FAST ALFVÉN WAVE PROPAGATION IN MILTICOMPONENT NONUNIFORM PLASMAS

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The problem of the conversion, reflection and transmission of the fast Alfvén wave propagating in multi-component nonuniform plasmas is studied. The dependences of the wave scattering characteristics on the plasma composition and parallel wave number have been obtained. The results can be used to control the part of the launched power converted to the slow short wavelength mode and to prevent the essential reflection of the wave power back to the antenna.

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1. INTRODUCTION

The problems of the wave-particle interaction become especially interesting when the electromagnetic wave propagates in multi-component nonuniform plasmas. The plasma inhomogeneity leads to the appearance of the ion-ion hybrid resonance and the evanescence layer during the wave propagation when the plasma consists of the ion species with different charge-to-mass ratios [1-3]. When the plasma contains more than two ion species the number of the ion-ion hybrid resonances in plasmas can become greater. The launched electromagnetic wave will be transmitted through, reflected from and converted near the evanescence layer in the vicinity of each of the resonances. The interference picture of the fast waves reflected back to the antenna will give the distribution of the wave field along the plasma column. The information about the wave field distribution is important to study the wave-particle interaction for both ion and electron subsystems. At the same time, part of the fast wave power will be converted to the slow short wavelength mode, which is effectively damped on electrons. In such a way the power launched by the antenna to the plasma will be distributed over different wave modes. The fast wave is mainly absorbed by ions through the cyclotron damping mechanisms under the appropriate resonance conditions. The slow wave will be absorbed locally by electrons if the plasma temperature is of the order of the present day fusion experimental data. If the wave-particle interaction is not enough effective the essential part of the launched power can be transmitted through and reflected from the plasma column. When the reflection coefficient is large the experimental conditions could be dangerous for the antenna operation due to the large power flux back to the antenna (the problem of power coupling). In such a way, the antenna operation with the multi-component nonuniform plasmas needs predicting the power distribution between the different modes in plasmas as a function of the experimental conditions. It will allow to choose the safety antenna operation regime and the dominant channel of the wave-particle interaction to provide heating or pinch velocity for the selected plasma component.

The simplest model of the fast wave propagation through the nonuniform plasma column was proposed by Budden [4]. The real dispersion curve was approximated by the simplified dependence, which did not take into consideration the plasma density inhomogeneity. The transmission, reflection and conversion coefficients were obtained for two cases of the wave incidence. It was shown that the conversion coefficient can not exceed the value 25% when the fast wave is launched from the Low Magnetic Field Side (LMFS). Below only this case will be discussed as more realistic for the present day fusion experiments.

The more sophisticated model [5-7], which usually is called as “the triplet configuration”, takes into consideration the plasma density inhomogeneity at the edge of the plasma column. The Budden dispersion curve was corrected to take into account the wave reflection from R-cutoff layer at the High Magnetic Field Side (HMFS). Due to the reflection the second backward propagating wave appears in the plasma column. As a result the interference of two fast waves issues the wave field distribution in the plasma column between the evanescence layer and the antenna. But the interference picture depends on the phase difference between the two reflected waves. In the two limiting cases the launched power can be either completely converted to the short wavelength mode or completely reflected to the antenna. The model considers R-cutoff at the HMFS as the non-transparent barrier and, therefore, does not allow the fast wave to be transmitted through the plasma column. In the framework of the model the phase difference can be controlled by choosing the parallel wave number, the wave frequency or the plasma composition. But the position of the barrier provided by R-cutoff can be determined only approximately (it is always difficult to measure and control the plasma density at the edge). It leads to the uncertainty in the phase difference calculating. Since the barrier position can not be controlled externally the model restricts the possibility to predict the reflection and conversion fractions for the given experimental conditions.

