NONLINEAR MECHANISM OF ELECTROMAGNETIC WAVES GENERATION IN SPACE DUST PLASMA

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We consider a parametric excitation of electromagnetic waves induced by inertial Alfvén wave (IAW) in space dust plasma. The nonlinear dispersion equations describing decay of upper-hybrid wave (UHW) into the IAW and the ordinary electromagnetic wave as well as decay of UNW into the IAW and the left-hand circularly polarized (LHCP) wave are obtained using the three-fluid magnetohydrodynamics. The instability growth rates which depend on dust plasma parameters are found. It is shown that the LPCH wave is preferably excited by the UH pump wave for the parameters of the Saturn's F-ring.

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1. INTRODUCTION

The physical processes in dust plasmas have been studied intensively because of their importance for a number of application in space and laboratory plasmas. Generally, dust particles in plasma are charged by plasma current, photoemission, secondary emission, etc. The interplanetary space, the rings of the giant planets, comet tails, Earth's magnetosphere and ionosphere are Solar system objects with significant amount of dust particles. In recent years there has been much interest in new wave mode that results from the presence of micron-sized charged dust particles in plasma. About a decade ago it was recognized that the dust component may not only modify the usual plasma waves, such as ion acoustic waves and Alfvén waves, but leads to the appearance of new wave type [1,2], e.g., a low frequency mode, in which the inertia is provided by the massive dust component. Most of the studies on waves in a magnetized plasma have dealt with linear theories [3]. However in dust plasma the set of processes take place, for which there are important nonlinear effects, in particular nonlinear wave-wave interaction [4,5].

In the present paper we consider decay of the upper-hybrid pump wave into the IAW and the o-mode and into the IAW and the LHCP wave in dust plasma, which consist from electrons, protons and negatively charged dust particles.

2. BASIC EQUATIONS

We consider the upper-hybrid pump wave with:

$$E_0 = \left(E_{0x}\breve{e}_x + E_{0z}\breve{e}_z\right)\exp[i(-\omega_0 t + k_{0x}x + k_{0z}z)] + k.c.,$$
$$k_{0z} << k_{0x},$$

propagating in an homogeneously magnetized plasma $\left(B_0 = B_0\breve{e}_z\right)$. The pump wave decay into IAW with wave vector k and frequency ω and electromagnetic waves with wave vectors k_j and frequencies ω_j. It is assumed that the following wave resonance conditions are satisfied:

$$\omega_0 = \omega + \omega_j, \quad k_0 = k + k_j,$$

where $j=1, 2$ corresponds of o-mode and left-hand circularly polarized wave, respectively. Also we assume that all wave vectors are situated in the xz plane.

For studying parametric interaction, we use the three-fluid magnetohydrodynamics:

$$\frac{\partial V_\alpha}{\partial t} = \frac{1}{m_\alpha}\left(Z_\alpha e_\alpha \overset{\circ}{E} + \overset{\circ}{F}_\alpha\right) + \left(\acute{V}_\alpha \times \acute{\omega}_{B\alpha}\right) - \frac{T_\alpha}{m_\alpha n_\alpha}\acute{\nabla} n_\alpha,$$

$$\frac{\partial n_\alpha}{\partial t} = -\acute{\nabla}\left(n_\alpha \overset{\circ}{V}_\alpha\right),$$

$$\nabla \times \overset{\circ}{B} = \frac{4\pi}{c}\overset{\circ}{j} + \frac{1}{c}\frac{\partial E}{\partial t}, \quad \nabla \times \overset{\circ}{E} = -\frac{1}{c}\frac{\partial B}{\partial t}, \qquad (1)$$

$$\nabla \cdot E = 4\pi\rho,$$

where $j = e\left(n_i V_i - n_e V_e - Z_d n_d V_d\right)$, $\rho = e\left(n_i - n_e - Z_d n_d\right)$, $\overset{\circ}{F}_\alpha = \frac{Z_\alpha e_\alpha}{c}\left(\overset{\circ}{V}_\alpha \times \breve{\tilde{B}}\right) - m_\alpha\left(\overset{\circ}{V}_\alpha \nabla\right)\overset{\circ}{V}_\alpha$. The index $\alpha=i, e, d$ corresponds to the protons, electrons and dust particles, respectively.

