EVOLUTION OF OSCILLATOR SPECTRUM IN PERIODIC POTENTIAL

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Spectral power of the oscillator radiation, which moves in periodically inhomogeneous potential, is investigated analytically and numerically. Spectrum of the nonrelativistic oscillator can have a maximum at high numbers of harmonics of basic frequency. Amplitudes of potential at which the motion of oscillator become irregular are found. In relativistic case a small value of potential practically not influence on character of spectrum. The dependence of high-frequency range of spectrum from value of potential inhomogeneity period is investigated. With decreasing of inhomogeneity period the spectrum maximum is shifting into short-wave range. In linear approximation the dispersion equation for oscillation excitation by ensemble of oscillators at frequency, which corresponds to the maximum of radiation spectrum of single oscillator is found.

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1. INTRODUCTION

In the most of nonlinear Hamilton systems, which describe the dynamics of particles, it is possible to sort regions of phase space where trajectories have regular character and where they are stochastic. If we throw away stochastic trajectories (i.e. trajectories which lies near separatrix) then all the rest of trajectories will be periodic. In our case regular trajectories will be periodic. Particles, which will have a chaotic dynamics, will radiate random fields.

We restrict ourselves with particles, which have a regular dynamics because only those ones can radiate intensive coherent radiation.

We'll specify and realize such conditions, fulfillment of which leads to minimum of particles with stochastic dynamics. Therefore in further we'll orient, first of all, by particles with regular dynamics. The influences of stochastic particles at this stage of analysis we'll be neglect, although we will make estimate their number and minimize it.

2. RADIATION OF PARTICLE WHICH MOVES IN PERIODIC POTENTIAL

We'll describe the mechanism of high numbers harmonics generation with nonrelativistic oscillators. Let a charged particle moves in time-periodic electric field $E(t)=E_{ext}\cdot\sin\left(\omega_{ext}\cdot t\right)$ and in field of periodic potential $U(z)=U_0+g\cdot\cos\left(\kappa\cdot z\right)$. For simplicity we'll consider that motion occur only along z-axis. In general the equation of electron motion will be write:

$$\frac{d\overset{\circ}{P}}{dt}=e\overset{\circ}{E}-e\overset{\circ}{E}_U,\ \frac{d\overset{\circ}{r}}{dt}=\overset{\circ}{V},\quad \overset{\circ}{V}=\overset{\circ}{P}/\sqrt{1+P^2}, \qquad (1)$$

where $E_U=-\nabla U$, e - electron charge.

As far as we, first of all, are interested in particle motion along z axis, so we'll get dimensionless form of equation (1) for given component

$$\begin{cases}\dfrac{d}{d\tau}p+\Omega_0^2\cdot\sin\left(\zeta\right)=\varepsilon\cdot\sin\left(\Omega_\kappa^{-1}\cdot\tau\right),\\ \dfrac{d}{d\tau}\zeta=\dfrac{p}{\sqrt{1+p^2}}=\beta,\end{cases} \qquad (2)$$

where $p=p_z/mc$, $\zeta=\kappa z$, $\tau=\kappa c\cdot t$, $\Omega_0^2=eg/mc^2$, $\varepsilon=eE_{ext}d/2\pi mc^2$, $\beta=V_z/c$, $\Omega_\kappa=\lambda_{ext}/d$, $\kappa=2\pi/d$.

When intensities of the fields are small enough it is possible to consider nonrelativistic motion of particles:

$$\ddot{\zeta}+\Omega_0^2\cdot\sin\left(\zeta\right)=\varepsilon\cdot\sin\left(\Omega_\kappa^{-1}\cdot\tau\right). \qquad (3)$$

Moreover, let us consider case $E>>\kappa g$. While passing into moving coordinate frame $\zeta=\xi-\varepsilon\cdot\Omega_\kappa^2\cdot\sin\left(\Omega_\kappa^{-1}\cdot\tau\right)$ equation (3) takes form

$$\ddot{\xi}=-\Omega_0^2\cdot\sum_{n=-\infty}^{\infty}J_n\left(\mu\right)\cdot\sin\left(\xi-\left(n\cdot\Omega_\kappa^{-1}\right)\cdot\tau\right), \qquad (4)$$

where $J_n\left(\mu\right)$ - Bessel function, $\mu=\varepsilon\cdot\Omega_\kappa^2$.

Equation (4) describe changing of "particle" phase ξ, at which many of waves acts on. Amplitudes of those waves $\Omega_0^2\cdot J_n\left(\mu\right)$ are increasing with growing of harmonic's number and in region $n\sim\mu$ have a local maximum. Amplitudes of harmonics with number $n>\mu$ are exponentially decrease [1].