In this paper, the method is proposed to control the wave field distribution in plasma by creating the additional semi-transparent barriers for the fast wave. New barriers can be created by puffing the additional ion species into the plasma. The launched fast wave will be partially transmitted through and reflected from each of the barriers. The interference picture will issue the wave field distribution in some regions of the plasma column. As a result, the general conversion, reflection and transmission coefficients can be obtained. The proposed model is preferable for wave field distribution control in comparison with the triplet configuration model where the second barrier is non-transparent and its position depends on the plasma edge density profile. On the contrary, the properties of the second barrier in the proposed model depend on the concentration of the second ion species. Both the barrier transparency and the interference picture can be controlled by choosing the parallel wave number and the wave frequency. In such a way the most preferable wave field distribution (to create the desired radial profiles of the ion pinch velocity and the electron/ion heating) can be provided by changing the concentration of the second ion species and choosing the launched wave parameters.

2. MODE CONVERSION IN PLASMAS WITH TWO ION-ION HYBRID RESONANCES

The three ion component plasma will be studied here to show the effect of the semi-transparent barriers provided by the ion-ion hybrid resonances on the fast wave propagation. The problem can be generalized for the greater number of ion species in plasma but the more complicated interference picture will not change a nature of the physics processes. The propagation of the fast Alfvén wave (FAW) through the fusion plasma column is usually considered within a slab approximation. In this model the confining magnetic field is assumed to be directed along the z-axis, the wave vector has the components $(k_\perp, 0, k_\parallel)$ with $k_\parallel$-spectrum fixed by the Ion Cyclotron Resonance Frequency (ICRF) antenna phasing. The radial inhomogeneity of the plasma density and the nonuniformity of the magnetic field lead to the partial conversion of the launched FAW to the slow wave near the ion-ion hybrid resonances. The slow wave is effectively absorbed by electrons. In general, the propagation of the FAW through the resonances and cutoffs in nonuniform plasmas in the ion cyclotron frequency range is described by the wave equation

$$\frac{d^2 E_y}{d x^2} + Q(x) E_y = 0 \qquad (1)$$

for the electric field component E_y. The potential Q(x) is proportional to the square of perpendicular refraction index, $Q(x) = \frac{\omega^2}{c^2} n_\perp^2$, given by the cold plasma dispersion relation

$$n_\perp^2 = \frac{(R - n_\parallel^2)(L - n_\parallel^2)}{S - n_\parallel^2}, \qquad (2)$$

where S, L=S–D and R=S+D are the components of the plasma dielectric tensor in the Stix notation [8]:

$$S = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2 - \omega_{cs}^2}, \qquad (3^a)$$

$$D = \sum_s \frac{\omega_{cs}}{\omega} \frac{\omega_{ps}^2}{\omega^2 - \omega_{cs}^2}. \qquad (3^b)$$

The equation (2) gives the ion-ion hybrid resonances $(S = n_\parallel^2)$ and L-cutoffs $(L = n_\parallel^2)$ which form the evanescence layers. The relation $(R = n_\parallel^2)$ defines R-cutoffs located at low plasma densities near the plasma edge. The equation (1) is derived neglecting the electron inertia and assuming the longitudinal component of the electric field E_z is small. The cold-plasma approximation can be used to describe the mode conversion if the layers of the fundamental cyclotron resonances and ion-ion hybrid resonance are well separated. When the mode conversion is the dominated process the cyclotron damping and direct minority heating are small, but the enhanced electron damping is observed.

Historically, the first equation used to study the propagation of the FAW through the single cutoff-resonance pair was the Budden equation [4]. The wave scattering characteristics depend on the tunneling parameter $\eta = k_A \Delta$, the product of the wave vector far from the resonance and the width of the evanescence layer. The transmission coefficient does not depend on the incidence side and is given by $T = e^{-\pi\eta}$. It was shown that in case of the LMFS incidence, when the fast wave first approaches the L-cutoff, the maximal mode conversion is 25% provided that η=0.22.

