HEATING OF CHARGED PARTICLES AT THOMSON SCATTERING OF A MONOCHROMATIC ELECTROMAGNETIC WAVE

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Plane monochromatic electromagnetic wave scattering by charged particles is investigated theoretically taking into account the influence of scattered incoherent electromagnetic radiation on particles motion. The spread in the particles-velocity due to interaction of particles with each other via the fields of spontaneous incoherent radiation is shown to appear in the process of scattering of incident wave.
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1. INTRODUCTION

Investigation of processes of electromagnetic waves interaction with charged particles is of considerable interest for many applications of plasma physics and charged particles beams, such as heating of plasma by the external fields [1], stability of plasma in the external electromagnetic fields [2], its diagnostics [3], accelerations of electrons in the fields of laser radiation [4], increase of frequency in plasma [5]. The scattering of external electromagnetic waves by the charged particles is also concerned with generation of narrow-band ultrashort wavelength radiation at motion of relativistic electron beams in the external periodic fields [6]. The peculiar significance for these applications has finding out effects of radiation reaction on particles motion.

In the given paper the results of theoretical research of the plane monochromatic electromagnetic wave Thomson scattering process by charged particles taking into account the influence of scattering radiation on particles motion are presented, the incoherent scattering of external wave by the charged particles being discussed.

The charged particles motion in the total electromagnetic field in the limit case of a small value of unit-less wave strength parameter is considered. The spread in particles momentum is shown to appears in the process of incident wave scattering. The dependences of a mean-square value of particles longitudinal velocity on time are found and investigated by analytically and numeral methods.

2. FORMULATION OF THE PROBLEM

Let us consider the ensemble of the identical charged particles with the charge of q, mass of m and homogeneous average density n_0, moving in a field of a plane monochromatic electromagnetic wave (EMW)

$$\mathbf{E}^{(ext)} = \mathbf{E}_0 \cos(\omega t - \mathbf{kr}), \quad (1)$$

where $\mathbf{E_0}$ is the amplitude of the wave electric field; ω, **k** are its frequency and wave vector.

Assume that the $\mathbf{E_0}$ and **k** vectors are directed along axes 0X and 0Z accordingly Cartesian coordinates.

Let us find the spread in the longitudinal velocity of particles, due to the interaction of particles via electromagnetic fields, produced by these particles at motion in the external field (1). Let us call the direction parallel to that of scattered wave propagation a longitudinal one (viz. along the OZ axis).

In order to find solution in explicit analytical form let us consider $a = eE_0/mc\omega$ parameter as a small one which characterises a relative size of transverse oscillatory velocity of particle in the external field.

Solution of a problem will be searched by the following method: let us consider the ensemble of charges as large number of the individual charged particles. Having found the field produced by the individual charged particle, moving in the external field, in the limit case of a small value of *a* parameter let us find the total longitudinal force, acting on the individual (test) particle, the mean-square longitudinal velocity being expressed via the ensemble average of the product of pair interaction forces of particles.

3. RESULTS AND DISCUSSION

In the external field the charged particles oscillate in the direction parallel to the $\mathbf{E_0}$ vector. The equations for coordinate and velocity of particles can be written as follows:

$$\mathbf{r}_i(t) = \mathbf{r}_{0i} - \mathbf{e}_x (ca/\omega)\cos(\omega t - kz) + \mathbf{e}_z \Delta_i(t),$$
$$\mathbf{v}_i(t) = \mathbf{e}_x ca \sin(\omega t - kz) + \mathbf{e}_z \, \mathrm{v}_z,$$

where $\mathbf{r}_{oi} = \{x_{0i}, y_{0i}, z_{0i}\}$ is the coordinate of i-th particle in the initial moment of time t_0 (let t_0=0), $z_i(t) = z_{0i} + \Delta_i(t)$ be the longitudinal displacement of particle trajectory relative to the initial position.

