SPATIAL AND TIME DYNAMICS OF NON-LINEAR VORTICES IN HIGH-CURRENT PLASMA LENS

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The spatial shape and non-linear dynamics of vortices in the high-current plasma lens have been investigated.
PACS: 52.40.-w

INTRODUCTION

It is known from numerical simulations and experiments that vortices are long-lived perturbations in vacuum. However, the acceleration of evolution of vortices in electron plasma was observed in laboratory experiments. Same dynamics of vortices should take place in near wall turbulence of the nuclear fusion installations, where the crossed configuration of electrical and magnetic fields is also realized.

The charged plasma lens, intended for focussing of high-current ion beams, has the same crossed configuration of fields [1]. It is important to know properties of vortices at non-linear stage of their evolution. It is shown theoretically in this paper that at achievement of a quasi-stationary state the electrons in a field of a vortex are rotated around its axis with the greater velocity in comparison with velocity of an azimuth electron drift in fields of the lens. Slow and fast vortices are contacting combinations of two vortices rotating in the opposite directions.

The instability development in initially homogeneous plasma causes that the vortices are beard in kind of pairs. Namely, if a vortex - bunch of electrons is generated, a vortex - cavity of electrons comes up near it. It has been shown, that with taking into account of a weak non-uniformity of electron density in an actual experimental lens the preference in behavior of vortices is realized. Namely, the vortex - bunch propagates to region of greater electron density n_e, and vortex - cavity propagates in region of smaller n_e.

It has been shown that the vortex - bunch can result in to formation of spiral distribution of electron density.

The physical mechanism of coalescence of vortices - bunches of electrons has been proposed.

SPATIAL STRUCTURE OF VORTICES

Let's describe structure of a fast vortex in a rest frame, rotated with angular rate $\omega_{ph} \equiv V_{ph}/r_q$. Let's consider a chain (on θ) of interleaving vortices - bunches and vortices - cavities of electrons. Neglecting non-stationary and non-linear on ϕ terms, we receive the following equation

$$\mathbf{V}_\perp = -(e/m_e\omega_{He})[\mathbf{e}_z, \mathbf{E}_{ro}] + (e/m\omega_{He})[\mathbf{e}_z, \nabla\phi], \quad (1)$$

describing quasistationary dynamics of electrons in fields of the lens and vortical perturbation. From (1) we receive expression for radial and azimuth velocities of electrons

$$V_r = -(e/m_e\omega_{He})\nabla_\theta\phi\,,\ \ V_\theta = V_{\theta o} + (e/m_e\omega_{He})\nabla_r\phi\,,$$
$$V_{\theta o} = -(e/m_e\omega_{He})E_{ro} = (\omega^2_{pe}/2\omega_{He})(\Delta n/n_{oe})r \quad (2)$$

V_θ can been presented as the sum of the phase velocity of the perturbation, V_{ph}, and velocity of azimuth oscillations of electrons, δV_θ, in the field of the perturbation, $V_\theta = V_{ph} + \delta V_\theta$. As $V_\theta = r d\theta/dt$, we present $d\theta/dt$ as $d\theta/dt = d\theta_1/dt + \omega_{ph}$, here $\omega_{ph} = (\Delta n/n_{oe})(\omega^2_{pe}/2\omega_{He})|_{r=rv}$, r_v is the radius of the vortical perturbation location. Then from (2) we receive

$$d\theta_1/dt = (\omega^2_{pe}/2)(\Delta n/n_{oe})[1/\omega_{He}(r) - 1/\omega_{He}(r_v)] +$$
$$+ (e/rm_e\omega_{He})\partial_r\phi\,,\ dr/dt = -(e/m_e\omega_{He}r)\partial_\theta\phi \quad (3)$$

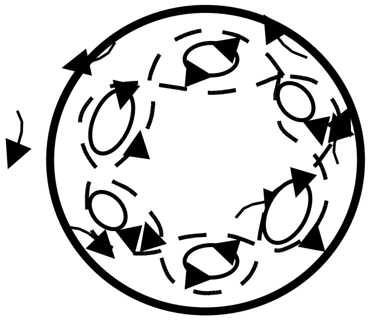

Fig. 1. The boundary of the chain of the quick vortices and the electron trajectories in its field in the rest frame, rotating with ω_{ph}, in the cylindrical plasma lens

At small deviations r from r_v, decomposing $\omega_{He}(r)$ on $\delta r \equiv r - r_v$ and integrating (3), we receive

$$(\delta r)^2 - 2\omega_{He}(r_v)\phi/\pi e\Delta n r_v(\partial_r\omega_{He})|_{r=rv} = \text{const} \quad (4)$$

