ABERRATIONS IN ELECTROMAGNETIC PLASMA LENSES PROPOSED FOR INTENSE LARGE-APERTURE ION BEAM FOCUSING

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It is shown, that in electromagnetic plasma lenses moment aberrations obey the conservation law of the moment of the ion generalized momentum; they disappear when an ion source is placed in a zero magnetic field. Geometrical aberrations can be removed by choice of the optimal electric field intensity distribution along the radius that have been calculated. As a result, the ion current density and beam compression can achieve in the focus a value up to 10^3 A/cm^2, and 10^5, accordingly. Then influence of discrete distribution of the external (boundary) electrical potential upon ion focusing was studied, and an example of two-lens achromatic system have been investigated as well.

PACS: 52.40.Mj

1. INTRODUCTION

The problems of intense ion beam focusing are important for the controlled fusion, nuclear physics, beam technologies, etc. [,]. The essential feature of intense ion beams is that they should be charge compensated during the focusing to prevent their destruction. In this case, the application of plasmaoptic focusing systems which development is initiated by A.I. Morozov and co-workers [-, 8] and recently mainly developed by A.A. Goncharov group [–] is expedient.

In the plasma lens of Morozov type the magnetic surfaces are the equipotentials of the electrical field. In practice the electrical potentials are entered in plasma by a discrete manner, using "basic" ring electrodes. The experimental researches [–7] confirm the theoretical model [-, 8] but some problems remain, in particular, aberrations and methods of their elimination. For this aim, computer modeling of plasma-optic focusing devices is considered in the proposed paper.

2. THE PROBLEM DEFINITION, MAIN EQUATIONS

Following [], we enter function of a magnetic flow:

$$\Psi(r,z) = rA_\varphi(r,z)\,, \tag{1}$$

where A_φ is the azimuth component of vector potential.

The equation of magnetic surfaces have the form:

$$rA_\varphi = const\,. \tag{2}$$

The connection between Ψ(r, z) and magnetic flow N passing through cross-section (r,z) is (see []):

$$N = \int_S H\cdot\vec{n}dS = 2\pi rA_\varphi(r,z) = 2\pi\Psi \tag{3}$$

In a Morozov lens equipotentiality of magnetic surfaces is determined by a relation [-]

$$\Phi(r,z) = F(\Psi) \tag{4}$$

where Φ is a potential of an electric field.

Let's express components of an electric and magnetic fields through Ψ:

$$B_r = -\frac{1}{r}\frac{\partial\Psi}{\partial z} \qquad B_z = \frac{1}{r}\frac{\partial\Psi}{\partial r} \tag{5}$$

$$E_r = -\frac{\partial\Phi}{\partial r} = -\frac{dF}{d\Psi}\frac{\partial\Psi}{\partial r} = -r\frac{dF}{d\Psi}B_z \tag{6}$$

$$E_z = -\frac{\partial\Phi}{\partial z} = -\frac{dF}{d\Psi}\frac{\partial\Psi}{\partial z} = r\frac{dF}{d\Psi}B_r\,. \tag{7}$$

Substituting these expressions in the equations of particle motion in the form of Newton, with account of Lorentz force, we have:

$$\frac{dV_r}{dt} = \frac{q}{mr}\frac{\partial\Psi}{\partial r}\left(\frac{1}{c}V_\varphi - r\frac{dF}{d\Psi}\right) + \frac{V_\varphi^2}{r} \tag{8}$$

$$\frac{dV_\varphi}{dt} = -\frac{q}{mcr}\left(V_z\frac{\partial\Psi}{\partial z} + V_r\frac{\partial\Psi}{\partial r}\right) - \frac{V_rV_\varphi}{r} \tag{9}$$

$$\frac{dV_z}{dt} = -\frac{e}{mr}\frac{\partial\Psi}{\partial z}\left(r\frac{dF}{d\Psi} - \frac{1}{c}V_\varphi\right) \tag{10}$$

In experimental works [–] the configuration of a magnetic field with counter connection of solenoids is proposed, that allows to locate basic electrodes near to the central plane of a lens. In the given work the lens is simulated by three coils with opposite currents. The central coil is located at $z = 0$, lateral ones at $z=\pm5.0$ cm. In the central part of the lens (-2.8 cm $< z <$ 2.8 cm) the potentials of basic ring electrodes are applied to magnetic surfaces, and the magnetic surfaces at the left and right of the central area are considered as grounded.

