MATHEMATICAL MODELING OF GAS FLOW IN PLASMA TORCH VORTEX CHAMBER

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Mathematical model of moving gas in vortex chamber (inter-electrode gas supply insert) of plasma torch channel was developed. Effect of moving electric arc on the motion of gas is considered. Software for solving problems of three dimensional non-ideal gas dynamics in complex areas was developed.

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1. INTRODUCTION

Nowadays several important problems of environmental control (ecology problems) take place. These include waste processing (municipal, medical, radioactive waste), efficient burning of low-calorific coal, some plasma-chemical processes (decomposition of hydrogen sulphide, gasification of coal and heavy hydrocarbons). These processes require to heat up to 3000…5000ºC for waste processing and up to 10000…12000ºC for plasma-chemical processing. These problems can be solved by the using of plasma technology. The direct current arc plasma torch is the more applicable source of heat gas in low temperature plasma technologies.

The plasma torch with tangential gas blowing is wide used because of its design simplicity and stability of operating parameters. There are two main parameters to control plasma torch power these are arc the current and the gas flow rate. To provide the wide range of plasma torch working parameters it is necessary to minimize the turbulization of gas flow in plasma torch channel.

Numerical experiment of physical processes (gas flow, gas heating by electrical arc) in plasma torch channel gas flow enables us to determine the correlation of plasma torch parameters such as geometry of plasma torch channel, working arc current and working gas flow rate at the stage of plasma torch design.

The most important factor that affects on the gas flow in the plasma torch channel and on the working power of plasma torch is the electric arc. The parameters of the arc-heated plasma are described by complicated structure of the gas dynamics, heat transfer and electromagnetic phenomena. Thus, it is necessary to solve the whole system of gas dynamic equations and the electrodynamics equations (Maxwell equations).

We try to find the solution of the whole system of non-ideal compressible magneto hydrodynamics equations (MHD) in complex 3D areas.

2. PHYSICAL FORMULATION OF THE PROBLEM

The gas is injected tangentially through inter-electrode gas supply insert (the insert between cathode and anode parts of plasma torch cannel) of the plasma torch (Fig.1). The gas flow is supplied in vortex chamber (inter-electrode insert) and it begins the rotational movement along the plasma torch channel. After some time the electric voltage is turned on and the electric arc appears. The electric arc heats the gas up to high temperature.

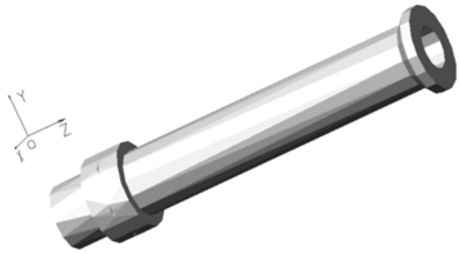

Fig.1. Plasma torch channel

We shall consider two types of tangential gas supply in vortex chamber: six cylindrical holes and three rectangular slots. We investigate the relation between temperature, pressure, arc voltage and the external current.

First, we solve non-ideal gas dynamic equation (without magnetic field). The problem is solved in the sector area (Fig.2). The solution is found up to the moment when the current is turned on. We suppose that this moment is known. The gas is assumed viscous, but not heat-conducting [1-3].

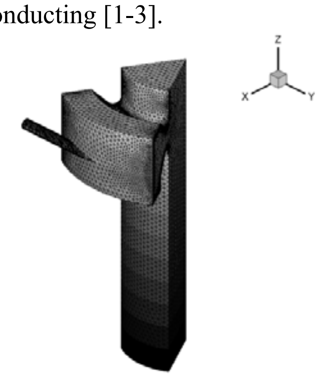

Fig.2. Sector area

After the current is on we will define the zone with the electric arc. The initial value of magnetic intensity is in this area. Then the whole system of non-ideal MHD is solved. The initial conditions for the system are taken from the solution of previous problem. The problem is solved in the whole area (Fig.3). The gas is now assumed viscous and heat-conducting.

