Интеллектуальное управление и системы

OBSERVABILITY ANALYSIS OF SPACECRAFTS' ATTITUDE MEASUREMENT SYSTEMS

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Одним из важных аспектов построения высокоточных систем управления космическими аппаратами (КА) является выбор такого бортового измерительного комплекса, который бы гарантировал наблюдаемость системы и не являлся бы избыточным. В работе проведен анализ наблюдаемости наиболее распространенных бортовых измерительных приборов космических аппаратов, а именно — магнитометра, построителя местной вертикали, астродатчика и датчиков угловых скоростей. Оценивание пригодности и качества различных по составу измерительных комплексов системы управления КА основано на существующих методах теории наблюдаемости динамических систем с учетом уравнений наблюдения и динамики углового движения КА. Исследования показали, что наиболее эффективным с точки зрения наблюдаемости системы КА, является измерительный комплекс, состоящий из астродатчиков.

Ключевые слова: оценивание состояния, наблюдаемость, кватернион, космический аппарат, магнитометр, астродатчик, построитель местной вертикали.

Одним з важливих аспектів побудови високоточних систем керування космічними апаратами (КА) є вибір такого бортового вимірювального комплексу, який би гарантував спостережуваність систем та не був би надлишковим. В роботі проведено аналіз спостережуваності найбільш поширених бортових вимірювальних пристроїв космічних апаратів, а саме — магнітометра, побудовувача місцевої вертикалі, астродатчика та датчиків кутових швидкостей. Оцінка придатності та якості різних за складом вимірювальних комплексів КА ґрунтується на наявних методах теорії спостережуваності динамічних систем з урахуванням рівнянь спостереження та динаміки кутового руху КА. Дослідження показали, що найбільш ефективним з точки зору спостережуваності є вимірювальний комплекс, який складається з астродатчиків.

Ключові слова: оцінювання стану, спостережуваність, кватерніон, космічний апарат, магнітометр, астродатчик, побудовувач місцевої вертикалі

INTRODUCTION

It is very important to choose a set of measurement equipment, which on the one hand is not excessive, and on the other — allows to estimate all attitude

parameters with required accuracy while designing precision spacecraft (SC) attitude control system (CS). The latter is achieved not only by proper selection of measurement system, but also by mathematical processing of measurement data, that ensures effective filtering of errors in the estimation of SC state parameters.

This article is focused on the problem of the minimization of onboard equipment, which is especially important task for small spacecrafts. Suitability and quality of different measurement systems is assessed using existing methods of dynamic systems observability theory based on observation equations and spacecraft's angular motion. Such approach to the determination of spacecraft's attitude (in case of incomplete measurements) is particularly considered in [1], where the vector of parameters that characterizes the attitude and angular velocity is determined using local geomagnetic field characteristics. These characteristics are obtained by calculation on measurements received from magnetometer with three orthogonal oriented probes without involvement of other measurements. Moreover, in [2] authors have shown the possibility to determine SC attitude using only two (arbitrary) out of three magnetometer probes. Evaluation of observability in both cases was carried out by dynamic filtering [3]. Sufficiently to show observability of the full attitude parameters vector in case when SC attitude quaternion, obtained from star sensor measurements, which is known in each point of SC trajectory. Determination of the attitude and angular velocity only on results of onboard measurements obtained from star sensor without involvement of other devices is regarded in [4]. Kalman filter, which allows to effectively evaluate both the quaternion attitude and angular velocity, was used in this research. Furthermore, article [4] provides other of publications devoted to the solution of the same problem by analogous as well as other possible methods.

PROBLEM STATEMENT

For observability evaluation of different measurement systems combinations we have used an approach which is slightly different from those mentioned above. The approach is based on the condition of solvability of nonlinear system of equations, which are obtained from observation equations with the use of Lyapunov differential operator [5].

We have considered not the full list of existing measuring sensors, but only those ones which are the most commonly used in practice [6]. In particular, the observability of various combinations of the following devices have been explored:

— Magnetometer with three orthogonally arranged magnetically sensitive probes;

— Star sensors or different astro-measurement systems [7, 8, 9];

— Sets of angular velocity sensors;

— Builder of the local vertical.

Since the most interest is focused on the combination of sensors, which provide incomplete direct or indirect information about the attitude parameters, we used the information on system dynamics, which is defined by the equations of SC angular motion. In addition, equations that describe the process of observation for the described above measuring systems are included.