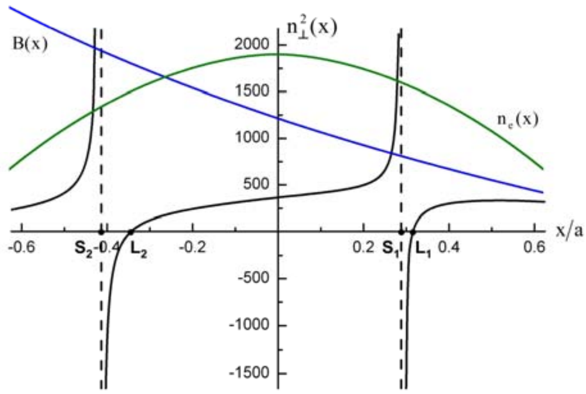

Fig.1. The real part of $n_\perp^2$ as a function of normalized distance x/a from the center of the plasma column. The plasma consists of H, D, and ^{3}He ions with the fractions 0.82, 0.14 and 0.02, respectively

In this paper, the wave propagation through the plasma column with two cutoff-resonance pairs is studied. This scenario can be realized in plasma which consists of three ion species with different charge-to-mass ratios. As an example, the hydrogen plasma with the fractions of deuterium and helium minorities, denoted as (D,^{3}He)H, is considered. Fig.1 shows the dispersion relation for FAW and the spatial variance of magnetic field and electron plasma density through the plasma column. The ion-ion hybrid resonances associated with ^{3}He and D minority are denoted as S_1 and S_2, respectively. The L-cutoffs are denoted as L_1 and L_2. The fundamental cyclotron resonances of ^{3}He and D species are located at $x/a = 0.2$ and $x/a = -0.67$, respectively. The central electron density is assumed to be $n_{e0} = 2.5 \cdot 10^{13}\ cm^{-3}$. The confinement magnetic field at the axis and the antenna operation frequency were chosen to be B_0=3.6 T and f=34.7 MHz, respectively.

To study the propagation of the FAW in such plasmas the potential Q(x) is modeled by the expression:

$$Q(x) = k_A^2 \left(1 - \frac{\Delta_1}{x - x_{S1}} - \frac{\Delta_2}{x - x_{S2}}\right). \quad (4)$$

The parameters of the potential Q(x) are found numerically to fit them with the dispersion relation $n_\perp^2$ (Fig.1). When the resonances are well separated the position of L-cutoffs are determined by the expressions:

$$x_{L1} \approx x_{S1} + \Delta_1 + \frac{\Delta_1 \Delta_2}{x_{S1} - x_{S2}}, \quad (5^a)$$

$$x_{L2} \approx x_{S2} + \Delta_2 - \frac{\Delta_1 \Delta_2}{x_{S1} - x_{S2}}. \quad (5^b)$$

In this case two back-to-back cutoff-resonance pairs are characterized by the tunneling parameters $\eta_{1,2} = k_A \Delta_{1,2}$. In general, the tunneling parameters must be evaluated numerically:

$$\eta_{1,2} = \int_{S1,S2}^{L1,L2} (-Q(x))^{1/2} dx. \quad (6)$$

The wave equation with the potential Q(x) in the form (4) was studied analytically using the phase-integral method [5,9]. It is the generalization of the WKB method to find the approximate solutions of wave equation for the given potential Q(x) in the complex plane. This method is based on the Stokes phenomena: the coefficient of the subdominant WKB term must be discontinuously changed upon crossing the so-called Stokes lines. The WKB solutions from each side of the resonances are matched not by passing along the real axis but tracing the evolution of the WKB solutions along the contour in the complex plane crossing the Stokes lines. Using this method, the mode conversion coefficient C was derived

$$C = T_1 T_2 (1 - T_1 T_2) + 4T_1 (1 - T_1)(1 - T_2) \sin^2(\Delta\varphi / 2). \quad (7)$$

In this formula $T_{1,2} = e^{-\pi \eta_{1,2}}$ are Budden transmission coefficients of each evanescence layer. The total transmission coefficient is $T=T_1T_2$.