The magnetic field of a ring current J_n (at radius of a coil a_c and coordinate l on an axis z) is described by azimuth component of vector potential:

$$A_{\varphi,n} = \frac{4J_n}{ck_n}\sqrt{\frac{a_c}{r}}\left[(1-\frac{k_n^2}{2})K(k_n) - E(k_n)\right] \tag{11}$$

$$k_n^2 = \frac{4a_cr}{(a_c+r)^2+(z-l_n)^2}$$

where c is the light velocity, **K** and **E** are the complete elliptic integrals of 1-st and 2 kind, n is a ring number. The sum field is $A_\varphi = \sum\limits_n A_{\varphi,n}$

Further we used the case of J_{center}=-1.5J_{side}. The equipotential surfaces topography is presented in Fig. 1.

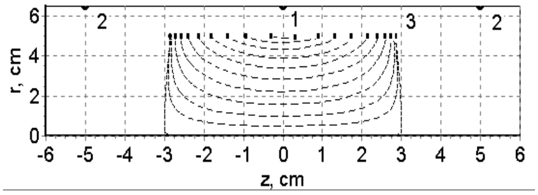

Fig. 1. (1-central coil, 2-side coils, 3-electrodes)

3. MOMENT ABERRATIONS

Computer modeling and analysis of ion trajectories, carried out in [], show that along the calculated ion trajectory, the conservation law for the ion moment of the generalized momentum, and the conservation law for the total ion energy are both satisfied to within five significant decimal digits.

If we are interested not by trajectories but only conditions at which moment aberrations are negligible, they can be obtained directly from the initial equations. To this effect, the equation of azimuth motion (9) after some transformations can present as:

$$m\frac{d(rV_\varphi)}{dt} = -\frac{q}{c}\frac{d\Psi}{dt}, \tag{12}$$

whence follows the conservation law for the moment M_φ of the generalized momentum of a particle P_φ:

$$\mathrm{M}_\varphi = rP_\varphi = r\left(mV_\varphi + qA_\varphi / c\right) = const\,. \tag{13}$$

With account of (3), it is follows from (12), that for Morozov axysimmetric static lenses the Busch theorem is valid as well as for vacuum magnetic ones:

$$rV_\varphi + \frac{q}{2\pi mc}\mathrm{N} = r_0V_{\varphi 0} + \frac{q}{2\pi mc}\mathrm{N}_0, \tag{14}$$

where N corresponds to a magnetic flow which is passing through a circle of radius r, on which at the given moment of time there is a particle. Functions N(r,z) and Ψ(r, z) can be measured with help of the electromagnetic induction law and suitable diagnostics.

As follows from (13), (14), in order for a parallel particle beam to be focused into the focal point of a lens, it suffices that the initial azimuth velocities of the particles be zero and that the magnetic field vanish in the injection region and in the focal plane. The lenses used in of [5–7] satisfy these conditions. Further we can consider minimization of geometrical aberrations.

4. GEOMETRICAL ABERRATIONS AT CONTINUOUS DISTRIBUTION OF POTENTIAL

4.1. PARALLEL BEAM FOCUSING

Initial conditions of particle injection in a lens are:

at $t = 0$ - $V_z = V_0$, $V_r = V_\varphi = 0$, $z = z_i$, $r = r_i$, (15)

where z_i is the injector coordinate, r_i is the radius at which an ion is injected; r_i is varied from zero to a value somewhat smaller than the radius R of the basic electrodes, which, in turn, is smaller than the radius a_c of the current-carrying coils.

The calculations of ions trajectory were made at parameters comparable to the Kiev lens [–]: energy of protons W=20 keV, initial beam radius r_0=3.5 cm, beam is parallel, radii of current rings r_a=6.5 cm, coordinate of protons injector z_i=-30 cm, current of protons 1 A.