This will be presented in the first two sections below. In the third section will

be described observability assessment procedures which are used in the study along with their adaptation to the mathematical models. Results for the observability of various combinations of measurement systems, the simplest possible estimator based on astro-sensor and conclusions resulting from the carried out analysis are presented in the final section.

MATHEMATICAL MODEL DESCRIPTION

Model of Spacecrafts' Angular Motion. Mathematical model of the SC angular motion may be written by using different coordinate systems and positional parameters of attitude (Krylov angles or normalized quaternion components). Establishment of observability or non-observability conditions does not depend on the choice of models, attitude parameters or coordinate system.

However, due to significant nonlinearity of these models, there may be some difficulties in analysis of the observability using approximation of the models. Thus, if in the kinematic equations Krylov angles are used, then under certain parameters a mathematical singularity occurs, but when someone moves to another description, for example, using a normalized quaternion, this feature is eliminated.

Nevertheless, preliminary studies have shown similar results for the parameters domain without singularity. Therefore, model with positional parameters as components of the quaternion is used in this paper. As coordinate systems we use the following orthogonal coordinate systems: SC related coordinate system (RCS) $Oxyz$ and certain inertial coordinate system (ICS) $Ox^u y^u z^u$.

Then, according to [10], under SC attitude we assume the orientation of RCS with respect to the orientation of ICS. We use the results of [11, 12, 13] and write the equation of angular motion of the spacecraft in case when angular coordinates are the components of the quaternion

$$\dot{\Lambda} = \frac{1}{2} B(\Lambda)\omega,$$
$$\dot{\omega} = J^{-1} m_u - J^{-1} \cdot \breve{\omega} J \omega + m_p,$$
(1)

where Λ is a quaternion with components $\Lambda^T = (\lambda_0, \lambda^T)$, $\lambda^T = (\lambda_1, \lambda_2, \lambda_3)$ which satisfies the normalization condition:

$$\|\Lambda(t)\|^2 = \lambda_0^2(t) + \lambda^T(t) \cdot \lambda(t) = 1,$$
(2)

vector $\omega = (\omega_1, \omega_2, \omega_3)^T$ is composed of the projections ω_i, $i = 1, 2, 3$ of absolute SC angular velocity on RCS axis, J — symmetric positive definite matrix $J = J^T > 0$, which is a representation of SC inertia tensor in RCS with the respect to the center O of RCS ; m_u is a vector of control moments, and m_p is a vector of disturbing moments, which are defined by projections on the axis of the RCS; matrix $B(\Lambda)$ of 4×3 dimension and $\breve{\omega}$ with 3×3 dimension have the following form:

$$\breve{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}, \; B(\Lambda) = \begin{bmatrix} -\lambda^T \\ \lambda_0 I_3 + \breve{\lambda} \end{bmatrix},$$

where I_3 is an identity matrix 3×3, $\breve{\lambda}$ — matrix which has the same representation as $\breve{\omega}$ but with $\lambda_1, \lambda_2, \lambda_3$ components.

More detailed description of model (1) characteristics along with the normalization condition (2) are described in [11, 12, 13].

Observation Equations. To write the observations equations for definite measurement systems, we need the vectors transfer matrix for transition from ICS to RCS, expressed in terms of the quaternion Λ. If v^u and v are column-vectors with components that are their projections on axis $Ox^u y^u z^u$ and $Oxyz$ respectfully, then according to [13, 14] we have

$$v^u = S(\Lambda)v, \tag{3}$$

where $S(\Lambda)$ is an orthogonal $\left(S^{-1}(\Lambda) = S^T(\Lambda), \; \det S(\Lambda) = 1 \right)$ 3×3 direction cosines matrix of the axes $Oxyz$ in $Ox^u y^u z^u$ which has the form

$$S(\Lambda) = I_3 - 2\lambda_0 \breve{\lambda} + 2\breve{\lambda}\breve{\lambda}, \tag{4}$$

with matrix $\breve{\lambda}$ having the described above representation.

MEASUREMENT SYSTEMS DESCRIPTION

Magnetometer. Magnetometer is certained to measure projections of the Earth magnetic field induction vector on three orthogonally arranged magnetic probes during the SC motion on the orbit. Typically, the magnetometer is made in a form of monoblock and is mounted on the SC so that the measurement axes are parallel to the axes of the base coordinate system (BCS), which may be the same as RCS or can be associated with it by (3) and (4). In this case the transition quaternion from the BCS to the RCS is strictly fixed, but under condition of flight may slightly be different from those that were installed on the Earth. For simplicity and due to the fact that the observability conditions are not affected by magnetometer disposition, we consider the case of coincidence of BCS and RCS.