In the vicinity of L_1-cutoff the FAW is partially reflected and partially transmitted through the first layer. The transmitted wave is partially reflected from L_2-cutoff. This reflected wave tunnels through the first layer. Finally, there is the interference of two waves with the different amplitudes and phases, which determines the reflection R and the mode conversion C coefficients. The phase difference $\Delta\varphi$ is the sum of three terms:

$$\Delta\varphi = 2\Phi + \Psi_2 - \Psi_1, \quad (8)$$

$$\Phi = \int_{x_{L2}}^{x_{S1}} Q(x)^{1/2} dx,$$

$$\Psi_{1,2} = \mathrm{Arg}\left(\frac{2\pi i \exp(2k_{1,2}(\ln k_{1,2} - 1))}{\Gamma(k_{1,2})\Gamma(1 + k_{1,2})}\right), \; k_{1,2} = -i\eta_{1,2}/2.$$

The phases $\Psi_{1,2}$ appear due to the additional phase shifting under the reflection from L-cutoffs. It can be shown for small values of tunneling parameters $\eta_{1,2}$, that

$$\Psi_{1,2} \approx \eta_{1,2}(1.116 - \ln(\eta_{1,2})). \quad (9)$$

For large values $\eta_{1,2}$ these phases tend to $\pi/2$ as for the case of the isolated cutoff. The phase 2Φ appears due to the double pass of the second reflected wave from S_1 resonance to L_2 cutoff.

As a function of a phase, the mode conversion will be maximal possible

$$C_{opt} = T_1 T_2 (1 - T_1 T_2) + 4T_1 (1 - T_1)(1 - T_2), \quad (10)$$

if the phase difference $\Delta\varphi$ is equal to the odd values of π

$$\Delta\varphi = \pi(2n + 1), \; n \in Z. \quad (11)$$

The contours of the constant C_{opt} as a function of the tunneling parameters η_1 and η_2 are plotted in Fig.2. It is clear that the mode conversion can be effective in plasmas with $\eta_1 \approx 0.22$ and $\eta_2 \sim 1.0$. In such plasmas the second evanescence layer acts similar to R-cutoff in the theory of triplet configuration [5-7]. If $\eta_2 >> 1$ our expression reduces to that obtained in [5]:

$$C_{trip} = 4T_1(1 - T_1)\sin^2(\Delta\varphi / 2). \quad (12)$$

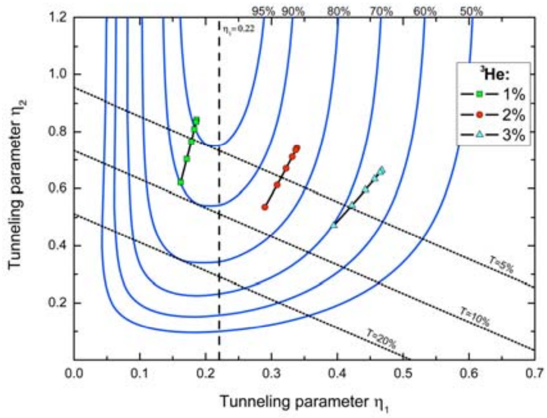

Fig.2. The contours of the maximal possible mode conversion coefficient C_{opt} as a function of the tunneling parameters η_1 and η_2

The physics of the enhancement of the mode conversion is the same: ***due to the interference of the reflected waves the total reflection coefficient R can be minimized***. Compared to the triplet configuration case, the proposed scenario has the certain advantage: the location of the cutoff is primarily determined by the plasma composition rather than the edge plasma density profile.