$J_n\left(\mu\right)\text{W}\left(2/n\right)^{1/3}Ai\left(z\right)\text{W}\left(2/n\right)^{1/3}(1/2\sqrt{\pi}\sqrt[4]{z})e^{-\varsigma}$.

Thus, it is possible to expect, that radiation field will contain harmonics with frequencies up to $n\omega_{ext}$.

Radiation intensity into space angle unit do with frequency $\omega=n\omega_{ext}$ is equal to [2]:

$$dI_n=\frac{c}{2\pi}\left|\overset{\circ}{H}_n\right|^2R_0^2do, \qquad (5)$$

where $\bar{H}_\omega=i\left[\bar{k}\ \bar{A}_\omega\right]$, and Fourier component of vector potential defined by

$$\overset{\circ}{A}_n=e\frac{\exp(ikR_0)}{cR_0}\frac{1}{T}\int_0^T\overset{\circ}{v}(t)\exp\left[in\left(\omega_{ext}t-\overset{\circ\circ}{kr}(t)\right)\right]dt, \qquad (6)$$

where $T=2\pi/\omega_{ext}$, $r(t)$, $v(t)$ - particle's radius-vector and velocity, k - wave vector, R_0 - distance to point of observation. In the common case, it isn't seemed possible to get the analytical dependencies of spectral density from parameters of external fields.

3. RADIATION SPECTRUM OF PARTICLE MOVING IN PERIODIC POTENTIAL

The investigation of spectral characteristic of fields radiated by charged particle moving in external electrical field and in the field of potential, was carried out by the numerical solution of equations (3) and substitution of its solutions into (5) and (6).

For a case of nonrelativistic motion the amplitude of an external electrical field is equal to $E_{ext}=10^4$ V/cm, for relativistic - $E_{ext}=10^6$ V/cm. Frequency of the external electromagnetic field was fixed.

Investigation was carried out for two value of potential period $d=0.0025\cdot\lambda_{ext}$ and $d=0.00125\cdot\lambda_{ext}$ with λ_{ext}=10cm. Value of potential amplitude varied within $g=(0-0.125)E_{ext}\kappa^{-1}$. Initial conditions for particles were equal to $\zeta(\tau=0)=\zeta_0$; $\dot{\zeta}(\tau=0)=0$. For that the right-hand of (3) had been presented as: $\varepsilon\cdot\cos(\Omega_\kappa^{-1}\cdot\tau)$.

Calculation accuracy was controlled with the help of motion integrals

$$I=\frac{\beta^2}{2}-\Omega_0^2\cdot(\cos(\zeta)-\cos(\zeta_0))+\varepsilon\cdot\int_0^\tau\beta(\tau)\cos(\Omega_k^{-1}\tau)d\tau ,$$

$$I_r=\frac{p^2}{2}+\Omega_0^2\int_0^\tau p(\tau)\sin(\zeta(\tau))d\tau+\varepsilon\int_0^\tau p(\tau)\cos(\Omega_k^{-1}\tau)d\tau ,$$

their absolute values was less than $|I|<10^{-12}$.

In absence of the periodic potential influence $\Omega_0^2=0.0$ the equation (3) has a simple analytical solution. Motion of the particle is periodic with frequency $\omega=\omega_{ext}$, and spectrum of its speed and spectrum of the radiation field are linear.

Presence of space-periodic potential with amplitude of $\Omega_0^2=0.025\varepsilon$ ($d=0.0025\cdot\lambda_{ext}$) (Fig.1) qualitatively changes the picture of charged particle motion and radiation. At phase plane the trajectory is not strictly periodic curve (Fig 1,a), because particle motion is determined by acting, both external electrical field, and periodic potential. In velocity spectrum (Fig. 1,b) the base frequency dominates. Components on its harmonics occur under acting of the periodic potential.

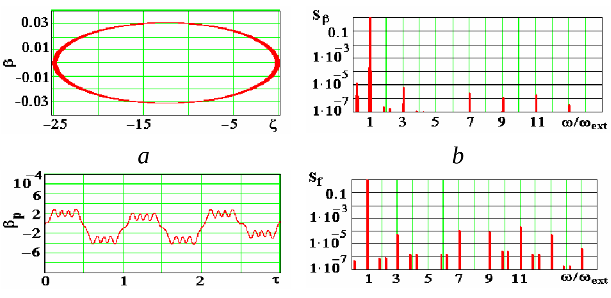

Fig.1. a) Phase space; b) Spectrum of velocity; c) Influence of potential on particle trajectory; d) Spectrum of radiated field

For the analysis of influence of the periodic potential on particle motion it is convenient to introduce variable $\beta_p(\tau)=\beta(\tau)-\varepsilon\,\Omega_\kappa\sin(\tau)$. It is visible, that under acting of the potential the particle performs high-frequency oscillations (Fig. 1,c), which results to appearance of harmonics in the spectrum of velocity. At that in power spectrum of radiated field (Fig.1,d) also appear the components on harmonics of external field with relative maximum on harmonic number n=11, that in order of magnitude is in very good accord with position of relative maximum $n\sim\mu=\varepsilon\cdot\Omega_k^{-2}\approx12$.