Such magnetometer allows to determine positional parameters of the attitude: Krylov angles or transition quaternion from ICS to RCS. It is assumed that at each point of the orbit trajectory components of the magnetic field in the ICS are known. Moreover, to determine the attitude it is sufficient to know only the direction of that vector, i.e. its direction cosines or vector $b_0 = (\cos\alpha(t), \cos\beta(t), \cos\gamma(t))^T$ provided that $\cos^2\alpha(t) + \cos^2\beta(t) + \cos^2\gamma(t) = 1$. Then the equation of observation may be written as

$$y(t) = S(\Lambda(t)) \cdot b_0(t), \tag{5}$$

where $y(t)$ is a vector of direction cosines of the unit vector, which has the same direction as a vector of the measured magnetic field induction. The quaternion normalization condition (2) should be added to the (5) in order to determine current quaternion $\Lambda(t)$ from the corresponding nonlinear equation using calculated $b_0(t)$ and obtained from the measurements $y(t)$. However, as it is well known, this problem cannot be solved uniquely.

Astro-measurement system. Astro-measurement system is composed of one or more star sensors and each sensor in its field of vision registers n stars ($n \geq 2$). Usually astro-sensor measures angular coordinates of the axes of internal coordinate system (IntCS), which is directly tied to the sensor's line of sight in the ICS. The number of used astro-sensors does not affect the observability conditions. Therefore, we consider a single device, for which IntCS coincides with the RCS. We assume that as a result of observations of several space objects (stars, the Sun) and use of point algorithms described, for example, in [7, 15], on the output of the astro-measurement system we get a normalized quaternion $\Lambda(t)$, which contains some component-wise errors.

Angular Velocity Sensors. One of the methods of construction a complete set of the angular velocity sensors is a monoblock, comprising four identical fiber-optic gyroscopes, which allows the most effective estimation of all three components of SC angular velocity. Therefore, the equations of observation of this set are quite simple. They contain values of angular velocity $\omega(t)$ components measured with some error.

However, when estimating the observability, for the sake of research completeness, we will use individual components of the angular velocity.

Local vertical builder. On a number of satellites local vertical builder (LVB) is widely used as a position sensor, which effectively measures the projection of the Oy_u axis unit vector on Ox and Oz axis, while the projection on Oy axis is not measured. Then the observation equations that describes the process of measurements carried out with LVB, can be written as

$$y(t) = G_v S(\Lambda) v_0 + \zeta(t), \qquad (6)$$

where $y(t)$ is a measurement vector, v_0 is a vector with components $v_0 = (0, 1, 0)^T$, and $\zeta(t)$ is a vector of measurement error, G_v — matrix of 2×3 dimension written as follows

$$G_v = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

OBSERVABILITY CONDITIONS

The above equations of spacecraft angular motion and some observation equations are nonlinear. Obtaining the overall global observability conditions for such systems is a complex problem today. Therefore, the most appropriate and constructive approach to calculate the output time derivatives [5] is approximate

analysis using linearization procedure and local observability conditions. Consider a system described in state-space form as

$$\dot{x} = f(x, u, t),$$
$$y = h(x, u, t),$$
(7)

with state vector $x \in R^n$, measurement vector $y \in R^m$; f and h are continuous and differentiable as many times as required functions of their arguments — namely this is the case considered in the problem. In order to calculate (or measure) time derivative, we may write the local observability conditions. Let's write Lyapunov differential operator, defined on the system trajectory (7)

$$L[*] = \frac{\partial[*]}{\partial x} f + \frac{\partial[*]}{\partial u} \dot{u} + \frac{\partial[*]}{\partial t},$$
(8)

where $[*]$ denotes any differentiable vector-function of its arguments. As such function we take y written in the form of (7) and differentiate the observation equation with respect to of n-1 times. We get

$$y = h \quad \dot{y} = L[h] = Lh, \ldots, y^{(n-1)} = L^{n-1} \cdot h.$$
(9)

