RESONANT INTERACTION BETWEEN FUSION PRODUCTS AND MAGNETIC FIELD PERTURBATIONS IN TOROIDAL MAGNETIC TRAP WITH THE ROTATIONAL TRANSFORM

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The resonance condition for "particle – magnetic field perturbation" interaction is obtained under the solving of the drift kinetic equation. The resonance phenomenon in the particle motion in dependence on the radial position in the vertical cross-section of torus is studied. Both numerical and the analytical treatment for the cold α-particles motion near resonant condition in toroidal magnetic trap are presented.

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1. MOTIVATION

The resonant interaction between the particle motion and magnetic field perturbations is being studied intensively experimentally and theoretically [1-5]. The most reason of such interest to this problem is connected with the physics essence of these phenomena and that it is proposed that the "particle – magnetic field perturbation" causes the prompt losses of the particles from the magnetic confinement volume. The periodic increase of the outward particle fluxes is observed in experiments on tokamaks (JET) [3] and torsatrons/heliotrons (Large Helical Device) [4]. The problem of the resonant interaction right now does not give the answer on the many questions concerning the connection between the amplitudes, phases and frequencies of the observed magnetic field perturbations and particle fluxes. Especially important this interaction can be the reason of losses of the fusion products in the modern fusion devices and future reactors. The model of the magnetic field is chosen as to describe the magnetic field with the rotational transform of the magnetic field lines as it takes place in tokamaks (Section 2.1). That is why we consider here the resonance condition of the particle-magnetic field interaction analytically (Section 2.3) and we analyze the satisfaction the resonance phenomena on the cross-section radial position (Section 2.4). The particle motion under and near resonance condition is demonstrated numerically with the solving of the guiding centre equations (Section 3). Here also the analytical treatment of the resonance condition is given. Principal conclusions are presented in Section 4.

2. RESONANCE CONDITION IN TOROIDAL CONFIGURATION WITH THE ROTATIONAL TRANSFORM

2.1. MAGNETIC FIELD MODEL

The coordinates used in the present calculation are the quasi-toroidal coordinates (r, ϑ, φ), connected with the circular axis of torus (Fig.1), r is the radial variable, a is the minor radius, R_0 is the major radius of the torus, ϑ is the poloidal angle counted from the direction opposite to the principal normal to circular axis of torus, φ is the toroidal angle.

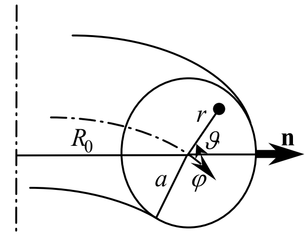

Fig.1. Quasi-toroidal coordinates

The main magnetic field is introduced in the following form

$$\mathbf{B} = \frac{B_0 R_0}{R}\left\{0,\ \frac{r}{R_0}\iota(r^2),\ 1\right\}, \qquad (1)$$

where

$$R = R_0 + r\cos\vartheta\,. \qquad (2)$$

The rotational transform is taken as it is described for tokamak JET

$$\iota = 1 - \frac{3}{4}\left(\frac{r}{a}\right)^2. \qquad (3)$$

To find the resonance the additional magnetic field perturbation in the following form is added

$$\delta B_r = \sum_{m,n,\omega} \frac{B_0 R_0}{R} b_{m,n,\omega}\left(\frac{r}{a}\right)^{m-1} \sin(m\vartheta - n\varphi + \omega t), \quad (4,a)$$

$$\delta B_\vartheta = \sum_{m,n,\omega} \frac{B_0 R_0}{R} b_{m,n,\omega}\left(\frac{r}{a}\right)^{m-1} \cos(m\vartheta - n\varphi + \omega t), \quad (4,b)$$

$$\delta B_\varphi = 0\,. \qquad (4,c)$$

Here $b_{m,n,\omega}$ is the perturbation field amplitude, m and n are the wave number, ω is the frequency of the perturbation field.

Magnetic field structure of this type is observed in the experiment [4].

2.2. ELECTRICAL FIELD MODEL

The ambipolar electrical field is introduced in the following form

$$\mathbf{E} = E_0\left\{-\frac{d\Phi_E}{dr}, 0, 0\right\}, \qquad (5)$$

where

$$\Phi_E = \Phi_{E0}\left(1 - \left(\frac{r}{a}\right)^{2\alpha}\right)^{\beta_E}. \qquad (6)$$

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The electric field corresponding to the assumed radial dependence (Fig.2) has the characteristic feature of the electric field observed experimentally.