The vortex boundary separates the trapped electrons, formed the vortex and moving on closed trajectories (see Fig. 1), and untrapped electrons moving outside the boundary of the vortex and oscillating in its field. For vortex boundary we receive the following expression from the condition $\delta r|_{\phi=-\phi o} = \delta r_{cl}$

$$\delta r = \pm[2(\phi+\phi_o)\omega_{He}(r_v)/\pi e\Delta n r_v(\partial_r\omega_{He})|_{r=rv} + (\delta r_{cl})^2]^{1/2} \quad (5)$$

Here δr_{cl} is the radial width of the vortex - bunch of electrons. From (5) the radial size of the vortex - cavity of electrons follows

$$\delta r_h \approx 2[\phi_o\omega_{He}(r_v)/\pi e\Delta n r_v(\partial_r\omega_{He})|_{r=rv}]^{1/2} \quad (6)$$

From the equation of electron motion and Poisson equation it is possible to receive approximately expression for the vorticity $\alpha \equiv \mathbf{e}_z \mathrm{rot}\mathbf{V}$, which is characteristic of the vortical motion of electrons

$$\alpha \approx -2eE_{ro}/rm\omega_{He} + (\omega^2_{pe}/\omega_{He})\delta n_e/n_{eo}$$

From here it follows that up to certain amplitude of vortices the structure of electron trajectories in the field of the chain on θ of fast vortices in the rest frame, rotated with $\omega_{ph} \equiv V_{ph}/r_q$, looks like, shown in Fig. 1 and is similar to the structure, presented in [2].

For large amplitudes of fast vortices in the region of electron bunches the reverse flows are formed, shown in Fig. 2. The vortex - cavity is rotated in the rest frame, rotated with frequency $\omega_{ph}\equiv V_{ph}/r_q$, in the same direction (see Fig. 2) as nonperturbed plasma. The vortex - bunch is rotated in the opposite direction of rotation of nonperturbed plasma at $\delta n_e>\Delta n\equiv n_{oe}-n_{oi}$ (see Fig. 2).

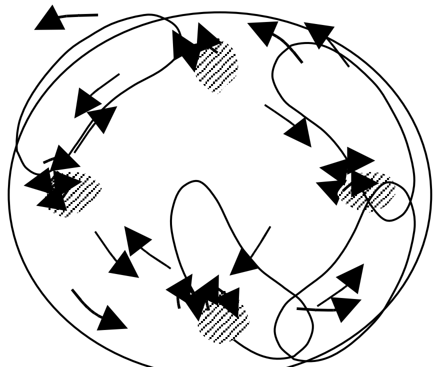

Fig. 2. The electron trajectories in rest frame, rotating with $\omega_{ph}\equiv V_{ph}/r_q$, in the field of the chain of the vortices-bunches and vortices-holes of electron density with $V_{ph}\approx V_{\theta o}(r_q)$ at $l_\theta=4$ in the cylindrical electric plasma lens

It is visible, that the size of the vortex is inversely proportional to $[(\Delta n/n_{oe})(\omega_{pe}/\omega_{eH})\partial_r\omega_{He}]^{1/2}$ and is proportional to $\phi^{1/2}_o$. That is the size of the vortex essentially depends on a gradient of the magnetic field. At small $\Delta n/2n_{oe}$ and ω_{pe}/ω_{He} already at small perturbations of electron density the sizes of the vortex, δr_h, can reach $\delta r_h\approx R/2$, R is the radius of the plasma lens.

(3) can be integrated without decomposing $\omega_{He}(r)$ on $\delta r\equiv r-r_v$. For this purpose we approximate $\omega_{He}(r)=\omega_{Ho}(1+\mu r^2/R^2)$. Then, integrating (3), we receive

$$2\phi+\pi e\Delta nr^2[1-\omega_{Ho}/2\omega_{He}(r_v)-\omega_{He}(r)/2\omega_{He}(r_v)]=const \quad (7)$$

From the condition $r|_{\phi=-\phi_o}=r_v+\delta r_{cl}$ and (7) we derive the expression, determining the boundary of the vortex - cavity of electrons,

$$[r^2-(r_v+\delta r_{cl})^2][1-\omega_{Ho}/\omega_{He}(r_v)]- \quad (8)$$
$$-[r^4-(r_v+\delta r_{cl})^4]\omega_{Ho}\mu/2R^2\omega_{He}(r_v)+2(\phi+\phi_o)/\pi e\Delta n=const$$