The distribution of potential $\Phi\,(r,0) \propto r^2$ is applied to minimization geometrical aberrations in [5]. But, as it is shown in [10], at focusing large-aperture beams it does not give satisfactory results. In works [-] the distribution $\Phi \propto \Psi$ is considered also. In Fig. 2 at distribution $\Phi = KrA_\varphi$, where $K = 4.8 \cdot 10^{-4}$ V/Gs cm^2 the trajectories of protons are submitted. Thus maximal density of a current of protons reaches only 5 A/cm^2 at z_f =17.5 cm.

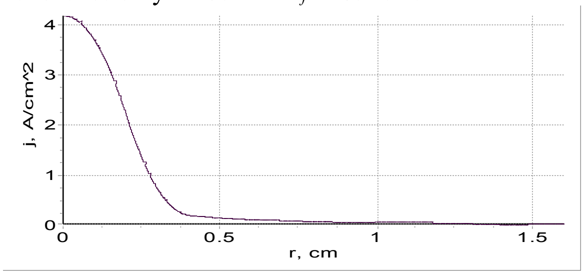

Fig. 2.

For reduction geometrical aberrations we will set the optimized distribution of potential on radius (in GS) as polynomial in which denominate factors are picked up such that the focusing will be best:

$$\Phi(\mathrm{r},0) = 0.75\, r^2 - 0.0143\, r^4 + 0.00014\, r^6 \tag{16}$$

For this case the trajectories of protons and distribution of current density of protons on radius in focal plane are calculated. As a result of optimization the current density has increased up to 9 kA/cm^2 (see Fig. 3), and compression factor of a beam (i.e., ratio between initial current density to final) - up to $3.6 \cdot 10^5$. In work [] best focusing (with compression factor about 30 at a total ion current 0.24 A) was experimentally obtained at distribution of potential proportional to intensity of a longitudinal magnetic field on the lens axis.

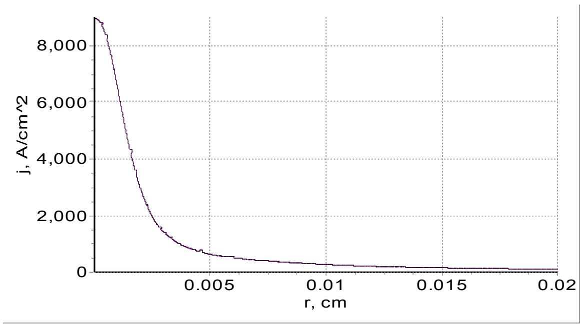

Fig. 3.

For such distribution the trajectories of protons and current density distribution on radius in focal plane have been modeling by us. In this case the current density in focus $J_{\max}$=4.2 A/cm^2 is obtained at average beam radius 0.5 cm and compression factor about 50. These values considerably concede the calculated results, see above.

4.2. FOCUSING OF IONS EMERGING FROM POINT SOURCE

As an example, it have been simulated the focusing of protons emerging from a point source at z_i = -30 cm. Beam divergence is 0.1 rad, so at the lens center the beam radius not exceeds 3.0 cm. Optimum distribution of potential is $\Phi\,(r,\,0) = 1.20r^2 - 0.0180r^4 + 0.000228r^6$. The trajectories of protons and current density in focal plane z_f = 27.18 cm are designed up to 75 кA/cm^2 at the spot average radius 0.002 cm (see Fig.4). So, in this case spherical aberrations are negligible (see also [2]).

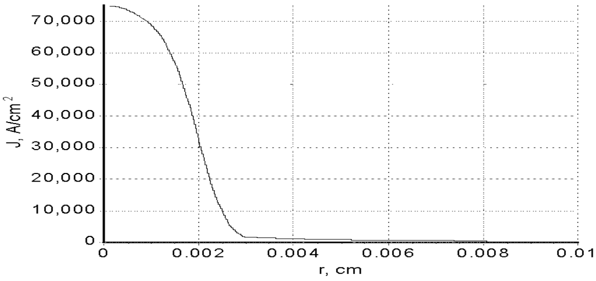

Fig. 4.

