PARAMETRIC INSTABILITY OF SURFACE WAVES AT THE SECOND HARMONIC OF ION CYCLOTRON FREQUENCY

M.R. Belyaev, V.O. Girka, V.I. Lapshin

Kharkov National University, 61077 Kharkov, Ukraine

Parametric instability of surface waves at the second harmonic of ion cyclotron frequency (SWCF) in plasma filled dielectric planar waveguide is examined in a kinetic approximation. SWCF are extraordinary polarized modes propagating across the external steady magnetic field. Simple analytical expressions for the SWCF frequencies, their damping rates due to collisions between plasma particles and interactions between plasma particles and plasma interface, increments of the parametrical instabilities are found. Increments of the SWCF parametric instabilities are slightly dependent on the transverse dimensions of the plasma layer and they strongly decrease with the SWCF wavelength. The obtained results can be used in controlled fusion researches in order to avoid undesirable regime of the plasma periphery heating in fusion devices during.

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1. INTRODUCTION

Application of more and more powerful sources of electromagnetic energy for plasma heating in fusion devices makes investigations of edge plasma events as an important problem of controlled fusion. There are some experimental datum see e.g. [1-3] which identify just the surface type modes propagating at range of ion cyclotron resonance as the most possible cause of the edge plasma heating, impurity plasma generation and other undesirable processes in fusion devices. But till now edge plasma phenomena are not well understood. To make a definite contribution to creation of a theory of these phenomena is the goal of this work.

The slow extraordinary polarised electromagnetic surface waves on the harmonics of ion and electron cyclotron frequencies (SCXM) are eigenmodes [4] of a planar waveguide structure consisted of metal wall, dielectric sheath and plasma filling. These waves exist under the condition of weak plasma spatial dispersion. They are characterised by straight dispersion under the condition of inequality validity $\Omega_\alpha^2 >> \omega_\alpha^2$ (here Ω_α and ω_α are Langmuir and cyclotron frequencies, respectively, index α identifies the plasma particles kind: $\alpha = i$ for ions and $\alpha = e$ for electrons). Their skin depth into plasma is approximately equal to their wavelength. Parametric instability of the SCXM at the second harmonic of electron cyclotron frequency was considered in [5] and [6] for the cases of monochromatic and non-monochromatic uniform pumping electric field, respectively. Scenario of the parametric instability of the plasma affected by non-monochromatic alternating electric field is essentially distinguished from the case of monochromatic pumping field [7,8]. Therefore it is interesting to study the ion SCXM parametrical instability caused by a non-monochromatic electric field.

Theoretically the problem of the parametric excitation of the bulk waves at harmonics of ion cyclotron frequency has been investigated in [9] and parametric instability of plasma affected by two alternating electric fields at different frequencies related to range of ion cyclotron resonance was studied in [10]. There it was analytically shown that development of the parametric instability of bulk type modes is strongly depended on correlation between frequencies of the pumping fields. Change of this correlation leads, for example, to the modification of the bulk ion cyclotron kinetic instability growth rate and under the definite conditions even to suppression of this instability. Therefore investigation into the ion SCXM parametric instability is actual task. Obtained result will be useful for making recommendations to control the instability and to avoid the undesirable regime of the ion SCXM excitation in fusion devices. It will be interesting also for investigation into gas discharges that can be applied for modern plasma microtechnologies wherein application of just surface type waves allows to elaborate more effective plasma sources [11,12]. The study is carried out in the framework of kinetic description, because plasma perturbation on the harmonics of electron cyclotron frequency cannot be described by hydro-dynamical methods [13].

2. THE CASE OF MONOCHROMATIC PUMPING ELECTRIC FIELD

Let's consider the case of monochromatic pumping electric field using the model of plasma layer in the space region determined by inequality $0 < x < A_p$, which is restricted by dielectric medium with dielectric constant ε_d. An external magnetic field B_0 is oriented along axis z. On the plasma interface $x = A_p$ there is external alternating electric field in the following form: $E_{0x}\cos(\omega_0 t)$. Solving Maxwell equations by Fourier method, one can obtain set of equations for Fourier coefficients E_1, E_2, H_3 of the considered wave's fields. Unlike the case of electron SCXM in sums over numbers of cyclotron harmonics in electric conductivity tensor one can take into account only $s = 0, \pm 1$, and for ion addenda one can account mentioned above addenda and the resonant $s = 2$ addendum. Other addenda can be neglected because the case of wave propagation at the second ion cyclotron harmonic is under the consideration. Dependence of Fourier coefficients of electric current density $j_n^{(p)}$ on the coefficients of electric fields $E_n^{(p+l)}$ under the condition of weak plasma spatial dispersion ($k_2^2 \rho_i^2 << 1$) is as follows,