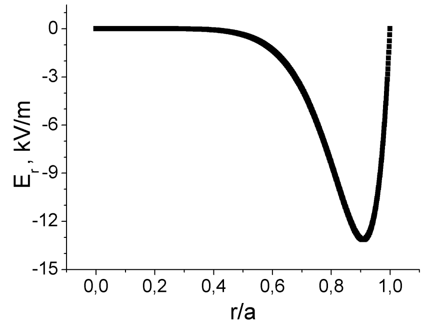

Fig.2. Electric field as the function of the radial variable

2.3. ANALYTICAL FORM OF RESONANCE CONDITION

We obtain the resonance condition under the solving of the drift kinetic equation [7-8] The usual drift kinetic equation for a guiding center distribution function f (E,μ,**r**), where E is the particle kinetic energy, μ is the magnetic moment and **r** is a guiding center coordinates has the form [6]

$$\frac{d\mathbf{r}_{g.c.}}{dt}\frac{\partial f}{\partial \mathbf{r}}+\frac{d\upsilon_{\parallel}}{dt}\frac{\partial f}{\partial \upsilon_{\parallel}}+\frac{d\upsilon_{\perp}^{2}}{dt}\frac{\partial f}{\partial \upsilon_{\perp}^{2}}=0\,. \quad (7)$$

Further we shall consider only the dependence of the distribution function of the spatial variables. Assuming that the solution of this equation can be found as $f = f^0 + f^1$, where f^0 satisfies the equation

$$\mathbf{V}^{0}_{g.c.}\frac{\partial f^{0}}{\partial \mathbf{r}}=0\,, \quad (8)$$

where $\mathbf{V}_{g.c.}\equiv\frac{d\mathbf{r}_{g.c.}}{dt}$ and $f^0 = f^0(\psi^*)$ is the function of the drift surfaces

$$\psi^{*}=\int\left(\upsilon_{\parallel}\frac{r}{R}\iota-\frac{cE_{r}(r/a)}{B_{0}}\right)d\left(\frac{r}{a}\right)-\left(\frac{r\cos\vartheta}{R}\frac{\upsilon_{\perp}^{2}}{2a\omega_{c}}\right), \quad (9)$$

where ω_c is the cyclotron frequency. The equation $\psi^* = const$ describes the cross-sections of the drift surfaces. The expression for ψ^* is determined with the accuracy linear relatively r/R_0 .

In the first order on r/R_0 :

$$\mathbf{V}^{0}_{g.c.}\frac{\partial f^{1}}{\partial \mathbf{r}}+\mathbf{V}^{1}{}_{g.c.}\frac{\partial f^{0}}{\partial \mathbf{r}}=0\,. \quad (10)$$

Distribution function correction we can find in the following form:

$$f^{1}=A_{\perp}\cos((m-1)\vartheta-n\varphi+\omega t)+ A_{\parallel}\cos(m\vartheta-n\varphi+\omega t), \quad (11)$$

where

$$A_{\perp}=\frac{D_{\perp}A_{m}-D_{\parallel}B_{m}}{\Delta}\left(\frac{a}{r}\right)^{m-1}, \quad (12,a)$$

$$A_{\parallel}=\frac{D_{\perp}B_{m-1}-D_{\parallel}A_{m-1}}{\Delta}\left(\frac{r}{a}\right)^{m}, \quad (12,b)$$

$$D_{\perp}=\upsilon_{\parallel}\overline{\alpha}_{m,n,\omega}\overline{r}^{\,m-1}\frac{\upsilon_{\perp}^{2}}{2a\omega_{c}}, \quad (13,a)$$

$$D_{\parallel}=\upsilon_{\parallel}\overline{\alpha}_{m,n,\omega}\overline{r}^{\,m-1}\left(\overline{r}\upsilon_{\parallel}\iota-\overline{R}_{0}\upsilon_{DE}\right), \quad (13,b)$$

$$A_{m}=\frac{\overline{r}^{\,m-1}}{\overline{R}_{0}}\left[\overline{r}\upsilon_{\parallel}(\iota m-n)-m\overline{R}_{0}\upsilon_{DE}\right], \quad (13,c)$$