From (8) and $r|_{\phi=\phi_o}=r_v+\delta r_h$ we derive the expression, determining the radial width of the vortex - cavity of electrons,

$$\phi_o 4R^2\omega_{He}(r_v)/\pi e\Delta n]\omega_{Ho}\mu=$$
$$=(\delta r_h-\delta r_{cl})(2r_v+\delta r_h+\delta r_{cl})[r_v(\delta r_h+\delta r_{cl})+(\delta r_h^2+\delta r_{cl}^2)/2] \quad (9)$$

Let's consider the vortex with the small phase velocity V_{ph} in comparison with drift velocity of electrons, $V_{ph}<<V_{\theta o}$. The spatial structure of electron trajectories in its field for small amplitudes of the vortex looks like, shown in Fig. 3. It is determined by that in all lens α has the identical sign, $\alpha>0$. In other words, radial electrical field created by the vortex is less, than electrical field of the lens, $E_{rv}<E_{ro}$. Then in all lens the azimuth velocities of electrons have the identical sign and there are no reverse flows of electrons. For the description of spatial structure of electron trajectories we use (2). Using in them $V_\theta=rd\theta/dt$ and excluding θ, we receive for vortex boundary $r(\theta)$

$$r=[r^2_s+(\phi_o-\phi)2/\pi e\Delta n]^{1/2} \quad (10)$$

In the case of small amplitudes (10) becomes

$$\delta r\equiv r-r_s=(\phi_o-\phi)/\pi e\Delta nr_s \quad (11)$$

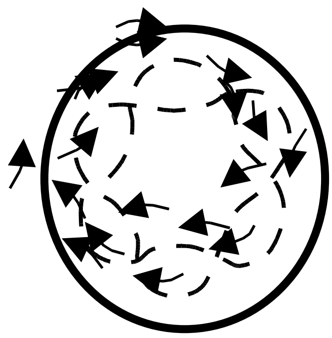

Fig. 3. The electron trajectories in the rest frame, rotating with ω_{ph}, in the field of the chain of the small amplitude slow vortices with $V_{ph}<<V_{\theta o}$ at $l_\theta=6$, in the cylindrical electric plasma lens

From (10) we receive the radial size of the slow vortex

$$\delta r_s\equiv r|_{\phi=-\phi_o}-r_s=[r^2_s+4\phi_o/\pi e\Delta n]^{1/2}-r_s \quad (12)$$

In the case of small amplitudes (12) becomes

$$\delta r_s\approx 2\phi_o/\pi e\Delta nr_s \quad (13)$$

For the description of the slow vortex structure one can also use the equation

$$d_t\omega_{He}/n_e\approx 0,\ d_t=\partial_t+(\mathbf{V}_\perp\nabla_\perp)-V_{ph}\nabla_\theta \quad (14)$$

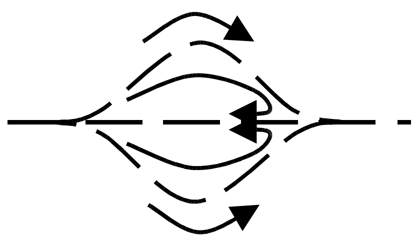

Fig. 4. The electron trajectories in the rest frame, moving with V_{ph}, in the field of the nonlinear slow vortex, with $V_{ph}<<V_{\theta o}$, and its boundary in a plane approximation

At large amplitudes, $\delta n_e>\Delta n$ (or $E_{rv}>E_{ro}$), in the region, where the electron cavities place, the characteristic of the vortical motion α receives the inverse sign, $\alpha<0$. In other words, on the axis, connecting the vortex - cavity and the vortex - bunch, the inequality $E_{rv}>E_{ro}$ is executed and there is an azimuth reverse flow of electrons. Then in some regions the electrons are rotated in the direction inverse to their rotation in crossed fields of the lens. At large amplitudes the structure of the separate slow vortex looks like, shown in Fig. 4. The slow vortex is a dipole perturbation of electron density, disjointed on radius. At $\delta n_e>\Delta n$ the structure of the slow vortex is similar to the structure of Rossby vortex.

NON-LINEAR DYNAMICS OF VORTICES

The development of instability in an initially homogeneous plasma lens causes that the vortices are beard in kind of pairs: if the vortex - bunch of electrons is generated, the vortex - cavity of electrons occurs near it.