The particles density, velocity, electric and magnetic fields are written in the forms:

$$n_\alpha = n_0 + \tilde{n}_0 + \tilde{n}_A, \; V = V_0 + V_A + V_j,$$

$$E = E_0 + E_A + E_j, \; B = B_0 + b_A + b_j,$$

where n_0 and B_0 are the average values of the plasma number density and magnetic field. The subscript A in these expressions corresponds to the IAW.

3. O-MODE GENERATION

3.1. NONLINEAR DISPERSION RELATION FOR DUST IAW

Nonlinear dispersion relation for the IAW is (see [6] for the details):

$$\varepsilon_A \varphi = P_{NL}, \qquad (2)$$

where $\varepsilon_A = \omega^2 - \frac{k_z^2 V_{Ad}^2}{1+\kappa_e}$, $\kappa_e = k_x^2\delta_e^2$, $\delta_e = c/\omega_{pe}$,

$$P_{NL} = i\frac{V_{Ad}^2}{1+\kappa_e}\left(\frac{m_e\omega}{n_{0e}e^2}\frac{\partial j_{ez}^{NL}}{\partial z} + \frac{1}{e}\frac{\partial F_{ez}}{\partial z}\right),\ V_{Ad} = \frac{B_0}{\sqrt{4\pi n_{od} m_d}},$$

$$j_{ez}^{NL} = -e\left(\tilde{n}_1^* V_{0z} + \tilde{n}_0 V_{1z}^*\right).$$

Using linear expressions for the electron velocity components, electron density perturbations and magnetic field perturbations for the pump wave and o-mode:

$$V_{0x} = -\frac{e}{m_e}\frac{\omega_0 k_{0x}}{\left(\omega_0^2 - \omega_{Be}^2\right)}\varphi_0,\quad V_{0y} = -i\frac{e}{m_e}\frac{\omega_{Be} k_{0x}}{\left(\omega_0^2 - \omega_{Be}^2\right)}\varphi_0,$$

$$V_{0z} = -\frac{e}{m_e}\frac{k_{0z}}{\omega_0}\varphi_0,\quad \tilde{n}_0 = -\frac{e}{m_e}n_{0e}\left(\frac{k_{0x}^2}{\omega_0^2 - \omega_{Be}^2} + \frac{k_{0z}^2}{\omega_0^2}\right)\varphi_0,$$

$$V_{1z} = -i\frac{e}{m_e}\frac{E_{1z}}{\omega_1},\quad b_{1y} = -\frac{ck_{1x}}{\omega_1}E_{1z},$$

we get following expression for the nonlinear dispersion relation for the IAW:

$$\varepsilon_A\varphi = \mu_A\varphi_0 E_{1z}^*, \tag{3}$$

where nonlinear term is

$$\mu_A = -i\frac{e}{m_e}\frac{V_{Ad}^2}{1+\kappa_e}k_z\frac{\omega}{\omega_1}\frac{k_{0x}^2}{\omega_0^2}.$$

3.2. NONLINEAR DISPERSION RELATION FOR O-MODE

The o-mode propagates along the x axis and has an electric field parallel to the ambient magnetic field. Excluding the magnetic field from Maxwell's Eq. (1), we obtained an equation for the electric field of the ordinary electromagnetic wave:

$$\varepsilon_1 E_{1z} = -\frac{\omega_{pe}^2}{e}F_{1z} + i4\pi e\omega_1\left(nV_z\right)_{NL},$$

where $\varepsilon_1 = \omega_1^2 - k_1^2c^2 - \omega_{pe}^2$, $\left(n\mathrm{V}_z\right)_{NL} = \tilde{n}_0 V_z^* + \tilde{n}_{eA}^* V_{0z}$.