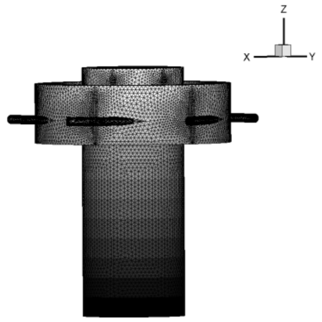

Fig.3. Whole area

3. NOTATION

Throughout the paper the following notation is used t is the time; $\mathbf{r} = (x, y, z)$ is the point of the area; ρ is the density; $\mathbf{V} = (V_r, V_\varphi, V_z)$ is the gas velocity of the cylindrical coordinate system (r, φ, z); $\mathbf{V} = (u, v, w)$ is the gas velocity of the rectangular coordinate system; $e = \rho\varepsilon + \rho\frac{\mathbf{V}^2}{2}$ is the volume density of the total energy, where ε is the mass density of the specific internal energy; p is the pressure; T is the temperature; R is the universal gas constant; D is the area; γ is the index of the adiabat; S the boundary of the area D; a the velocity of sound; $\mathbf{I}$ is the identity tensor; $\boldsymbol{\sigma}$ is the stress tensor; η is the coefficient of dynamic viscosity; ρ_0 is the initial density; $\mathbf{V}_1$ is the boundary speed (inflow speed); T_0 is the initial temperature; $\mathbf{H}$ is the magnetic field strength; ν is the magnetic viscosity; $\mathbf{q}$ is the heat-flux vector; λ is the coefficient of heat conduct.

4. MATHEMATICAL PROBLEM FORMULATION

4.1. THE SYSTEM OF EQUATIONS BEFORE THE CURRENT IS TURNED ON

The base system of gas dynamic equations is solved. There are continuity equation, momentum equation and energy equation.

$$\frac{\partial\rho}{\partial t} + \operatorname{div}(\rho\mathbf{V}) = 0, \quad (1)$$

$$\frac{\partial\rho\mathbf{V}}{\partial t} + \operatorname{div}(\rho\mathbf{V}\otimes\mathbf{V} + p\mathbf{I} - \boldsymbol{\sigma}) = \mathbf{0}, \quad (2)$$

$$\frac{\partial e}{\partial t} + \operatorname{div}((e+p)\mathbf{V} - \boldsymbol{\sigma}\mathbf{V}) = 0. \quad (3)$$

Stress tensor has the following components is $\boldsymbol{\sigma} = \{\sigma_{ij}\}$ [4, 5]:

$$\sigma_{ij} = \eta\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial V_l}{\partial x_l}\right). \quad (4)$$

The state equations are:

$$p = \rho RT,\ \varepsilon = \frac{RT}{\gamma - 1}. \quad (5)$$

The initial conditions are:

$$\rho\big|_{t=0} = \rho_0,\ \mathbf{V}\big|_{t=0} = \mathbf{0},\ T\big|_{t=0} = T_0.$$

The boundary conditions are:

- gas inflow: $\rho = \rho_1$, $\rho\mathbf{V} = \rho_1\mathbf{V}_1$, $p = p_1$, $T = T_0$ on S_1;
- adherence condition: $\mathbf{V} = 0$ on S_2;
- gas outflow conditions on S_3;
- periodicity condition:

$$\begin{aligned}
&\rho(r,\ \varphi_1 + \varphi_0, z) = \rho(r,\ \varphi_1, z),\\
&V_n(r,\ \varphi_1 + \varphi_0, z) = V_n(r,\ \varphi_1, z),\\
&V_r(r,\ \varphi_1 + \varphi_0, z) = V_r(r,\ \varphi_1, z),\\
&V_z(r,\ \varphi_1 + \varphi_0, z) = V_z(r,\ \varphi_1, z),\\
&e(r,\ \varphi_1 + \varphi_0, z) = e(r,\ \varphi_1, z),
\end{aligned} \quad (6)$$

where φ_0 is the sector angle (in our case $\varphi_0 = \pi/3,\ 2\pi/3$), φ_1 is the angle between bottom and top faces, V_n (in our case V_ϕ) is normal velocity component, V_r is radial velocity component.