As the real plasma is nonhomogeneous, we consider, how the non-uniformity of electron density effects on behavior of vortices.

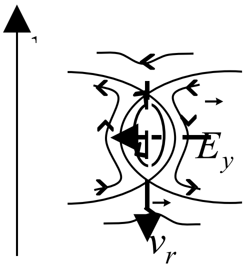

Fig. 5. The radial shift of the vortex-bunch of the electron density in the plasma lens with electron density, decreasing on radius

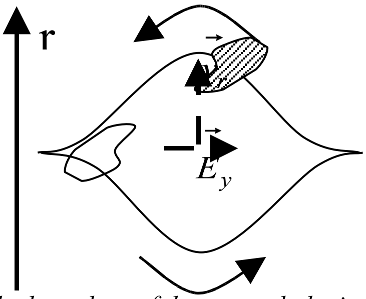

Fig. 6. The boundary of the vortex-hole, its radial shift, the electron trajectories and spatial distribution of the perturbations of the electron density in the field of the excited vortex-hole in the rest frame, moving with V_{ph}, in a plane approximation

Finiteness of time of the vortices symmetrization and also the reflection of resonant electrons from vortices - bunches result that the vortices are asymmetrical. Namely, on the inverse on θ parties of vortices the small bunches and cavities are derivated. It results in formation of polarization azimuth electrical fields E_θ, directed along $\mathbf{e}_\theta$. The formation of fields E_θ causes the radial drift and spatial separation of vortices (see Fig. 5, 6). In other words, the property of preference of motion of the vortex - cavity on peripherals of the plasma column and of the vortex - bunch to its axis shows. As the electron rotations in the vortex - cavity and vortex - bunch are directed to the opposite directions, the polarization electrical fields in the vortex - cavity and vortex - bunch have the inverse sign. Then the velocities of radial drift of the vortex - cavity and vortex - bunch have inverse signs. Namely, the vortex - cavity moves to the region of smaller electron density (see Fig. 6), and the vortex - bunch moves to the region of greater electron density (see Fig. 5).

The resonant electrons, i.e. electrons arranged inside a separatrix of the vortex - bunch of electrons, generated in the plasma lens, are reflected from it. Thus the distribution of the electron density is formed to be asymmetrical on azimuth at radially nonuniform distribution of the electron density in the plasma lens. It results in radial motion of the vortex - bunch of electrons and to simultaneous formation of spiral distribution of the electron density, shown in Fig. 7.

The width of the spiral equals to radial width of the vortex in the case of its large radial velocity. In the case of small radial velocity of the vortex the width of the spiral is less, than radial width of the vortex.

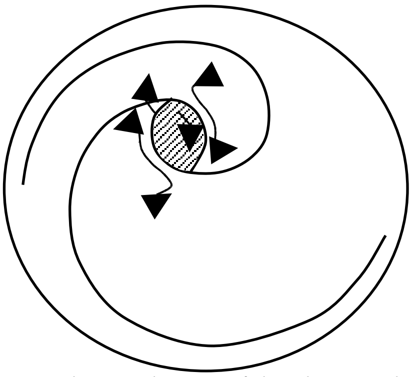

Fig. 7. The spiral perturbation of the electron density in the field of the vortex-bunch in the electric cylindrical plasma lens, for focusing of the high-current ion beams, with electron density, decreasing on radius

When two vortices - bunches of electrons start to concern each other, the electrons of each vortex located near to its boundary, are reflected from an adjacent vortex. Thus the distribution of electron density in a neighborhood of each vortex is derivated asymmetrical on azimuth. It results in occurrence of relative speed of vortices.

The similar behavior of electrons was observed in experiments in purely electron plasma, in charged plasma of the lens [1, 3] and in the plasma, placed in crossed radial electrical and longitudinal magnetic fields.

This work was supported by STCU under grant 1596.

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ДИНАМІКА В ПРОСТОРІ І ЧАСІ НЕЛІНІЙНИХ ВИХОРІВ У СИЛЬНОСТРУМОВІЙ ПЛАЗМОВІЙ ЛІНЗІ

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Досліджені властивості і збудження вихорів в електростатичній плазмовій лінзі, що призначена для фокусування сильнострумових іонних пучків.

ПРОСТРАНСТВЕННАЯ И ВРЕМЕННАЯ ДИНАМИКА НЕЛИНЕЙНЫХ ВИХРЕЙ В СИЛЬНОТОЧНОЙ ПЛАЗМЕННОЙ ЛИНЗЕ

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