$$\begin{cases} j_1^{(p)} = \sigma_1(m,l,p)E_1^{(p+l)} + \sigma_2(m,l,p)E_2^{(p+l)}, \\ j_2^{(p)} = \sigma_1(m,l,p)E_2^{(p+l)} + \sigma_2(m,l,p)E_1^{(p+l)}, \end{cases} \quad (1)$$

where

$$\sigma_1 \approx \sum_{m,l=-\infty}^{+\infty} \frac{\Omega_\alpha^2 s^2 I_s(y_\alpha) J_m(g_1) J_{m-l}(g_1)}{4\pi i y_\alpha (s\omega_\alpha - \omega_{p+m})\exp(y_\alpha)},$$

$$\sigma_1 \approx \sum_{m,l=-\infty}^{+\infty} \frac{\Omega_\alpha^2 s[I_s'(y_\alpha) - I_s(y_\alpha)] J_m(g_1) J_{m-l}(g_1)}{4\pi i (s\omega_\alpha - \omega_{p+m})\exp(y_\alpha)},$$

I_s and I_s' are modified Bessel function and its derivative over the argument, $J_n(g)$ is Bessel function of the first type, $\omega_{p+m} = \omega + (p+m)\omega_0$, $\alpha_n = n\pi / A_p$, $y_\alpha = \alpha_n^2 \rho_\alpha^2 / 2 << 1$, ρ_α is Larmor radius, $g_1 = \alpha_n b_{E1} = \alpha_n e_i \omega_0 E_{0x} / (m_i \omega_0 |\omega_o^2 - \omega_i^2|)$.

Boundary conditions for ion SCXM fields on the interfaces separated the mediums are as follows: - impedance of the plasma is equal to impedance of dielectric on the interface $x = 0$; - on the interface $x = A_p$ there is continuity of the tangential electric field; - on the interface $x = A_p$ there is discontinuity of tangential component of magnetic field caused by nonlinear surface electric current flowing:

$$\left\{H_Z^{(n)}(A_p)\right\} = \sum_\alpha \sum_{m=-\infty}^{+\infty} \frac{i(iZ_{E1})^{|m|+|m-l|}}{ck_2 m!(m-l)!} \cdot \left[\frac{\Omega_\alpha^2 \omega_a E_y^{(n+l)}(A_p)}{\omega_{n+m}^2 - \omega_\alpha^2} - \frac{\Omega_i^2 E_y^{(n+l)}(A_p) k_2^2 \rho_i^2 / 4}{\omega + (n+m)\omega_0 - 2\omega_i}\right], \quad (2)$$

here $Z_{E1} = k_2 b_{E1}/2$, k_2 is the wave number. To obtain boundary condition (2) we have used assumption that the amplitudes of the pump fields are relatively small so that inequality $|Z_{E1}| << 1$ is taken place. Application of the indicated boundary conditions allows to obtain infinite set of equation for tangential component of ion SCXM:

$$D(p+l)E_y^{(p+l)}(A_p) + F(m,l)E_y^{(p+l)}(A_p) = 0, \quad (3)$$

where

$$D(l+p) = -\delta_{0,l} + \left[\sum_{j=0}^{1} \frac{[ik_2\varepsilon_{12}^l + i\alpha_j \varepsilon_{11}^l - |k_2|\delta_{0,l}\varepsilon_d]}{tg(\alpha_j a)(\partial\Delta/\partial\alpha_j)}\right]^2 - \left[\sum_{j=0}^{1} \frac{[ik_2\varepsilon_{12}^l + i\alpha_j \varepsilon_{11}^l + |k_2|\delta_{0,l}\varepsilon_d]}{[\sin(\alpha_j a)\partial\Delta/\partial\alpha_j]}\right]^2. \quad (4)$$

Relation $D(p) = 0$ is dispersion equation for the ion SCXM propagating in the considered waveguide structure. And the following expressions are valid for the considered case:

$$\varepsilon_{12}^l(\alpha_0) = i(\varepsilon_2^l - \varepsilon_1^l), \ \alpha_0 = i\frac{2\omega}{V_{Ti}}\sqrt{\frac{1 - 2\omega_i/\omega}{3}}, \quad (5)$$

$$F(m,l) = \sum_\alpha \sum_{\substack{m,l=-\infty \\ l \neq 0}}^{+\infty} \frac{k_2(iZ_{E1})^{|m|+|m-l|}}{2m!(m-l)!\omega_p} \cdot \quad (6)$$