Using linear expressions for the electron velocity components, electron density perturbations and magnetic field perturbations for the pump wave and IAW:

$$V_x = \frac{e}{m_e}\frac{k_x\omega}{\omega_{Be}^2}\varphi,\quad V_y = i\frac{e}{m_e}\frac{k_x}{\omega_{Be}}\varphi,$$

$$V_z = -\frac{e}{m_e}\frac{k_z\kappa_e}{\omega}\varphi,\quad \tilde{n}_{eA} = -n_{0e}\kappa_e\frac{e}{T_e}\frac{V_{Te}^2}{V_f^2}\varphi,$$

$$b_y = -i\frac{k_x}{k_z}\frac{c\omega}{V_{Ad}^2}\varphi$$

we find the dispersion relation for the o-mode:

$$\varepsilon_1 E_{1z} = \mu_1\varphi_0\varphi^*, \tag{4}$$

where nonlinear term is

$$\mu_1 = -i\frac{e}{m_e}\omega_{pe}^2\frac{\omega k_x}{k_z V_{Ad}^2}\frac{\omega_0 k_{0x}}{\omega_0^2 - \omega_{Be}^2}.$$

3.3. NONLINEAR GROWTH RATE

From Eqs. (3) and (4), we obtained the nonlinear dispersion relation for the parametric instability:

$$\varepsilon_A\varepsilon_1^* = \mu_A\mu_1^*\left|\varphi_0\right|^2. \tag{5}$$

When we allow for a dissipative part in the wave frequencies, $\omega = \omega_r + i\gamma_1$ and $\omega_1 = \omega_{1r} + i\gamma_1$ in Eq.(5), we can obtaine the expression for the instability growth rate:

$$\gamma_1 = \frac{\sqrt{W}}{2}\frac{\omega_{pe}^2 V_{Te}}{\omega_1\omega_0}\left(\frac{k_x k_{0x}\omega}{\omega_0}\right)^{\frac{1}{2}},$$

where $W = \dfrac{\left|E_{0x}\right|^2}{4\pi n_{0e}T_e}$.

4. LHCP WAVE GENERATION

4.1. NONLINEAR DISPERSION RELATION FOR LHCP WAVE

Excluding the magnetic field from Maxwell's Eq. (1), we obtained an equation for the electric field of the LHCP wave:

$$\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial z^2}\right)\overset{\circ}{E}_2 - 4\pi e n_0\frac{\partial V_2^L}{\partial t} = -4\pi\frac{\partial j_{2NL}}{\partial t}, \tag{6}$$

where the nonlinear current density is determined by the beating of the pump wave and the IAW

$$j_{2NL} = -e\left(\tilde{n}_{eA}^* V_0 + \tilde{n}_0 V^* + n_{0e}V_{2e}^{NL}\right).$$

If we use the linear expressions for the electron velocity components, electron density perturbations and magnetic field perturbations for the pump wave and IAW in Eq. (6) we obtained the nonlinear dispersion relation:

$$\varepsilon_2 E_{2x} = \mu_2\varphi_0\varphi^*, \tag{7}$$

where $\varepsilon_2 = \omega_2^2 - k_2^2c^2 - \omega_{pe}^2\dfrac{\omega_2}{\omega_2 + \omega_{Be}}$,

nonlinear term is

$$\mu_2 = -i\frac{e}{m_e}\omega_{pe}^2\kappa_e\omega_2\omega_0\frac{k_z^2}{\omega^2}\frac{k_{0x}}{\omega_0^2 - \omega_{Be}^2}.$$

4.2. NONLINEAR DISPERSION RELATION FOR DUST IAW

The dispersion relation for the IAW is given by Eq. (2) in Sec. 3.1, where components of ponderomotive force F_{ez} and nonlinear current j_{ez}^{NL} are determined by the interaction of UHW and LHCP wave. Using linear expressions for the electron velocity components, electron density perturbations and magnetic field perturbations for the pump wave and LHCP wave, we get the following expression for the nonlinear dispersion relation for the IAW:

$$\varepsilon_A \varphi = \mu_3 \varphi_0 E_{2x}^*, \quad (8)$$

where nonlinear term is

$$\mu_3 = -i \frac{V_{Ad}^2}{1+\kappa_e} \frac{e}{m_e} \frac{k_{0x}}{\omega_0} \frac{k_z^2}{\omega_2}.$$