$$A_{m-1}=\frac{(m-1)}{\overline{r}^{\,m}}\left[\frac{\overline{r}\upsilon_{\parallel}(\iota-n/(m-1))}{\overline{R}_{0}}-\upsilon_{DE}\right], \quad (13,d)$$

$$B_{m}=\frac{\overline{r}^{\,m-1}}{\overline{R}_{0}}m\frac{\upsilon_{\perp}^{2}}{2a\omega_{c}}, \quad (13,e)$$

$$B_{m-1}=\frac{1}{\overline{r}^{\,m}\overline{R}_{0}}(m-1)\frac{\upsilon_{\perp}^{2}}{2a\omega_{c}}. \quad (13,f)$$

The analytical expression for Δ in the denominator of (12) after the transformation takes the form

$$\Delta=\frac{Wm(m-1)}{2M}\left\{\begin{array}{l}\frac{2W}{M(a\omega_{c})^{2}}\left(1-\frac{\upsilon_{\parallel}^{2}}{\upsilon^{2}}\right)^{2}-\\ -\left(\frac{r}{a}\frac{\upsilon_{\parallel}}{\upsilon}\left(\iota-\frac{n}{m}\right)-\frac{R_{0}}{a}\frac{\upsilon_{DE}}{\upsilon}\right)\times\\ \times\left(\frac{r}{a}\frac{\upsilon_{\parallel}}{\upsilon}\left(\iota-\frac{n}{m-1}\right)-\frac{R_{0}}{a}\frac{\upsilon_{DE}}{\upsilon}\right)\end{array}\right\}, \quad (14)$$

where $\upsilon_{DE} = cE_r/B_0$ is the electric drift velocity. The equation Δ=0 gives us the resonance condition. The resonance condition is obtained under the assumption that $\omega = 0$.

2.4. DEPENDENCE OF RESONANCE CONDITION ON THE VARIOUS PARAMETERS

The analysis of the resonance condition is presented on the Fig.3.

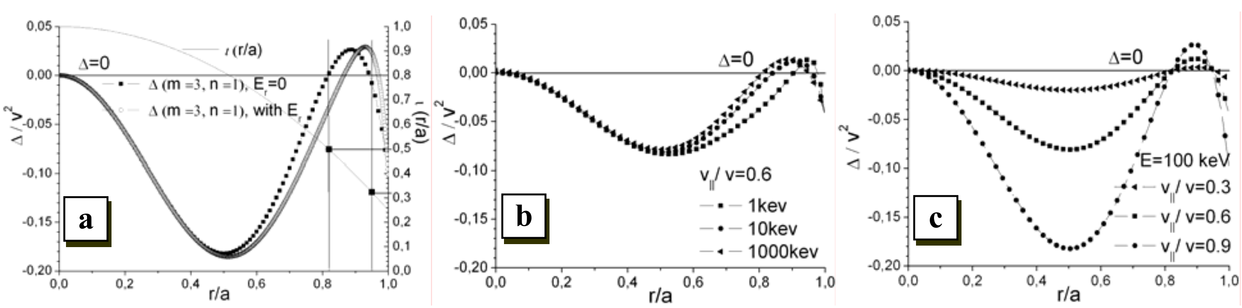

Fig.3. Resonance condition versus the radial variable in the cases $E_r = 0$ and $E_r \neq 0$ (a), $V_\parallel / V = 0.6$ and $W = 1\,keV, 10\,keV, 100\,keV$ under $E_r \neq 0$ (b), $W = 100\,keV$ and $V_\parallel / V = 0.3, 0.6, 0.9$ under $E_r \neq 0$ (c)

As we see from the plots on Fig.3 under the ambipolar electric field the resonance shifts to the larger radial variable (Fig.3,a). The resonance condition takes place at two values of the radial positions (there exist two roots) under the perturbations with “wave” numbers $m = 3,\ n = 1$. Position of the resonance weakly depends on the energy particle in the range $10\,keV \le W \le 1000\,keV$ (Fig.3,b) and almost does not depend on the pitch-velocity $\upsilon_{\parallel}/\upsilon$ under the fixed value of the energy W.