$$\left[\frac{4\omega_\alpha \Omega_\alpha^2}{\omega_{p+m}^2 - \omega_\alpha^2} - \frac{\Omega_i^2 k_2^2 \rho_i^2}{\omega_{p+m} - 2\omega_i}\right] \cdot \left[\sum_{j=0}^{1} \frac{ctg(A_p \alpha_j)}{\partial\Delta/\partial\alpha_j} \cdot \sum_{j=0}^{1} \frac{ik_2\varepsilon_{12}^l - i\alpha_j\varepsilon_{11}^l - |k_2|\delta_{0,l}\varepsilon_d}{tg(\alpha_j A_p)\partial\Delta/\partial\alpha_j} - \sum_{j=0}^{1} \frac{(\partial\Delta/\partial\alpha_j)^{-1}}{\sin(\alpha_j A_p)} \sum_{j=0}^{1} \frac{ik_2\varepsilon_{12}^l - i\alpha_j\varepsilon_{11}^l + |k_2|\delta_{0,l}\varepsilon_d}{\sin(\alpha_j A_p)\partial\Delta/\partial\alpha_j}\right],$$

here ε_1 and ε_2 are components of the plasma dielectric permeability tensor in approach of a cold magneto-active plasma, see e.g. [13,14].

Let's consider resonance of the following type: $\omega = 2\omega_i + \Delta_{Ti} + n\omega_0 + \gamma$, here $\Delta_{Ti} \approx 0.52\omega_i k_2^2 \rho_i^2$, $|\gamma| << 2\omega_i$. Analytical solutions of eq.(3) can be obtained in the limiting cases of thin ($A_p k_2 < 1$) and thick ($A_p k_2 >> 1$) plasma layers. Taking into the account only the main $E_y^{(p)}$ harmonic of electric field and its neighboring satellites $E_y^{(p\pm 1)}$ one can reduce an infinite determinant constructed from coefficients at these harmonics to the third order determinant and solve it. Its solutions can be written in the following form:

$$\begin{gathered} \gamma \approx -\Delta_{Ti} + i0.86\Delta_{Ti} Z_{E1}\left(1 - 0.41k_2^2 A_p^2\right), \quad (A_p k_2 < 1) \\ \gamma \approx -\Delta_{Ti} + i0.81\Delta_{Ti} Z_{E1}, \quad (k_2 A_p >> 1) \end{gathered} \quad (7)$$

Analyzing expressions (7), one can make a conclusion that growth rates of the ion SCXM parametric instability $\mathrm{Im}(\gamma)$ are weakly depended on A_p and characterized by approximately cubic dependence on the wave number. To increase the $\mathrm{Im}(\gamma)$ one can strengthen the B_0, enlarge amplitude of the pumping electric field or decrease the plasma layer. Comparing the $\mathrm{Im}(\gamma)$ with their damping rates caused by collisions between plasma particles one can find threshold value b_{th} of the external alternating electric field.

$$b_{th} > 2.3\nu / (k_2^3 \rho_i^3 \omega_i). \quad (8)$$

This inequality can be easily realized in weakly collisional plasma where $\nu / \omega_i < k_2^3 \rho_i^3$.

3. THE CASE OF NON-MONOCHROMATIC PUMPING ELECTRIC FIELD

Let's consider the case when external alternating electric field has the following form:

$$E_0(t) = E_{01}\sin(\omega_{01}t) + E_{02}\sin(\omega_{02}t), \quad (9)$$

here E_{0i} are amplitudes of the pumping fields, ω_{0i} are their frequencies. Expressions for utilised components of plasma conductivity tensor are similar to the (1), but one can multiply them by factor: $J_{u_1-n_1}(g_2)J_{u_2-n_2}(g_2)$. It is connected with the presence of the second pumping field. Because of that there is some changing in the boundary conditions. The first and the second ones of them are the same as that are applied in the previous case. The third condition is only similar to the (2), but there is additional factor in the right hand side of it: $(iZ_{E2}/2)^{|m_2|+|m_2-l_2|}$.

Taking reverse Fourier transform with the help of the indicated boundary conditions one can obtain the infinite set of equations for (n_1, n_2) - th harmonics of the SCXM tangential electric field $E_y(x = A_p, n_1, n_2)$ on the plasma interface. Coefficients $F(n_1, n_2, l_1, l_2)$ are also differed from that are applied in the previous case by factor $(iZ_{E2}/2)^{|m_2|+|m_2-l_2|}$. The obtained set of equations is relatively more difficult than that in the case of one pumping field. It can be analytically studied in the case when relation of the pump frequencies ω_{01}/ω_{02} is a relation of simple natural numbers, see [7,8,10]. Using the method applied there one can reduce this set to infinite set of algebraic equations which is similar to well known sets from the theory of the parametrical instabilities in non-bounded plasmas, see e.g. [15]. Algebraic set of equations has solutions when the determinant constructed from the coefficients located at the $E_y(x = A_p, N, l_1, l_2)$ is equal to zero.