3. NUMERICAL RESULTS AND DISCUSSIONS

The dependence of the maximal possible mode conversion coefficient on both tunneling parameters is shown in Fig.2. Theoretically the fast wave can be converted completely to the slow short wavelength mode if the second barrier is non-transparent and the phase difference (8) is equal to odd values of π. In Fig.2 it corresponds to the vertical dashed line at η_1=0.22 (the optimal mode conversion condition for two-component plasmas according to the Budden model) when η_2 goes to an infinity. The solid lines show the levels of the maximal possible mode conversion coefficient when the second barrier is semi-transparent. The value of the mode conversion coefficient lays somewhere in the range from $T_1T_2(1 - T_1T_2)$ to C_{opt} (the precise value is defined by the phase difference). The dot lines are the constant levels of the transmission coefficient T. Since the power conservation law can be written in the form T+R+C=1 the fraction of the wave power reflected back to the antenna can also be obtained from the figure. As a result, the range of the experimental conditions can be obtained when the reflection coefficient R is enough high to damage the antenna. Also the operational paths for the waves with the parallel wave vectors in the range from 0 to 5 m^{-1} are shown in Fig.2 for three concentrations of ^{3}He species. It can be seen that for 2% concentration of ^{3}He the mode conversion coefficient for all waves from the range can reach but does not exceed the value of 85%. But the real value of the mode conversion coefficient for each wave with the particular $k_{||}$ will be defined by the phase difference between the reflected waves. This difference depends on the experimental conditions and will be discussed below.

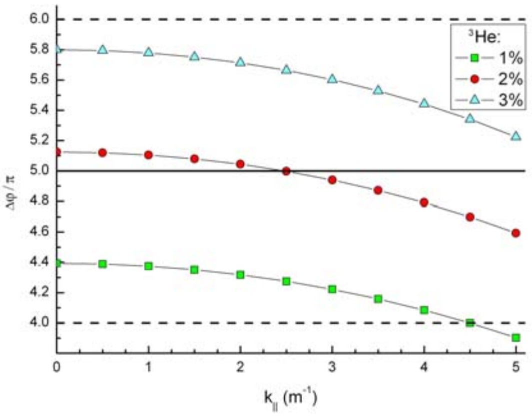

Fig.3. The phase difference Δφ as the function of the parallel wave number $k_{||}$ for different ^{3}He fractions

When the maximal possible value of the mode conversion coefficient is known, the dependence of the phase difference on the experimental conditions should be discussed. Fig.3 shows the dependence of the phase difference between two reflected waves (8) on the parallel wave vector for different values of ^{3}He concentration. The phase difference (the vertical axis) is normalized on π. When the phase difference is equal to odd values of π the mode conversion coefficient is equal to the maximal possible value C_{opt} (presented in Fig.2). When the phase difference is equal to even values of π the mode conversion coefficient is minimal possible $T_1T_2(1-T_1T_2)$ and goes to zero for the considered conditions. In other words, the phase difference defines a multiplier (Fig.4) for C_{opt} that must be used to find the mode conversion coefficient C.

Fig.4 shows this multiplier for three values of ^{3}He concentrations as a function of the parallel wave vector. The shaded areas correspond to different concentrations (1%, 2%, 3%, from the left to the right) and cover the $k_{||}$ range from 0 to 5 m^{-1}. As it can be seen from Fig.4, the multiplier approaches to 1 for the case of 2% concentration and it is enough small for concentration of 1% for the selected experimental conditions. The value of the multiplier for 3% concentration changes from 0.1 for $k_{||}$=0 m^{-1} to 0.9 for $k_{||}$=5 m^{-1}.

Figs.2-4 demonstrate three typical but different cases of the launched power distribution between the modes in the plasma column.

The first case is optimal for the wave conversion (2% of ^{3}He concentration). The maximal possible value of the conversion coefficient C_{opt} slightly depends on the parallel wave vector and is equal to $C_{opt} \approx 85\%$.