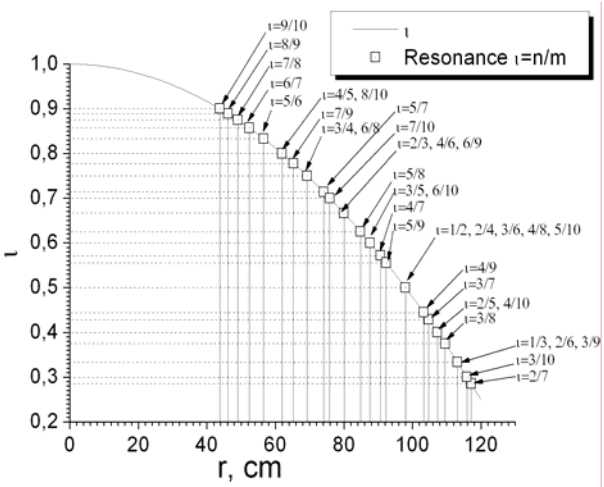

Fig.4. Rotational transform and the rational values as the function of the radial variable

The rotational transform can accept the rational values as it is shown on Fig.3. Here the azimuthal numbers $n, m = 1,...10.$ If the higher numbers are taken into account the resonance map will be more dense. Under the high shear and dense spectrum of the magnetic field harmonics the overlapping of the adjacent resonances can take place.

The drift transform angle of the particle can be expressed through the rotational transform in the following way

$$\iota^* = \iota \cdot \left\{ \frac{ |B| - \dfrac{R}{r\iota}\left[\dfrac{cE_r}{\upsilon_{\parallel}} - \rho_L \dfrac{\upsilon_{\perp}}{\upsilon_{\parallel}} \dfrac{\partial |B|}{\partial r} \right] }{ |B| + \dfrac{r\iota}{R}\left[\dfrac{cE_r}{\upsilon_{\parallel}} - \rho_L \dfrac{\upsilon_{\perp}}{\upsilon_{\parallel}} \dfrac{\partial |B|}{\partial r} \right] } \right\}, \qquad (15)$$

where $\rho_L = \dfrac{\upsilon_{\perp}}{\omega_c}$. In the process of the ion motion the parallel velocity $\upsilon_{\parallel}$ varies relatively to the total velocity υ and the position of the resonance can be changed. Drift transform (rotational transform of the particle trajectories) curves are not shown on this plot but we demonstrate their properties in the particle motion below. The numerical integration of the particle motion equations demonstrates the resonance condition mechanism and the escape of the resonance particle outward the magnetic confinement volume. The negative electric field deteriorates the ion confinement.

3. FUSION PRODUCTS SIMULATION NEAR THE RESONANCE WITH THE MAGNETIC FIELD PERTURBATION

3.1. BASIC EQUATIONS

Guiding centre equation is used in the standard form [6]

$$\mathbf{V}_{g.c.} = \upsilon_{\parallel}\frac{\mathbf{B}}{|\mathbf{B}|} + \frac{c}{|\mathbf{B}|^2}\left[\mathbf{E}\times\mathbf{B}\right] + \frac{mc\upsilon_{\parallel}^2}{e|\mathbf{B}|^4}\left[\mathbf{B}\times(\mathbf{B}\nabla)\mathbf{B}\right] + \frac{mc\upsilon_{\perp}^2}{2e|\mathbf{B}|^3}\left[\mathbf{B}\times\nabla|\mathbf{B}|\right], \qquad (16,a)$$

$$\dot{p}_{\parallel} = (e\mathbf{E} - \mu\nabla|\mathbf{B}|)\mathbf{h}, \text{ where } \mathbf{h} = \frac{\mathbf{B}}{|\mathbf{B}|}, \qquad (16,b)$$

$$\frac{d\mu}{dt} = 0. \qquad (16,c)$$

Here **E** is the electric field, **B** is the magnetic field, $\upsilon_{\parallel}, \upsilon_{\perp}$ are the parallel and transverse (relatively the direction of the magnetic field) particle velocities. The first term in (16,a) describes the motion along the magnetic field line, the second term describes the electric drift, the third one – the centrifugal drift and the forth one – the drift in the inhomogeneous magnetic field.

3.2. GUIDING CENTER TRAJECTORIES

We simulate the fusion production motion in Joint European Torus (JET), EURATOM, Great Britain. The subject of our study is the features of the particle motion the near the resonance structures of the magnetic configuration and the resonance processes in the trajectories and the stochastization of the drift trajectories.