5. GEOMETRICAL ABERRATIONS AT DISCRETE DISTRIBUTION OF FOCUSING POTENTIALS

Till now in calculations we accepted continuous distribution of potential on coordinates. As against it, in experiments [–] potentials in plasma are entered with the help of finite number (5 or 9) cylindrical electrodes.

Let's consider a case of 9-electrode lens, that corresponds to the preset of 6 discrete values of potential on half of lens (6-th potential corresponds to zero potential on an axis). If these 6 values to set on the optimized curve corresponded to Fig. 3, and then to smooth by the splines, the satisfactory concurrence of these curves will turn out. However in experiments the electrodes of finite length specifying step distribution of potential were applied which in plasma smoothed out. Characteristics of this smoothing are not yet investigated experimentally. In calculations this smoothing was simulated by **B**-splines of 3-rd order, and the degree of smoothing was defined by a ratio of an effective length of electrodes and an effective gap between them. At the effective gap between electrodes of 2 mm (that is close to the real experimental value) the distribution with smoothed stairs is obtained (Fig. 5). The trajectories of protons which correspond to this case are submitted on Fig. 6 and current density in the focus region on Fig. 7. Current density (about 0.1 A/cm^2) and half width of a focal spot (about 1 cm) under the order coincide with the experimental results [–]. In this case because of step distribution of potential in plasma bad focusing of a beam takes place. The current density (~ 0.3 A/cm^2) and the focal spot half length (~1 cm, see Fig. 7) are correspond to the experimental results by the order of the values. More details for this problem see in [11].

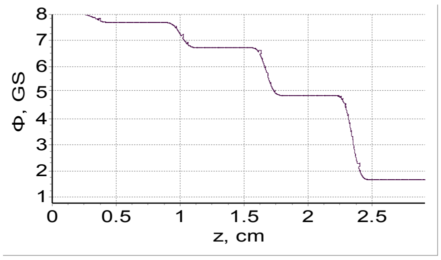

Fig. 5.

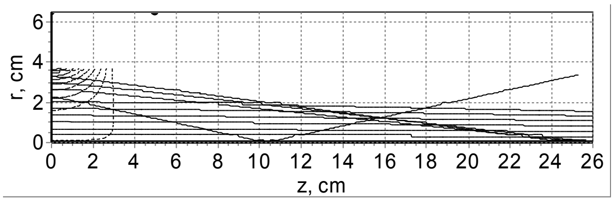

Fig. 6.

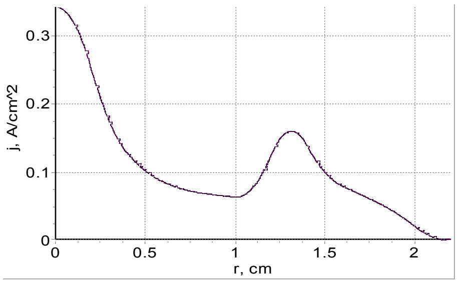

Fig. 7.

6. CHROMATIC ABERRATIONS

Accordingly to [4], in Fig. 8 it is presented the system of two Morozov lenses for study and removing chromatic aberrations at a tube ion beam focusing. (It is used the rule: before joining, it's necessary to separate).

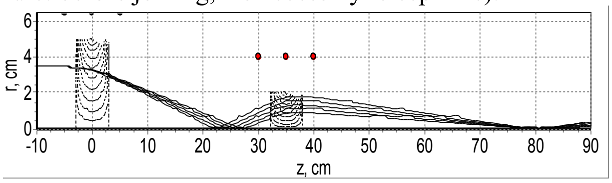

Fig. 8.