Let us express electric and magnetic fields produced by the individual particle via Lienard-Wiechert potentials [7]. In the nonrelativistically motion of particles and small *a* value the expression for the electric field, produced by the i-th particle in **r** coordinate in *t* time is:

$$\mathbf{E}_i(\mathbf{r},t) = q\frac{\mathbf{R}_i}{R_i^3} - \mathbf{e}_x aq\frac{k}{R_i}\left\{\left(1 - 3\frac{R_x^2}{R_i^2}\right)\frac{\sin\varphi}{kR_i} + \left[1 - \frac{1}{k^2R_i^2} - \left(1 - \frac{3}{k^2R_i^2}\right)\frac{R_{xi}^2}{R_i^2}\right]\cos\varphi\right\}, \quad (2)$$

where $\mathbf{R}_i = \mathbf{e}_x(x - x_{0i}) + \mathbf{e}_y(y - y_{0i}) + \mathbf{e}_z[z - z_i(t')]$,
$\varphi = \omega t - k[R_i - z_i(t)]$.

A total field in the observation point of **r** in the time *t* will be determined by charges initial coordinates of which satisfy condition:

$$ct > |\mathbf{r} - \mathbf{r}_{0s}| \quad (3)$$

Let us consider the motion of some individual (test) particle both in the field (1) and in the fields, produced by all other particles. Equations of longitudinal motion of such particle (let it be the i-th particle) in these fields can be written as follows:

$$\frac{d\,p_{zi}}{dt} = F_z\left[\mathbf{r}_i(t),t\right] = \sum_s F_z^{(s)}\left[\mathbf{r}_i(t),t;x_s\right], \qquad (4)$$

$$F_z^{(s)}(\mathbf{r},t;x_s) = e\left\{E_{zs}(\mathbf{r},t) + \frac{1}{c}\left[\mathbf{v}\,\mathbf{H}_s(\mathbf{r},t)\right]_z\right\}, \quad (5)$$

where $\mathbf{r}_i$, $\mathbf{p}_i$ are the position and momentum of the i-th particle at time *t*; $\mathbf{E}_s(\mathbf{r},t)$, $\mathbf{H}_s(\mathbf{r},t)$ are the strength of electric and magnetic fields, produced by the *s*-th particle at time *t* in $\mathbf{r}$ coordinate.

Let's the considered system consists of N charged particles the coordinate and momentum of which at the initial time are random values. We introduce the distribution function of system states $D_N(x_{01},\ldots, x_{0N}; t_0)$ at time t_0. This function is normalized as: $\int D_N(x_{01},\ldots, x_{0N}; t_0)dx_{01}\ldots dx_{0N}=1$, where $x=\{\mathbf{r}, \mathbf{p}\}$, $x_{0s}=x_s(0)$. By integrating of Eq. (4) we can obtain the deviation of longitudinal momentum from the mean value for test particle

$$\Delta p_{zi} = \int_0^t \delta F_z\left[\mathbf{r}_i(t'),t'\right]dt', \qquad (6)$$

and expression for rate of change of mean-square spread in the longitudinal momentum of particles

$$\frac{d}{dt}\left\langle(\Delta p_{zi})^2\right\rangle = 2\int_0^t \left\langle \delta F_z\left[\mathbf{r}_i(t'),t'\right]\delta F_z\left[\mathbf{r}_i(t),t\right]\right\rangle dt', \quad (7)$$

where $\Delta p_{zi}=p_{zi}-\langle p_{zi}\rangle$, $\delta F_z = F_z - \langle F_z\rangle$, angular brackets indicate average values,

$$\left\langle \mathbf{F}(\mathbf{r},t)\right\rangle = \int \mathbf{F}^{(1)}\left[\mathbf{r},t;x(t;x_0)\right]f_1(x_0)dx_0 ,$$

$$f_1(x_0) = N\int D_N(x_0,x_{02}\ldots,x_{0N};t_0)dx_{02}\ldots dx_{0N} ,$$

f_1 is the single-particle distribution function.