Below there are presented the most characteristic examples of the fusion product behavior. Under the magnetic field perturbations with the “wave” numbers $m = 3,\ n = 1$ the drift surfaces experience the splitting and the chains of the drifts islands occur (Fig.5). These resonances take place when the electric field in the form (5) is taken into account. The electric field reflects the fact that the ions escape from the confinement volume through the each magnetic surface faster than electrons.

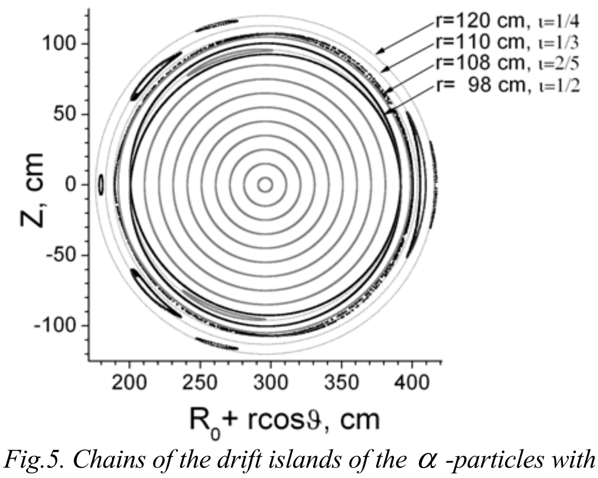

Fig.5. Chains of the drift islands of the α-particles with the energy W=100 keW and $V_{\parallel}/V$=0.9 on the background of the ***near****-rational unperturbed magnetic surfaces*

The drift islands are shown on the background of the unperturbed magnetic surfaces with the ***near***-rational values of the rotational transform. The drift islands with the rational values of the ***drift rotational transform*** are displaced relatively to the magnetic surfaces with the same values of the rotational transform.

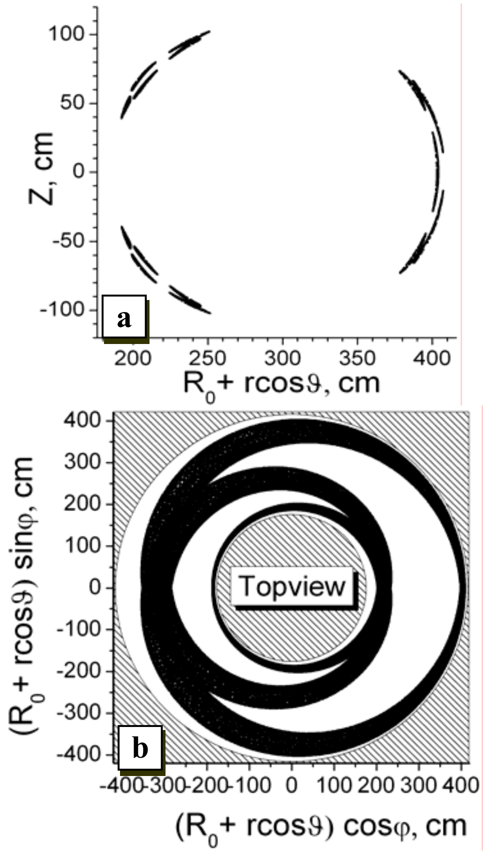

Fig.6. The drift islands with the drift transform $\iota^*=1/3$ *in the vertical (a) and horizontal (b) cross-sections of the torus*

The splitting of the drift surface with $\iota^* = 1/3$ on the 3 islands , which is seen on Fig.6,a,b, is accompanied with the splitting of adjacent drift resonances with $\iota^* = 3/15$ on the 5 islands. On the Fig.6,b there is shown the horizontal projection of the resonant trajectory and the vacuum chamber. The particle rounds the major circumference of the torus 7669 times and 2464 times around the minor circumference. The time of the particle motion in this simulation is 80 milliseconds.

Separately we consider the particles, which satisfy the resonance condition (14). The footprints of the particle trajectories (in the guiding center approach) fill some radial layer with the "white" gaps. The reason of this behavior is that chosen particle (α-particle with the energy $W = 100$ keV) is the resonant particle. As we see from the plot on Fig.7,a, near the drift island with ι=1/3 there takes place the overlapping of the adjacent resonances and the stochastic layer occurs. From Fig.7,b we see that this test particle during the period since the starting point till the time approximately 6.0 msec in the resonance interaction with the perturbing field. We mean that the equation $\Delta = 0$ is satisfied. After that time the particle escapes from the resonance condition $\Delta = 0$ (Fig.7,b) and deviates at some larger distance from the initial surface.