4.2. THE SYSTEM OF EQUATIONS AFTER THE CURRENT IS TURNED ON

After the current is on we define the zone with the electric arc. The system of the solved is the following:

$$\frac{\partial\rho}{\partial t} + \operatorname{div}(\rho\mathbf{V}) = 0, \quad (7)$$

$$\frac{\partial\rho\mathbf{V}}{\partial t} + \operatorname{div}(\rho\mathbf{V}\otimes\mathbf{V} + p\mathbf{I} - \boldsymbol{\sigma}) = \frac{1}{4\pi}\left[\operatorname{rot}\mathbf{H}, \mathbf{H}\right], \quad (8)$$

$$\frac{\partial e}{\partial t} + \operatorname{div}((e+p)\mathbf{V} - \boldsymbol{\sigma}\mathbf{V} + \mathbf{q}) = \frac{-1}{4\pi}\operatorname{rot}\mathbf{H}\cdot\left[\mathbf{H}, \mathbf{V}\right], \quad (9)$$

$$\frac{\partial\mathbf{H}}{\partial t} = \operatorname{rot}\left[\mathbf{V}, \mathbf{H}\right] - \operatorname{rot}\left(\nu_m \operatorname{rot}\mathbf{H}\right), \quad (10)$$

$$\operatorname{div}\mathbf{H} = 0. \quad (11)$$

Stress tensor components $\boldsymbol{\sigma} = \{\sigma_{ij}\}$ are taken from (4).

Heat-flux vector is [4, 5]:

$$\mathbf{q} = -\lambda\cdot\operatorname{grad}T. \quad (12)$$

The system (7)-(11) is supplemented with the state equations (5).

The initial conditions are taken from the solution of the last problem described in 4.1.

The boundary conditions are the same.

5. THE MESH

In the area of computation the tetrahedron is constructed with account of period conditions. The mesh is refined especially in the zone of the inflow (Fig.4).

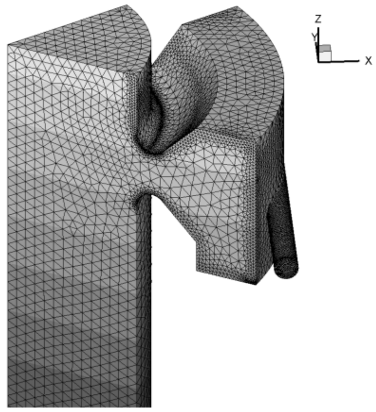

Fig.4. Tetrahedron meshing. Sector area. View near inflow area. H_max = 0.1

After we have the sector mesh we can get the whole area mesh using torsion (Fig.5).

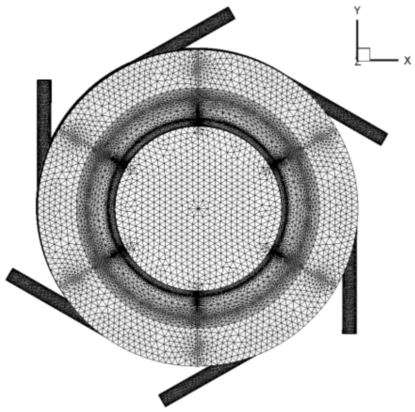

Fig.5. Tetrahedron meshing. Whole area. View near inflow area. H_max = 0.1

6. NUMERICAL METHOD

6.1. DESCRIBING SOFTWARE

We use our previously developed [2] software to solve the problem of non-ideal gas dynamics in the complicated 3D areas. The software is developed using FORTRAN language. We also plan to add the electrodynamics module (10)-(11). Data input is mesh parameters and gas parameters.