The right-hand side of Eq. (7) is founded in neglecting of influence of fields, produced by particles on its motion. Thus in the right-hand side of Eq. (7) $\mathbf{r}_i$ (t) can be replaced by unperturbed trajectory $\mathbf{r}_i^{(0)}(t) = \mathbf{r}_i(t;\Delta=0)$ of particle in the external field (1). Taking into account that particles are identical and neglecting initial correlation between them Eq. (7) can be expressed as

$$\left\langle \frac{d}{dt}(\Delta p_{zi})^2\right\rangle = 2\int_0^t dt' \iint_{V'} dx_{os}\, f_1(x_{os}) \times$$
$$\times F_z^{(s)}\left[\xi_i^{(0)}(t);x_s(t,x_{os})\right]F_z^{(s)}\left[\xi_i^{(0)}(t');x_s(t',x_{os})\right], \qquad (8)$$

where $\xi_i^{(0)}(t) = \{r_i^{(0)}(t),t\}$

Analytical expression for the force, acting on an individual particle in the electromagnetic field, produced by the other particle will be derived by substituting expression for field (2) in Eq. (5):

$$F_{zs} = q^2k^2G\,,\ G = \frac{\rho_z}{\rho_t^2}\left[\frac{1}{\rho_t} - a_0^2\left(\sin\psi + \frac{\cos\psi}{\rho_t}\right)\right], \quad (9)$$

where $\psi = \rho_t - \rho_z$, $\rho_t = \left(\rho_\perp^2 + \rho_z^2\right)^{1/2}$, $\rho_z = k(z_i - z_s)$,

$\rho_\perp = k\sqrt{(x_{0i}-x_{0s})^2 + (y_{0i}-y_{0s})^2}$, $a_0 = a/\sqrt{2}$.

Neglecting initial momentum spread the Eq. (8) for rate of change of root mean square longitudinal particles momentum is:

$$\frac{d}{dt}\left\langle(\Delta p_{zi})^2\right\rangle = 4\pi n_0(qk)^4\int_0^\tau dt' \int_{V_0(t')}\left[G(\rho_\perp,\rho_s)\right]^2 d\mathbf{r}_{0s}\ , \quad (10)$$

where V_0 is the integration region determined by condition (3).

Having integrated Eq. (10) taking into account the fact that particles are at some finite distance one from other, we find:

$$\left\langle(\Delta\beta_z)^2\right\rangle = \sigma_R^2 + \sigma_Q^2, \qquad (11)$$

where $\sigma_R = \frac{r_0}{3\lambda}a_0^2(N\omega t)^{1/2}\omega t$, $\sigma_Q = \sqrt{\frac{\pi}{3}}\frac{r_0}{3\lambda}N^{2/3}\omega t$,

$N = n_0\lambda^3$, $r_0 = q^2/mc^2$, $\beta_z = v_z/c$.

As a minimum distance between particles the value of $r_{\min} = n_0^{-1/3}$ was taken.

The first term in right hand side Eq. (11), that depends on parameter *a*, is the longitudinal particle velocity spread, due to interaction of particles via the electromagnetic waves. The second term in Eq. (11), that is not dependent on parameter *a*, describes the spread, due to the coulomb interaction of particles.

On the $\omega t >> N^{1/3}/\left(\pi^2K^4\right)$ times the particle velocity spread will be determined by interaction of particles with each other via electromagnetic waves. The expression for the momentum spread in this case may be presented as follows:

$$\left[\left\langle(\Delta\beta_z)^2\right\rangle\right]^{1/2} = \frac{\Delta\varepsilon_R^{(1)}(t)}{2\varepsilon_0}\sqrt{N_{eff}(t)}, \qquad (12)$$

where $N_{eff} = N\omega t/2\pi^2$, $\Delta\varepsilon_R^{(1)} = 2q^2a_0^2k\omega t/3$ is the energy losses of an individual charge on radiation, $\varepsilon_0 = mc^2$.

Now let us consider the EMW scattering on the clusters of the charged particles, having a form of a circular cylinder with the height of l_b and radius of r_b. Let the wave vector $\mathbf{k}$ be parallel to the axis of a cylinder (cluster). The spread in the longitudinal particle velocity will be described by Eq. (11) for particles present in the initial moment of time in the center of cluster on the cylinder axis and at the distance of $l_b/2$ from its base. Thus the first term in this equation, corresponding to the radiative interaction, is:

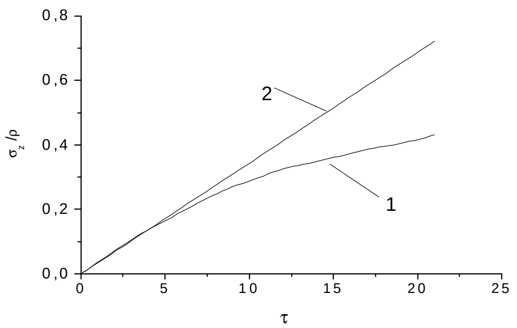
$$\sigma_R = \sigma_R^{(c)} = \frac{r_0}{\lambda}a_0^2\tau\left[\frac{\pi}{2}N\frac{l_b}{\lambda}\Phi\left(\frac{2r_b}{l_b}\right)\right]^{1/2}, \qquad (13)$$

where $\Phi(x)=x$ arcctg(x).

When Eq. (13) was derived a considered test particle was assumed to be located in the influence region of fields of all other particles of the cluster. Eq. (13) shows, that for the sufficiently wide charged particles cluster ($2r_b>>l_b$) longitudinal velocity spread due to the radiative interaction of particles will be proportional to the root square from the number of particles in the effective volume $V_{eff} = \pi\lambda^2 l_b$.

The dynamics of the change of particles longitudinal velocity spread was investigated by particle simulation of the EMW scattering process on the charged particles cluster. The cluster has the form of a cylinder, whose axis coincides with the direction of external wave propagation. The radius of a cluster is equal to 5λ, and its length equals 10λ. A cluster consists of 5000 particles, a_0=0.7, λ=10μm. The motion of particles was described by equations (4), (5), (9).

The dependence on the dimensionless time $\tau=\omega t\rho$ normalized value of the spread in the longitudinal velocity of N_1 particles, located in the center of a cluster at the initial time t=0, is shown in the figure, where $\rho = 2a_0(\pi r_0 l_b N/3)^{1/2} / \lambda$. Namely the initial coordinates of these particles (test particles) were in the region, limited by the surface of a cylinder with the radius of $r_1=0.15r_b$ and the height of $l_1=0.4l_b$. Axes of cylinders and planes of symmetry (a plane perpendicular to the axis of a cylinder and dividing its in two) coincide.



In this figure the curve 1 corresponds to the particle simulation, and curve 2 - the dependence of the longitudinal velocity spread, determined by Eq. (11), (13). Here $\sigma_z^2 = \overline{\beta_z^2} - (\overline{\beta_z})^2$, $\overline{A} = \frac{1}{N_1}\sum_{i=1}^{N_1} A_i$ - summation is carried on over all test particles, N_1 is their total number.

The figure shows that the spread in velocity is proportional to the *t* on the initial stage of scattering. The results of numeral simulation agree with the analytical estimations by Eqs. (11), (13). On the times τ>5 the rate of change in longitudinal velocity spread decreases.

Thus, the EMW scattering on the charged particles leads to the increase of the velocity spread of particles.

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НАГРЕВ ЗАРЯЖЕННЫХ ЧАСТИЦ ПРИ ТОМСОНОВСКОМ РАССЕЯНИИ МОНОХРОМАТИЧЕСКОЙ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ

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Теоретически исследовано рассеяние плоской монохроматической электромагнитной волны заряженными частицами с учетом влияния рассеянного некогерентного электромагнитного излучения на движение частиц. Показано, что в процессе рассеяния падающей волны появляется разброс по импульсам частиц, обусловленный их взаимодействием друг с другом через поля спонтанного некогерентного излучения.

НАГРІВАННЯ ЗАРЯДЖЕНИХ ЧАСТИНОК ПРИ ТОМСОНІВСЬКОМУ РОЗСІЯННІ МОНОХРОМАТИЧНОЇ ЕЛЕКТРОМАГНІТНОЇ ХВИЛІ

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Теоретично досліджено розсіяння плоскої монохроматичної електромагнітної хвилі зарядженими частинками з урахуванням впливу розсіяного некогерентного електромагнітного випромінювання на рух частинок. Показано, що в процесі розсіяння падаючої хвилі з'являється розкид по імпульсах частинок, обумовлений їх взаємодією одна з другою через поля спонтанного некогерентного випромінювання.