With growing of amplitude of periodic potential $\Omega_0^2=0.075\cdot\varepsilon$ ($d=0.0025\cdot\lambda_{ext}$) (Fig.2) occurs the growing of harmonics amplitudes in spectrum of velocity and its enriching at intermediate frequencies. Amplitudes of high-frequency oscillations grow up at the influence of periodic potential. Amplitudes of all harmonics in spectrum of radiated field are growing up too. Kind of the spectrum practically hasn't changes.

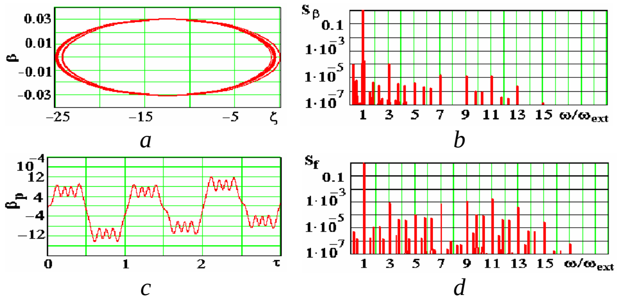

Fig.2. a) Phase space; b) Spectrum of velocity; c) Influence of potential on particle trajectory; d) Spectrum of radiated field

For potential amplitude $\Omega_0^2=0.125\cdot\varepsilon$ ($d=0.0025\cdot\lambda_{ext}$) (Fig.3) occur qualitative changes of phase plane – the particle motion isn't localized in limited region of space and represented series of oscillations near locally stabled state. Spectrum of particle velocity and, consequently, spectrum of radiated field lot enriching at all of intermediate frequencies.

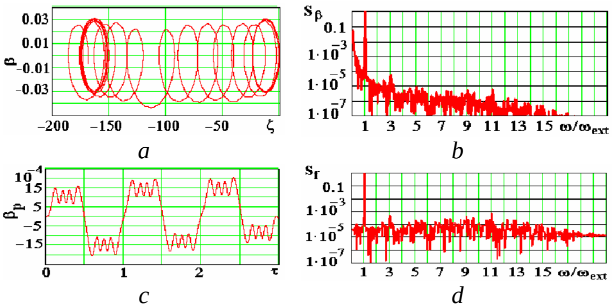

Fig.3. a) Phase space; b) Spectrum of velocity; c) Influence of potential on particle trajectory; d) Spectrum of radiated field

Further growing of potential amplitude leads to appearance of non-regular motion of particle hence the spectrum of radiated field also becomes non-regular.

For a potential with period $d=1.25\cdot10^{-3}\cdot\lambda_{ext}$ the number of harmonics, both in the spectrum of velocity and radiated field, are proportional to parameter μ .

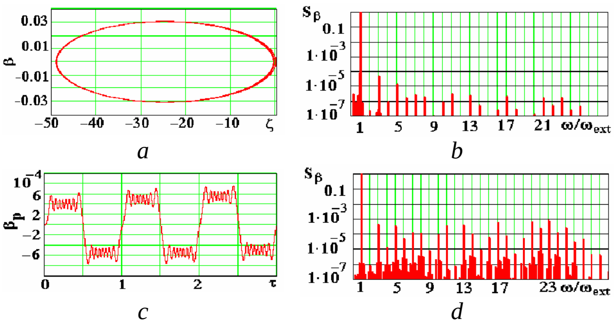

Fig.4. a) Phase space; b) Spectrum of velocity; c) Influence of potential on particle trajectory; d) Spectrum of radiated field

At amplitudes of potential $\Omega_0^2 = 0.07\cdot\varepsilon$ (Fig.4) the motion of particle is quasiregular. Spectrum of velocity and spectrum of radiated field have line structure. The local maximum of spectrum fall on harmonics with number $n_{\text{maч}}$=23, that is in very good accord with value $\mu = \varepsilon\cdot\Omega_k^{-2} \approx 25$.