Then $x(t)$, which should be found according to the observability problem statement, is the solution of equations system (9). For the system (9) unique solvability in these conditions is necessary and sufficient that

$$rankD = rank\left[\left(\frac{\partial h}{\partial x}\right)^T \left(\frac{\partial}{\partial x} Lh\right)^T \cdots \left(\frac{\partial}{\partial x} L^{n-1}h\right)^T\right] = n,$$
(10)

where $\frac{\partial h}{\partial x}$, $\frac{\partial}{\partial x} Lh, \cdots$ are Yakobi matrices, which are calculated on the solution $x = x(t)$ with known input (control) $u(t)$. As a result, using rank criteria (10) we can consider only a particular motion observability. Therefore, assessing systems observability on the base of rank criteria (10) is not a constructive approach. If we have a linear system

$$\dot{x} = Ax + Bu,$$

with linear observation equations

$$y = Hx + Cu,$$

where A, B, C, H are constant matrices of corresponding dimensions, then criteria (10) given that

$$L[*] = \frac{\partial[*]}{\partial x}(Ax + Bu) + \frac{\partial[*]}{\partial u}\dot{u},$$

takes the well-known form

$$rank\left[H^T \;\vdots\; A^T H^T \;\vdots\; \left(A^T\right)^2 H^T \;\vdots\; \cdots \;\vdots\; \left(A^T\right)^{n-1} H^T\right] = n \,. \tag{11}$$

If matrices A, B, C, H are functions of time, then (10) will take the form

$$rank\left[\Pi_0^T \;\vdots\; \Pi_1^T \;\vdots\; \ldots \;\vdots\; \Pi_{n-1}^T\right] = n \,, \tag{12}$$

where $\Pi_0 = H(t), \Pi_k = \overset{\bullet}{\Pi_{k-1}} + \Pi_{k-1} \cdot A, \ k = \overline{1, n-1}$.

Each block in (11, 12) has $n \times m$ dimension (where m is dimension of vector y), and overall dimension of matrix in square brackets equals to $n \times n \cdot m$. In contrast to (10) the criterion (11) is global condition and gives full or partial (incomplete) observability of the system on all system trajectories.

Assume that functions $f(\cdot)$ and $h(\cdot)$ from (9) can be written as :

$$\begin{aligned} f(x, u, t) &= Ax + Bu + f^{NL}(x, u, t), \\ h(x, u, t) &= Hx + Cu + h^{NL}(x, u, t), \end{aligned} \tag{13}$$

where A, B, C, H are matrices of corresponding dimensions. Vector-functions f^{NL} and h^{NL} have nonlinearity with respect to variables x, u of at least second-order (it is assumed that $f(0, 0, t) = h(0, 0, t) = 0$). The expansion (13) may be carried out not only near zero, but in the neighborhood of any stationary state x^s, u^s, with the same properties of functions f^{NL} and h^{NL}. Observability matrix Γ provided (13) can be represented as follows

$$\Gamma = \left[\left(HA + D_1(x, u, t)\right)^T \;\vdots\; \left(HA^2 + D_2(x, u, t)\right)^T \;\vdots\; \cdots \;\vdots\; \left(HA^{n-1} + D_{n-1}(x, u, t)\right)^T\right], \tag{14}$$

where matrices D_i, $i = \overline{1, n-1}$ have at least linearly dependent from x, u and $D_i = 0$ providing that $x = u = 0$. Since expressions for $D_i = D_i(x, u, t)$ are cumbersome and are not used in future sections, we will not provide them here.

Assume that the linearized system is fully observed, i.e.

$$rank\Gamma_0 = rank\left[A^T H^T \;\vdots\; \left(A^T\right)^2 H^T \;\vdots\; \cdots \;\vdots\; \left(A^T\right)^{n-1} H^T\right] = n \,. \tag{15}$$

It means that among $n \cdot m$ columns of matrix Γ_0 there are such n columns, that square matrix determinant composed from these columns, will not be equal to zero. Let us construct matrix $\Gamma^* = \Gamma_0^* + D^*$ from these n columns, where D^* consists of the same columns as matrix Γ_0^*. According to the properties of matrix D_i, $D^* = 0$ while $x = u = 0$. Since matrix Γ_0^* is nonsingular, we may write

$$\Gamma^* = \Gamma_0^*\left(I_n + \left(\Gamma_0^*\right)^{-1} \cdot D^*\right).$$

According to the perturbation theory, matrix $I_n + \left(\Gamma_0^*\right)^{-1} \cdot D^*$ is nonsingular for any matrix norm with condition $\left\|\left(\Gamma_0^*\right)^{-1} \cdot D^*\right\| < 1$. The condition $\left\|\left(\Gamma_0^*\right)^{-1} \cdot D^*\right\| < 1$ may be achieved by using small values of x and u. Thus, full observability condition for all trajectories defined by linearized equations and which ensure fulfillment of $\left\|\left(\Gamma_0^*\right)^{-1} \cdot D^*\right\| < 1$ follows from (15).