4.3 NONLINEAR GROWTH RATE

From Eqs. (7) and (8), we obtained the nonlinear dispersion relation for the parametric instability:

$$\varepsilon_A \varepsilon_2^* = \mu_2 \mu_3^* |\varphi_0|^2. \quad (9)$$

When we allow for a dissipative part in the wave frequencies, $\omega = \omega_r + i\gamma_2$ and $\omega_2 = \omega_{2r} + i\gamma_2$ in (9), we obtained the following expression for the instability growth rate:

$$\gamma_2 = \frac{\sqrt{W}}{2} \frac{\omega_{pe}^2 V_{Te}}{V_{Ad}} \left(\kappa_e \frac{\omega}{\omega_2} \frac{1}{\omega_0^2 - \omega_{Be}^2} \right)^{\frac{1}{2}}.$$

CONCLUSIONS

We have investigated the nonlinear mechanisms for the generation of electromagnetic waves in β-low space dust plasma. We have studied the nonlinear decay process of the upper-hybrid pump wave into the IAW and the o-mode as well as decay of pump wave into the IAW and the LHCP wave. The source of radiation of the left-hand polarized and ordinary electromagnetic waves is the nonlinear current produced by the resonant interaction of the pump wave with the low-frequency inertial Alfvén wave.

We find the magnitudes of the growth rate of instabilities, for typical wave and plasma parameters relevant to the Saturn's F-ring [4,7]: $Z_d \approx 10^4$, $n_{0d} \approx 1cm^{-3}$, $n_{0i} \approx 10^4 cm^{-3}$, $T_e \approx T_i \approx 10eV$, $B_0 \approx 0.02G$, $m_d \cong 10^{-12} g$, $n_{0e} \approx 10cm^{-3}$. For decay process UHW à IAW+O-mode the instability growth rate is $\gamma_1 = 5.9 \cdot 10^{-10} s^{-1}$. For decay process UHW à IAW+LHCP the instability growth rate is $\gamma_2 = 10^{-7} s^{-1}$. Our calculations show that the generation of the left-hand polarized electromagnetic wave is faster than that of the o-mode.

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НЕЛИНЕЙНЫЙ МЕХАНИЗМ ГЕНЕРАЦИИ ЭЛЕКТРОМАГНИТНЫХ ВОЛН В ПЫЛЕВОЙ КОСМИЧЕСКОЙ ПЛАЗМЕ

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Рассмотрено параметрическое возбуждение электромагнитных волн инерциальными альфвеновскими волнами (ИАВ) в пылевой космической плазме. На основе уравнений трехжидкостной магнитной гидродинамики получено нелинейное дисперсионное уравнение, описывающее как распад верхнегибридной волны на ИАВ и обыкновенную электромагнитную волну, так и распад верхнегибридной волны на ИАВ и левополяризованную электромагнитную волну. Найден инкремент развития неустойчивости, зависящий от параметров пылевой плазмы. Показано, что левополяризованная электромагнитная волна преимущественно возбуждается верхнегибридной волной накачки для параметров F-кольца Сатурна.

НЕЛІНІЙНИЙ МЕХАНІЗМ ГЕНЕРАЦІЇ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ В ПИЛОВІЙ КОСМІЧНІЙ ПЛАЗМІ

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Розглянуто параметричне збудження електромагнітних хвиль інерційними альфвенівськими хвилями (ІАХ) в пиловій космічній плазмі. На основі рівнянь трирідинної магнітної гідродинаміки отримано нелінійне дисперсійне рівняння, що описує як розпад верхньогібридної хвилі на ІАХ та звичайну електромагнітну хвилю, так і розпад верхньогібридної хвилі на ІАХ та на лівополяризовану електромагнітну хвилю. Знайдено інкремент розвитку нестійкості, який залежить від параметрів пилової плазми. Показано, що лівополяризована електромагнітна хвиля переважно збуджується верхньогібридною хвилею накачки для параметрів F-кільця Сатурна.