially filled by magnetoactive plasma were investigated. Those of them which may take part in chaotic decay processes are pointed out. The results of experimental investigations which qualitatively agree with theoretical conclusion are presented.

The results presented in [2] and in this article show a good agreement of theoretical views about chaotic decay and experimental data. It is necessary to note that obviously in [2] and in present work the process of chaotic decay was first investigated experimentally. We note that in experiment the conditions for chaotization of interaction are satisfied not only for wave-wave processes, but for the wave-particle processes too. This becomes obvious that simultaneously with stochastic decay stochastic heating of plasma particles takes place. For our goals this process is harmful. It can be removed by introducing nonuniformity of magnetic field.

It is also necessary to note the peculiarities of the dispersive properties of the electrodynamics structures with rare plasma. In spite of low plasma density the system dispersion essentially changes due to the resonance interaction of waves with plasma particles. In particular, the large number of the additional eigen modes arises, which can be used for chaotization of decay processes.

REFERENCES

1. V.A. Buts, I.K. Kovalchuk, E.A. Kornilov, D.V. Tarasov. Stabilization beam instability as a result of development of local instability at interaction of type a wave-wave // *Physics of plasma*. 2006, v.32, №6, p.1-10.
2. V.A. Buts, I.K. Kovalchuk, E.A.Kornilov, D.V. Tarasov. Dynamical chaos rise in the system of large number of nonlinear coupled oscillators // *Voprosy atomnoy nauki i tekhniki.* 2007, №3(2), p.260-264 (in Russian).
3 A.N. Kondrtenko. *Plazmenniye volnovody*. M.: "Atomizdat", 1976, 232 p. (in Russian).
4 *Entsiklopediya nizkotemperaturnoy plazmy.* M.: "Nauka", 2000, v.4, part X, p.132-151 (in Russian).
5 I.N. Onishchenko, G.V. Sotnikov. Dispersiya plazmennikh voln v konechnom magnitnom pole // *Phizika plazmy.* 1992, v.18, №3, p.335-345 (in Russian).
6. G.I. Zaginaylov, V.I. Shcherbinin, K. Shunemann. Nekotoriye osobennosty dispersionnikh svoystv volnovodov, zapolnennikh plazmoy v magnitnom pole // *Phizika plazmy*. 2005, v.31, №7, p.647-751 (in Rusian).

Статья поступила в редакцию 31.05.2008 г.

ХАОТИЧЕСКИЕ РАСПАДЫ В РЕЗОНАТОРАХ С РАЗРЕЖЕННОЙ ПЛАЗМОЙ

А.Н. Антонов, В.А. Буц, И.К. Ковальчук, О.Ф. Ковпик, Е.А. Корнилов, В.Г. Свиченский, Д.В. Тарасов

Изложены результаты теоретического и экспериментального исследования процессов формирования широкополосных СВЧ-колебаний в электродинамической структуре с разреженной плазмой. Они основаны на нелинейном распаде ВЧ-волны на новую ВЧ- и НЧ-волны. Определены собственные моды, которые могут принимать участие в таком взаимодействии. Между результатами теоретических и экспериментальных исследований имеется хорошее качественное соответствие.

ХАОТИЧНІ РОЗПАДИ В РЕЗОНАТОРАХ З РІДКОЮ ПЛАЗМОЮ

А.М. Антонов, В.О. Буц, І.К. Ковальчук, О.Ф. Ковпик, Є.О. Корнілов, В.Г. Свіченський, Д.В. Тарасов

Викладені результати теоретичного та експериментального дослідження процесів формування широкосмугових НВЧ-коливань в електродинамічній структурі з рідкою плазмою. Вони ґрунтуються на нелінійному розпаді ВЧ-хвилі на нову ВЧ- та НЧ-хвилі. Визначені власні моди, що можуть приймати участь в такій взаємодії. Між результатами теоретичних та експериментальних досліджень існує добра якісна відповідність.