The first lens parameters: current circle radii 6.5 cm, center circle current 30 kA, electrodes radii 5 cm, maximal potential 3 kV, central circle coordinate z = 0, half length of the lens is 5 cm. The second lens parameters: current circle radii 4 cm, center circle current 30 kA, electrodes radii 2 cm, maximal potential 1.5 kV, central circle coordinate z = 3.5 cm, half length of the lens is 5 cm. Injector coordinate z = -70 cm, initial radius of the tube proton beam 3.5 cm. In Fig. 9 it is presented the potential distribution in 2-nd lens that was selected from the condition of minimum dependence of focal distance on proton energy in the range 16–21 keV. In this case it was used the approximation of potential distribution by B-splanes of third order in the left half length of the electrodes system with control points amount n = 11. Trajectories of protons at focus region are shown on Fig. 10.

The authors would like to thank A.A. Goncharov and I.M. Protsenko for consultations on their lenses.

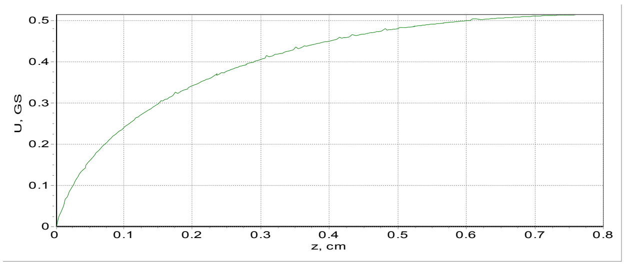

Fig. 9.

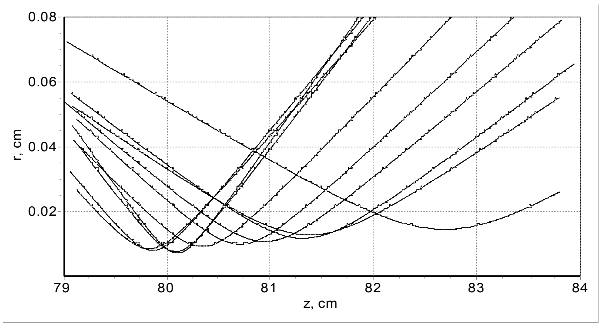

Fig. 10.

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АБЕРАЦІЇ У ЕЛЕКТРОМАГНІТНІЙ ПЛАЗМОВІЙ ЛІНЗІ, ЗАПРОПОНОВАНОЇ ДЛЯ ФОКУСУВАННЯ ІНТЕНСИВНИХ ШИРОКОАПЕРТУРНИХ ІОННИХ ПУЧКІВ

В.І. Бутенко, Б.І. Іванов

Досліджується плазмова лінза Морозова, у котрої магнітні поверхні є еквіпотенціалями електричного поля. Магнітне поле створюється центральним струмовим витком і двома бічними витками, включеними назустріч центральному. У роботі приведені результати комп'ютерного моделювання фокусування іонів з урахуванням їх поздовжнього, радіального й азимутального руху. Розглянуто засоби усунення моментних, геометричних і хроматичних аберацій. Проводиться оптимізація магнітного й електричного полів по величині і розподілу в просторі. Промодельоване вплив дискретного розподілу потенціалу на фокусування іонів і розглянуті пов'язані з цим аберації. Розглянуто комп'ютерну модель двухлінзової ахроматичної системи.

АБЕРРАЦИИ В ЭЛЕКТРОМАГНИТНОЙ ПЛАЗМЕННОЙ ЛИНЗЕ, ПРЕДЛОЖЕННОЙ ДЛЯ ФОКУСИРОВКИ ИНТЕНСИВНЫХ ШИРОКОАПЕРТУРНЫХ ИОННЫХ ПУЧКОВ

В.И. Бутенко, Б.И. Иванов

Исследуется плазменная линза Морозова, в которой магнитные поверхности являются эквипотенциалями электрического поля. Магнитное поле создается центральным токовым витком и двумя боковыми витками, включенными навстречу центральному. В работе приведены результаты компьютерного моделирования фокусировки ионов с учетом их продольного, радиального и азимутального движения. Рассмотрены способы устранения моментных, геометрических и хроматических аберраций. Проводится оптимизация магнитного и электрического полей по величине и распределению в пространстве. Промоделировано влияние дискретного распределения потенциала на фокусировку ионов и рассмотрены связанные с этим аберрации. Рассмотрена компьютерная модель двухлинзовой ахроматической системы.