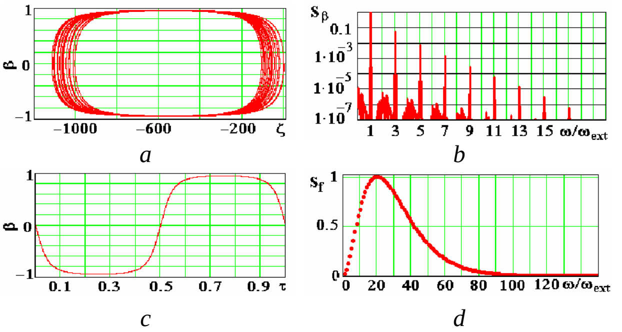

Fig.5. a) Phase space; b) Spectrum of velocity; c) Particle velocity as time function; d) Spectrum of radiated field

For the case of relativistic motion the amplitude $E_{ext}=10^6$ V/cm ($\Omega_0^2 = 0.15\varepsilon$, $d=0.0025\cdot\lambda_{ext}$) typical is motion of particle with almost constant velocity, close to velocity of light (Fig. 5), practically during all half period of the external electrical field. Spectrum of velocity has line nature. Spectrum of radiation has a maximum on the harmonics with frequency $\omega_{max}=\gamma^3/2$ that completely corresponds to analytical result [3].

4. DISPERSION EQUATION

For obtaining of dispersion equation it is necessary to solve self-consistent systems, which implies of the Maxwell equations for field and equations of charged particles' motion in exited fields. In linear approximation we choose a field of such type:

$$E = \mathrm{Re}\,\breve{\varepsilon}_0 \exp(i(kx+\kappa z)\exp(-i\omega t)\ .$$

Executing necessary transformations, we'll obtain a set of linear algebraic equations. The equality to zero of a determinant of this algebraic system represents a dispersion equation. We shall keep only resonant members $\omega \approx n\omega_{ext}$. In these conditions dispersion equation takes on enough simple form:

$$\left(1-\frac{\omega_b^2}{\omega^2}\right)\left(1-\frac{k^2\cdot c^2}{\omega^2}-\frac{\omega_b^2}{\omega^2}-\frac{1}{4}\frac{\omega_b^2}{(\omega-n\omega_{ext})^2}\left(\frac{\omega_0 J_n(\mu)}{n\,\omega\omega_{ext}}\right)^2\right)=0, \quad (9)$$

with increment $\mathrm{Im}\,\delta = \frac{\sqrt{3}}{4}\frac{\omega_0}{\omega}\left(J_n^2(\mu)\omega_b^2\omega_0\right)^{1/3}$.

CONCLUSIONS

The presence of space-periodic potential even small amplitude leads to generation of high-number harmonics of radiated field. At that, local maximum in spectrum of radiation lays at high-number harmonics of external electric field. Number $n_{\max}$ of harmonic, at which local maximum of radiation lays, is in accord with analytical results very well. Radiation frequency in its maximum $\omega_{\max} = n_{\max}\omega_{ext}$ is sufficiently higher than external field frequency ($n_{\max} >> 1$). Intensity of harmonic radiation at local maximum is high enough.

While, that for the case of nonrelativistic oscillator discussed in [3] the intensity of n-th harmonic radiation is proportional β^{2n}, so intensity of harmonic radiation with numbers equal to appropriate numbers of harmonics at local maximum (in our case $n \sim n_{\max} \approx 11$) will be negligible small.

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ЭВОЛЮЦИЯ СПЕКТРА ОСЦИЛЛЯТОРА В ПЕРИОДИЧЕСКОМ ПОТЕНЦИАЛЕ

В.А. Буц, А.М. Егоров, В.И. Мареха, А.П. Толстолужский

Аналитически и численно исследована спектральная мощность излучения осциллятора, который движется в периодически-неоднородном потенциале. Показано, что спектр нерелятивистского осциллятора может иметь максимум на высоких номерах гармоник основной частоты. Найдены амплитуды потенциала, при которых движение осциллятора становится нерегулярным. Показано, что в релятивистском случае малая величина потенциала практически не влияет на характер спектра. Показано, что с уменьшением периода неоднородности максимум спектра передвигается в коротковолновую область. В линейном приближении получено дисперсионное уравнение возбуждения колебаний ансамблем осцилляторов на частоте, соответствующей максимуму спектра излучения индивидуального осциллятора.

ЕВОЛЮЦІЯ СПЕКТРА ОСЦИЛЯТОРА У ПЕРІОДИЧНОМУ ПОТЕНЦІАЛІ

В.О. Буц, О.М. Єгоров, В.І. Мареха, О.П. Толстолужський

Аналітично й чисельно досліджена спектральна потужність випромінювання осцилятора, що рухається в періодично-неоднорідному потенціалі. Показано, що спектр нерелятивістського осцилятора може мати максимум на високих номерах гармонік основної частоти. Знайдено амплітуди потенціалу, при яких рух осцилятора стає нерегулярним. В релятивістському випадку мала величина потенціалу практично не впливає на характер спектра. Досліджено залежність високочастотної області спектра від величини періоду неоднорідності потенціалу. Зі зменшенням періоду неоднорідності максимум спектра пересувається в короткохвильову область. У лінійному наближенні отримане дисперсійне рівняння збудження коливань ансамблем осциляторів на частоті, що відповідає максимуму спектра випромінювання індивідуального осцилятора.