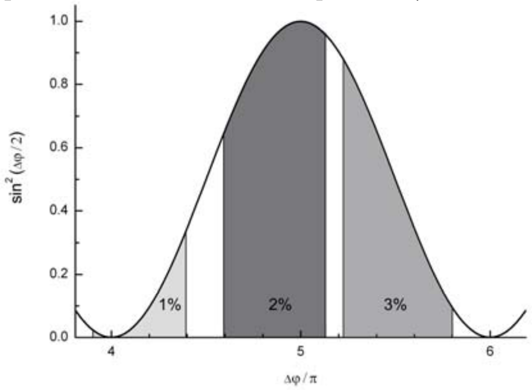

Fig.4. The multiplier for the maximal possible mode conversion coefficient C_{opt} for different ^{3}He fractions

The phase multiplier (Fig.4) is enough large (in the range from 0.6 to 1.0). As a result, the conversion coefficient is in the range from 50% to 85%. The main part of the launched power is converted to the slow mode. This case can be used to provide the local electron heating through the effective damping of the converted mode on the electrons.

The second case is optimal for the wave reflection (1% of ^{3}He concentration). The maximal possible value of the conversion coefficient changes from 95% for $k_{||}$=0 m^{-1} to 90% for $k_{||}$=5 m^{-1}. But due to the antiphase the phase multiplier is very small and lays in the range 0.0…0.3. Therefore the conversion coefficient is very small. It is in the range from 0% to 25%. The main part of the launched power is reflected back to the antenna. This case would be used to realize the fast wave interaction with plasma ions if there is the cyclotron damping mechanisms somewhere between the first evanescence layer and the antenna.

The third case is the intermediate between the first and the second cases (3% of ^{3}He concentration). The maximal possible value of the conversion coefficient is changed in the range 65…70% but the phase multiplier covers the wide range of values 0.1…0.85 giving the conversion coefficient somewhere between 5% and 60%. In this case there is a big difference in the processes of the reflection and conversion for the waves with different parallel wave vectors. Therefore it is important to know how the generator power is distributed over the $k_{||}$-spectra of the ICRF antenna. The relation between the converted and reflected fractions can be changed in the wide range.

The presented numerical data has been obtained for the enough wide second barrier. For all the discussed cases the transmission coefficient T does not exceed 10%. It allows to speak only about the conversion and reflection channels of the launched power distribution. But the cases with the essential power transmission can also be studied in a similar way. The transmission will become important if the D concentration will be decreased or the parallel wave number will be increased. The case of the substantial wave transmission could be interesting to provide the partial wave absorption near the cyclotron resonance layer behind the second ion-ion hybrid resonance.

CONCLUSIONS

Puffing the additional ion species into the plasma can be used to create the additional ion-ion hybrid resonance layers. These layers play an important role for the fast wave propagation through the plasma column. It can be used to control the processes of fast wave transmission through, reflection from and conversion in the plasma. These processes define the distribution of the launched power between the transmitted, reflected and converted modes. In turn, the modes will define the structure of the electromagnetic field in plasma. Finally, the local field polarization defines the efficiency of the wave-particle interaction. The properties of the semi-transparent wave barriers depend on the plasma composition and therefore they can be controlled by maintaining the needed concentration of the additional ions. From the other point of view, the barrier transparencies depend on the parallel wave vector value and the wave frequency. Choosing the antenna phasing and the operating frequency is a key to obtain the preferable distribution of the launched power between the modes and, as a result, the structure of the electromagnetic field through the plasma column.

The problem has an essential dependence on the experimental conditions: plasma density profile, nonuniformity of the magnetic field, plasma ion composition. Due to the obtained results the dominant channel of the launched power going (transmission, reflection or conversion) can be identified for the given experimental conditions and for the particular antenna phasing and operating frequency. The scenarios with large reflection have to be avoided because of the essential backward power flux to the antenna. The scenarios with large conversion can be used for the effective local electron heating through the slow wave damping on electrons. The scenarios with large transmission can be useful, for example, to realize (simultaneously with the partial mode conversion) the minority heating mechanism for the ions behind the second ion-ion hybrid resonance layer at the HMFS. The intermediate scenarios can be used to get a preferable partition of the launched power between the wave modes.