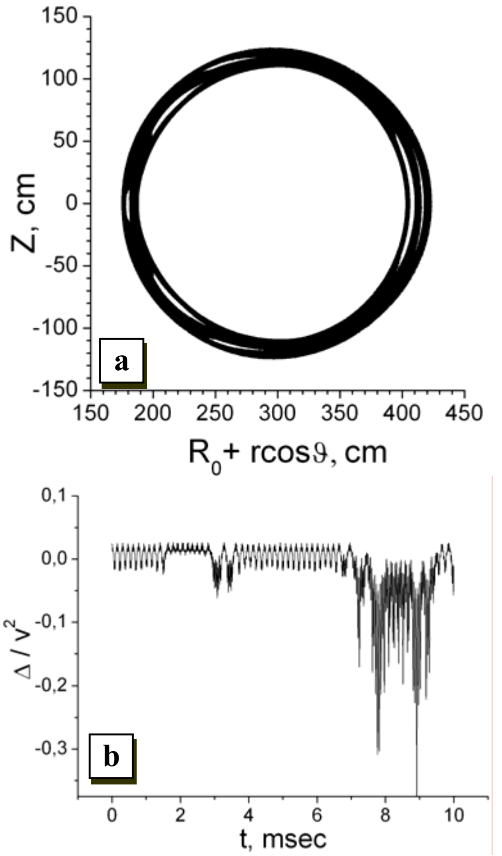

Fig.7. The projection of the resonant particle on the vertical cross-section of the torus (a) and the accompanied value of the resonance condition as the function of time (b)

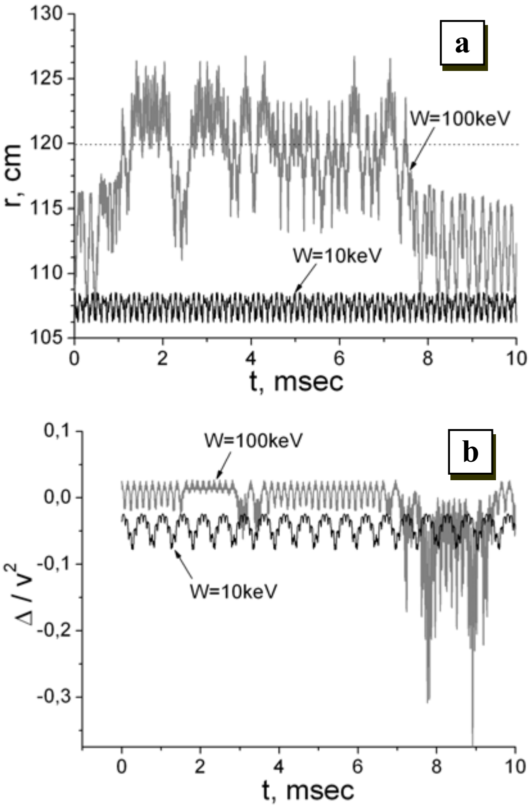

Fig.8. Radial position of the α-particles with the energy W=10 and 100 keV as the function of the time (a) and the resonance satisfaction for these particle during this motion (b)

The radial variable of the particle increases (Fig.8,a). Under the same conditions α-particle with the energy $W = 10$ keV moves quite differently: this particle does not deviate from the initial surface so far (Fig.8,a "black" curve) and does not satisfy the resonance condition (Fig.8,b "black" curve).

We see from the plots on Fig.7 and 8 that the higher energy α-particles escape from the confinement volume faster than the α-particle with the smaller energy, for example $W = 10$ keV and main plasma ions because their deviation in the stochastic layer accumulates and the α particle with W = 100 keV starting from the position r =108 cm comes to some position on the wall (r =120 cm) for the time near 1 msec.

4. SUMMARY AND DISCUSSIONS

1. One of the mechanisms effective for the removal of cold α-particles outward from the confinement volume is the resonant interaction of the particle with the perturbing magnetic field. The resonance condition is based on the equality $\iota^* = n/m$, where n, m are the "wave" numbers and ι^* is the drift rotational transform (drift trajectory twisting). The condition $\iota^* = n/m$ is satisfied on the different values of the radius. If the resonance conditions take place at the plasma periphery the removal of the cold α-particles is noticeable.