Analysis of the algebraic equations gives us mutually opposite results in the two different cases distinguished by correlation between pumping frequencies and ion cyclotron frequency. In the first case neither ω_{01} nor ω_{02} are not closed to the ω_i value. Under this condition algebraic set of equations has any complex roots. Therefore the ion SCXM parametrical instability is not realised when both pumping frequencies (ω_{01} and ω_{02}) are far from ion cyclotron frequency. In the second case one of the pump frequencies (for example ω_{01}) is approximately equal to ω_i. Then under this resonant condition one obtains algebraic equation of the third order for the parameter γ. Solving it one can find the following expression for the ion SCXM instability growth rate:

$$\mathrm{Im}\gamma \approx 0.3\Delta_{Ti}\left[3b_{E1}(b_{E1} + 0.7b_{E2})\right]^{1/3}. \qquad (10)$$

This value of SCXM parametric instability growth rate is differed from that in the case of one pumping field, because there was linear dependence of the parametric growth rate on pumping field amplitude. Hence under the condition of small pump fields' amplitudes the ion SCXM parametric growth rates value (10) is relatively greater as compared with that obtained in the case of one pumping field effect.

So unlike the case of one monochromatic pump field effect, the ion SCXM parametric instability caused by two pumping fields with different amplitudes and operating frequencies has two different versions of the development. Change of the correlation between the pumping frequencies ω_{01} and ω_{02} gives one possibility to enhance the ion SCXM parametric growth rates or to suppress the instability.

4. CONCLUSION

The article is devoted to the theoretical study of the surface waves at the second ion cyclotron harmonic parametric instability affected by monochromatic and non-monochromatic electrical pumping field. Simple analytical expressions for the instability growth rates are obtained. Using numerical analysis it is shown that the main impact on the growth rates values are executed by correlation between pumping fields' frequencies and their amplitudes. Just exchanging of these factors gives one possibility to enhance the instability, to decrease its growth rates or even to suppress this parametric instability. So application of the second pumping electric field allows controlling scenario of the parametric instability of the considered surface waves.

The effect of the plasma waveguide parameters on the considered instability is relatively weak as compared with the effect of mentioned above factors. The more significant role among the plasma waveguide parameters has been played by external magnetic field value.

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ПАРАМЕТРИЧНА НЕСТІЙКІСТЬ ПОВЕРХНЕВИХ ХВИЛЬ НА ДРУГІЙ ГАРМОНІЦІ ІОННОЇ ЦИКЛОТРОННОЇ ЧАСТОТИ

М.Р. Бєляєв, В.О. Гірка, В.І. Лапшин

Параметрична нестійкість поверхневих хвиль на другій гармоніці іонної циклотронної частоти (ПЦХ) в діелектричних хвилеводах з плазмовим заповненням досліджена в кінетичному наближенні. ПЦХ є незвичайно поляризованими модами, що поширюються поперек зовнішнього сталого магнітного поля. Знайдено прості аналітичні вирази для частот ПЦХ, їх декрементів загасання, обумовлених зіткненнями частинок плазми між собою та їх взаємодією з межею плазми, а також інкрементів їх параметричної нестійкості. Інкременти параметричної нестійкості ПЦХ слабко залежать від поперечних розмірів плазми та сильно зменшуються із збільшенням довжини хвилі ПЦХ. Здобуті результати можна використовувати в термоядерних дослідження для визначення небажаних режимів нагрівання периферії плазми, яких слід уникати при ІЦРН.

ПАРАМЕТРИЧЕСКАЯ НЕУСТОЙЧИВОСТЬ ПОВЕРХНОСТНЫХ ВОЛН НА ВТОРОЙ ГАРМОНИКЕ ИОННОЙ ЦИКЛОТРОННОЙ ЧАСТОТЫ

М.Р. Беляев, В.О. Гирка, В.И. Лапшин

Параметрическая неустойчивость поверхностных волн на второй гармонике ионной циклотронной частоты (ПЦВ) в диэлектрических волноводах с плазменным заполнением исследована в кинетическом приближении. ПЦВ являются необыкновенно поляризованными модами, распространяющимися поперек внешнего постоянного магнитного поля. Найдены простые аналитические выражения для частот ПЦВ, их декрементов затухания, обусловленных столкновениями частиц плазмы между собой и их взаимодействием с границей плазмы, а также инкрементов их параметрической неустойчивости. Инкременты параметрической неустойчивости ПЦВ слабо зависят от поперечных размеров плазмы и сильно уменьшаются при увеличении длины волны ПЦВ. Полученные результаты могут быть использованы в исследованиях по термоядерному синтезу для определения нежелательных режимов нагревания периферии плазмы, которых следует избегать при ИЦРН.