The software is based on physical process separation. The flux solver of ideal gas dynamic is HLLC [5,6]. The flux solver of non-ideal gas component is taken from [1,7-9]. The numerical procedures of the software exploit the idea of physical process separation.

6.2. PHYSICAL PROCESS SEPARATION

We use physical process separation procedure to obtain the approximation for the non-hyperbolic parts of system (7)-(11) [10-11].

The problem is separated into two parts [12-16]: Navier-Stokes equations and electrodynamics part (10), (11).

The algorithm of finding solution at the next time step is:

- solve ideal gas dynamics [6];
- solve the parabolic system with initial conditions from the previous time step;
- find the solution of (10), (11) ($\mathbf{V}$ is known);
- find the gas parameters ($\mathbf{H}$ is known).

Let's write the basic system in operation form.

$$\mathbf{U}_t + \mathbf{A}(\mathbf{U}) = \mathbf{F}(\mathbf{U}, \mathbf{H}),$$

$$\mathbf{H}_t = \mathbf{G}(\mathbf{U}, \mathbf{H}),$$

$\mathbf{U}$ – vector of conservative variables; $\mathbf{A}$ – system operator; $\mathbf{F}$ – right part of the gas dynamic equations; $\mathbf{G}$ – right part of the Maxwell equations.

We can write $\mathbf{A}$ as:

$$\mathbf{A} = \mathbf{A}_i + \mathbf{A}_v,$$

where $\mathbf{A}_i$ – hyperbolic operator, $\mathbf{A}_v$ – parabolic operator.

The solution of (7)-(11) at $(t_0, t_0 + \tau)$ is found by solving the four problems:

$$\mathbf{U}^{1/4}\Big|_{t=t_0} = \mathbf{U}(t_0),\ \mathbf{U}_t^{1/4} + \mathbf{A}_i(\mathbf{U}^{1/4}) = 0, \qquad (13)$$

$$\mathbf{U}^{2/4}\Big|_{t=t_1} = \mathbf{U}^{1/4}(t_1),\ \mathbf{U}_t^{2/4} = \mathbf{A}_v(\mathbf{U}^{1/4}), \qquad (14)$$

$$\mathbf{H}^{3/4}\Big|_{t=t_1} = \mathbf{H}(t_0),\ \mathbf{H}_t^{3/4} = \mathbf{G}(\mathbf{U}^{2/4}, \mathbf{H}^{3/4}), \qquad (15)$$

$$\mathbf{U}^{4/4}\Big|_{t=t_2} = \mathbf{U}^{2/4}(t_1),\ \mathbf{U}_t^{4/4} = \mathbf{F}(\mathbf{U}^{2/4}, \mathbf{H}^{3/4}), \qquad (16)$$

$$\mathbf{U}(t_0 + \tau) = \mathbf{U}^{3/4}(\tau), \qquad (17)$$

where t_1, t_2 are the intermediate time steps at $\left(t_0,\ t_0 + \tau\right)$.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ТЕЧЕНИЯ ГАЗА В ФОРСУНКЕ И В ОБЛАСТИ АНОДА КАНАЛА ПЛАЗМАТРОНА

А.Е. Бутырев, М.П. Галанин, А.В. Переславцев

Построена математическая модель движения газа в форсунке (межэлектродной вставке) и в области анода канала плазматрона. Модель учитывает влияние электрической дуги на течение газа. Разработан программный комплекс для решения задач трехмерной неидеальной газовой динамики в областях сложной геометрической формы.

МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ТЕЧІЇ ГАЗУ У ФОРСУНЦІ І В ОБЛАСТІ АНОДА КАНАЛУ ПЛАЗМАТРОНА

А.Є. Бутирєв, М.П. Галанін, А.В. Переславцев

Побудовано математичну модель руху газу у форсунці (міжелектродній вставці) і в області анода каналу плазматрона. Модель враховує вплив електричної дуги на течію газу. Розроблено програмний комплекс для рішення завдань тривимірної неідеальної газової динаміки в областях складної геометричної форми.