2. The obtained results can be used for the analysis of the experiments on the tokamak JET and torsatron Large Helical Device (LHD). The fluxes of the particles observed near the plasma boundary can be identified with the periodical high energy particle losses occurring on JET and LHD.

3. The characteristic time of the studied processes is of order 1…10 milliseconds. Coulomb scattering can effect on the resonance condition [9] however the real time of this effect and MHD activity (order is 1 seconds) larger than the considered here time of particle motion.

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REFERENCES

1. S.J. Zweben, D.S. Darrow, H.W. Herrmann, et al. Alpha particle loss in the TFTR DT experiments // *Nucl. Fusion.* 1995, v.35**,** p.893-917.
2. G.R. McKEE, R.J. Fonck, B.C.Stratton, et al. Transport measurements for confined non-thermal alpha particles in TFTR DT plasmas // *Nucl. Fusion.* 1997, v.37, p.501-516.
3. D. Testa, G.Y. Fu, A. Jaun, et al. Experimental test of damping models for n=1 toroidal Alfven eigenmodes in JET // *Nucl. Fusion.* 2003, v.43, p.479-482.
4. K. Toi, M. Takechi, S. Takai, et al. Global MHD modes excited by energetic ions in heliotron/torsatron plasmas // *17th IAEA Fusion Energy Conference. Research Report NIFS Series*. October 19-24, 1998, Yokohama, Japan, p.1-4.
5. V.S. Marchenko. α-particle transport induced by Alfven eigenmodes in the Helias reactor // *Nucl. Fusion.* 2002, v.42**,** p.1314-1317.
6. D.V. Sivukhin. Motion of Charged Particles in Electromagnetic Fields in the Drift Approximation // *Reviews of Plasma Physics*. Volume 1. Authorized translation from the Russian by Herbert Lashinsky, University of Maryland, USA. Edited by M.A. Leontovich. Published by Consultants Bureau, New York, 1965, p.1.
7. A. Shishkin, O. Shishkin, H. Wobig, et al. The effect of magnetic field modification on heavy ion movement in advanced stellarators and helical devices // *J. of Nucl. Materials*. 2003, v.313-316, p.1192-1196.
8. A.Shishkin, A. Sagara, O. Motojima, et al. Controlling the cross-field flux of clod α-particles with resonant magnetic perturbations in a helical fusion plasma // *Nucl. Fusion.* 2007, v.47, p.800-808.
9. O. Motojima and A.A. Shishkin. Drift island motion in helical plasma and its use for ash removal and high energy ion injection device // *Plasma Physics and Controlled Fusion.* 1999, v.41, p.227-242.

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РЕЗОНАНСНОЕ ВЗАИМОДЕЙСТВИЕ ПРОДУКТОВ СИНТЕЗА С ВОЗМУЩЕНИЯМИ МАГНИТНОГО ПОЛЯ В ТОРОИДАЛЬНОЙ МАГНИТНОЙ ЛОВУШКЕ С ВРАЩАТЕЛЬНЫМ ПРЕОБРАЗОВАНИЕМ

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При решении дрейфового кинетического уравнения получено резонансное условие для взаимодействия «частица – возмущающее магнитное поле». Изучены резонансные явления в движении частицы в зависимости от радиального положения в вертикальном сечении тора. Численным интегрированием уравнений движения получены траектории резонансных «холодных» α-частиц и дана аналитическая трактовка полученных результатов.

РЕЗОНАНСНА ВЗАЄМОДІЯ ПРОДУКТІВ СИНТЕЗУ ЗІ ЗБУРЕННЯМ МАГНІТНОГО ПОЛЯ У ТОРОЇДАЛЬНІЙ МАГНІТНІЙ ПАСТЦІ З ОБЕРТАЛЬНИМ ПЕРЕТВОРЕННЯМ

Ю.К. Москвітіна, О.О. Шишкін

При розв'язку дрейфового кінетичного рівняння отримана резонансна умова для взаємодії «частинка – збурювальне магнітне поле». Досліджені резонансні явища у русі частинки в залежності від радіального положення в вертикальному перерізі тору. Чисельним інтегруванням рівнянь руху отримано траєкторії резонансних «холодних» α-частинок і представлено аналітичне трактування отриманих результатів.