ESTIMATIONS OF RESTRICTIONS ON USE "STANDARD" SOLUTIONS OF VLASOV EQUATIONS FOR DRIFT WAVES

E.V. Kasilova, V.I. Khvesyuk

Bauman Moscow State Technical University,
2nd Baumanskaya Str. 5, 105005, Moscow, e-mail: khves@power.bmstu.ru

Some estimations of correctness of use "standard" solution of Vlasov equation for finite sistems are curried out.
PACS: 52.35.-g

In this work we analyse correctness of ordinary solution of Vlasov-equation for drift waves in infinite plasma for analysis of plasma instability in finite size. Such a problem occurs for example when processes in FRC-systems are investigated where instability evolves in finite size. The initial expression for perturbed ion density [1] is:

$$\delta n_i = -\frac{q\varphi}{k_B T_i} n_0 - i\frac{q\varphi}{k_B T_i}\times$$
$$\times\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\tau}^{0} f_0 v_\perp\left[\omega + \frac{k_B T_i}{m_i}\frac{k_y}{\Omega}\right]J_0^2\left(\frac{k v_\perp}{\Omega}\right)\times \quad (1)$$
$$\times\exp[i(k_z v_z - \omega)]d\tau dv_\perp dv_z,$$

("standard" time integration is taking from $-\infty$ to zero [1]).

Results of integration is:

$$\int_{-\tau}^{0}\exp[i(k_z v_z - \omega)]dt = \frac{1-\exp[-i(k_z v_z - \omega)]}{i(k_z v_z - \omega)}. \quad (2)$$

And

$$\int_0^\infty \exp\left(-\frac{m_i v_\perp^2}{k_B T_i}\right)J_0^2\left(\frac{k_\perp v_\perp}{\Omega}\right)v_\perp dv_\perp =$$
$$= I_0\left(\frac{k_\perp^2}{\Omega^2}\frac{k_B T_i}{m_i}\right)\exp\left(-\frac{k_\perp^2 k_B T_i}{\Omega^2 m_i}\right).$$

Problem comes to integration of

$$\omega\sqrt{\frac{m_i}{2\pi k_B T_i}}\int_{-\infty}^{\infty}\frac{\exp\left(-\frac{m_i v_z^2}{k_B T_i}\right)}{\omega - k_z v_z}[1-$$
$$-\exp(-i(k_z v_z - \omega)\tau)]dv_z,$$

when τ is constant.

Physical meaning of this integral is that the first item in square brackets corresponds to plasma function, the second item is correction for finite time of integration.

Let's transform the last expression

$$\frac{\omega}{k_z}\sqrt{\frac{m_i}{2\pi k_B T_i}}\int_{-\infty}^{\infty}\frac{\exp\left(-\frac{m_i v_z^2}{k_B T_i}\right)}{\frac{\omega}{k_z} - v_z}\left[1-\exp\left(-ik_z\left(\frac{\omega}{k_z} - v_z\right)\tau\right)\right]dv_z$$

Introduce dimensionless variables:

$$v_z \Rightarrow t = v_z\sqrt{\frac{m_i}{2k_B T_i}}, \quad \frac{\omega}{k_z} \Rightarrow z = \frac{\omega}{k_z}\sqrt{\frac{m_i}{2k_B T_i}},$$
$$k_z\tau \Rightarrow k_z\tau\sqrt{\frac{2k_B T_i}{m_i}}.$$

The first integral in (2) is

$$W(z) = \frac{i}{\pi}\int_{-\infty}^{\infty}\frac{\exp(-t^2)}{z-t}dt.$$

The second integral is

$$W''(z,a) = \frac{i}{\pi}\int_{-\infty}^{\infty}\frac{\exp(-t^2)}{z-t}\exp[ia(z-t)]dt.$$

Standard approach is acceptable to system of finite size if $W''(z,a) \ll W(z)$. Drift waves phase velocity must satisfy condition:

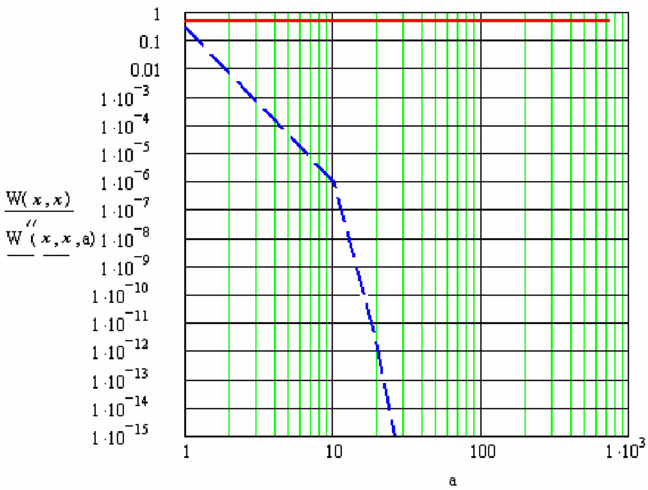
$$v_{ti} \ll \frac{\omega}{k_z} \ll v_{te}.$$

v_{ti} is the ion thermal velocity, v_{te} is the electron thermal velocity:

$$v_{ti} = \sqrt{\frac{2k_B T_i}{m_i}}, \qquad v_{te} = \sqrt{\frac{2k_B T_e}{m_e}}.$$

Parameter z is a complex value

$$z = x + iy.$$



Relation between $W''(a,z)$ and $W(z)$

In particular for $|x| = |y|$ corresponding approximately to so-called η_i mode (ITG - ion temperature gradient mode) we obtain restriction on z:

$$|z| > 1 \quad \text{or} \quad x \approx y > \frac{1}{\sqrt{2}}.$$

Relation between $W''(a,z)$ and $W(z)$ is presented in the Figure. As one can see condition $W''(z,a) << W(z)$ is satisfied when a >2.

Relation between $W''(a,z)$ and $W(z)$ is presented in the Figure above. As one can see condition $W''(z,a) << W(z)$ is satisfied when a >2. Therefore "standard" solution is satisfied when

$$a = k_z v_{ti} \tau = \frac{2\pi L}{\lambda_z} > 2.$$

REFERENCES

1. N.A. Krall, A.W. Trivelpiece. *Principles of Plasma Physics*. New York: "Mc Graw-Hill Co.", 1973.

ОЦЕНКИ ОГРАНИЧЕНИЙ НА ИСПОЛЬЗОВАНИЕ "СТАНДАРТНЫХ" РЕШЕНИЙ УРАВНЕНИЯ ВЛАСОВА ДЛЯ ДРЕЙФОВЫХ ВОЛН

Е.В. Касилова, В.И. Хвесюк

Выполнены оценки корректности использования результатов интегрирования уравнения Власова для систем конечных размеров.

ОЦІНКИ ОБМЕЖЕНЬ НА ВИКОРИСТАННЯ "СТАНДАРТНИХ" РІШЕНЬ РІВНЯННЯ ВЛАСОВА ДЛЯ ДРЕЙФОВИХ ХВИЛЬ

Є.В. Касилова, В.І. Хвесюк

Виконано оцінки коректності використання результатів інтегрування рівняння Власова для систем кінцевих розмірів.