The processes of the wave-particle interaction are not considered here. Therefore the used approximations will be realistic if there are not the effective mechanisms of the fast wave interaction with the plasma particles. Usually it is realized when there is not the effective ion cyclotron damping. When the damping mechanisms become essential and they can not be neglected the fast wave amplitude will be changed essentially along the propagation path. As a result, the interference picture will be the other but it can be calculated in a similar way taking into consideration the damping mechanisms. Here the possibility to control the launched power distribution between the wave modes has been demonstrated. It can be used to optimize the wave-particle interaction according to the goals of the experiments.

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REFERENCES

1. P.U. Lamalle, M.J. Mantsinen, J.-M. Noterdaeme, et al. Expanding the operating space of ICRF on JET with a view to ITER // *Nuclear Fusion*. 2006, v.46, №2, p.391-400.
1. M.-L. Mayoral, P.U. Lamalle, D. Van Eester, et al. Hydrogen plasmas with ICRF inverted minority and mode conversion heating regimes in the JET tokamak // *Nuclear Fusion*. 2006, v.46, №7, p.S550-S563.
2. Ye.O. Kazakov, I.V. Pavlenko, B. Weyssow, I.O. Girka. Enhanced mode conversion in fusion discharges with large concentration of minority ions // *Ukrainian Journal of Physics*. 2008, v.53, №5, p.443-450.
3. K.G. Budden. *The Propagation of Radio Waves*. Cambridge: "Cambridge University Press", 1985, 684 p.
4. V. Fuchs, A.K. Ram, S.D. Schultz, et al. Mode conversion and electron damping of the fast Alfvén wave in a tokamak at the ion-ion hybrid frequency // *Physics of Plasmas*. 1995, v.2, №5, p.1637-1647.
5. C.N. Lashmore-Davies, V. Fuchs, A.K. Ram, A. Bers. Enhanced coupling of the fast wave to electrons through mode conversion to the ion hybrid wave // *Physics of Plasmas*. 1997, v.4, №6, p.2031-2043.
6. R.Majeski, C.K.Phillips, J.R.Wilson. Electron heating and current drive by mode converted slow waves // *Physical Review Letters*. 1994, v.73, №16, p.2204-2207.
7. T.M. Stix. *The Theory of Plasma Waves*. New York: "McGraw-Hill", 1962, 283 p.
8. R.B. White. *Asymptotic Analysis of Differential Equations*. London: "Imperial College Press", 2005, 286 p.
9. J. Heading. *An Introduction to Phase Integral Methods*. New York: "Wiley", 1962, 160 p.

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РАСПРОСТРАНЕНИЕ БЫСТРОЙ МАГНИТОЗВУКОВОЙ ВОЛНЫ В МНОГОКОМПОНЕНТНОЙ НЕОДНОРОДНОЙ ПЛАЗМЕ

Е.О. Казаков, И.В. Павленко, И.А. Гирка, Б. Вейссов

Изучается задача конверсии, отражения и прохождения быстрой магнитозвуковой волны, распространяющейся в многокомпонентной неоднородной плазме. Получены зависимости характеристик прохождения волны от состава плазмы и от параллельного волнового вектора. Полученные результаты могут быть использованы для управления частью энергии волны, которая конвертируется в коротковолновую моду, и для предотвращения существенного отражения энергии волны назад к антенне.

ПОШИРЕННЯ ШВИДКОЇ МАГНІТОЗВУКОВОЇ ХВИЛІ У БАГАТОКОМПОНЕНТНІЙ НЕОДНОРІДНІЙ ПЛАЗМІ

Є.О. Казаков, І.В. Павленко, І.О. Гірка, Б. Вейссов

Вивчається задача конверсії, відбиття та проходження швидкої магнітозвукової хвилі, яка поширюється у багатокомпонентній неоднорідній плазмі. Отримано залежності характеристик проходження хвилі від складу плазми та від паралельного хвильового вектора. Отримані результати можуть бути використані для керування частиною енергії хвилі, яка конвертується у короткохвильову моду, та для запобігання суттєвого відбиття енергії хвилі назад до антени.