Although the resulting observability condition using linear approximation is local, but it remains valid for a set of solutions near the equilibrium. If we cover the entire set of acceptable $x \in X$ and $u \in U$ with close enough stationary points x^s, u^s such that in the neighborhood of each of them the system is observable, and the intersection of sets defined by the condition $\left\|\left(\Gamma_0^*\right)^{-1} \cdot D^*\right\| < 1$ fully contain X and U, then we have sufficient observability condition of the original nonlinear system. If exist such x^s, u^s, where linear system is not fully observable, then in order to find precisely the border between observable and non-observable area more complex analysis may be required. In this case, instead of rank criteria of observability, we will consider condition number of observability matrices Γ_0^* ($cond\,\Gamma_0^*$). The concept of practical observability, based on the concept of practical rank of the matrix is introduced [16]. We consider the system to be practically observable, if the inverse value of observability matrix condition number is greater than or equal to a specified value ε. The value of ε is usually agreed with existing uncertainties in the measurements and motion equations. Than areas of X and U where $\chi\left(\Gamma_0^*\right) = cond\,\Gamma_0^* \leq \varepsilon^{-1}$ are practically observable. Singular value decomposition may be used to evaluate practical observability of a rectangular matrix Γ_0,

$$\Gamma_0 = P\Sigma Q^T ,\qquad (16)$$

where $P = [p_1, \ldots, p_2] \in R^{n \times n}$ and $Q = [q_1, \ldots, q_{n \cdot m}] \in R^{nm \times nm}$ are orthogonal matrices, i.e. $p_i^T \cdot p_j = \begin{cases} 1, & \text{if} \quad i = j \\ 0, & \text{if} \quad i \neq j \end{cases}$ and $q_i^T \cdot q_j = \begin{cases} 1, & \text{if} \quad i = j \\ 0, & \text{if} \quad i \neq j \end{cases}$, $R^{n \times n}$ and $R^{nm \times nm}$ are matrix spaces of $n \times n$ and $nm \times nm$ dimensions respectfully, $\Sigma = diag\left(\sigma_1, \ldots, \sigma_n\right)$ is a diagonal matrix of singular values arranged in descending order, i.e. $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \geq 0$. Singular values of matrix Γ_0 are exactly half-lines of hyper ellipsoid E defined by the ratio $E = \left\{\Gamma_0 x : \|x\|_2 = 1\right\}$.

Condition number of matrix Γ_0^* according to [16] for Euclidean norm (Frobenius norm) is given by

$$\chi_2\!\left(\Gamma_0^*\right)=\frac{\sigma_1}{\sigma_n}\,.$$

Condition number for different norms are equivalent in sense that there are such constants c_1 and c_2, that

$$c_1\chi_2 \le \chi_\alpha \le c_2\chi_2\,,$$

where α defines other norms. For example, for $\alpha=1: c_1=\dfrac{1}{n}, c_2=n.$

Based on condition number we make judgment about sensitivity of the estimation algorithm to errors in measurement data and to disturbances equations in motion. As $cond\,\Gamma_0^* \ge 1$, then the closer $\chi(\Gamma_0^*)$ to 1, the more effective noise filtering is achieved for any estimation algorithm. Therefore, when evaluating observability using various combinations of measurement sensors not only the rank of the observability matrices of the linear approximation will be assessed, but also their condition numbers. The consistency issue in condition number and available errors in the data and equations should be taken into account when estimation algorithms are developed.

OBSERVABILITY ANALYSIS OF DIFFERENT MEASUREMENT SYSTEMS

Equation of spacecraft angular motion and observation equation, that describe the processes of measurement may be expressed as (14), i.e with separated linear part. First, it is necessary to select equilibrium state defined by nonlinear system of equations (1). Set of all possible equilibrium states is defined by the condition

$$\left(\Lambda^E,\,\omega^E\right)^T=\left(\Lambda^E,\,0\right)^T,$$

i.e. any quaternion which satisfies the normalization condition with zero angular velocity components satisfy the equations of equilibrium. Quaternion Λ^E defines position of RCS with respect to ICS that is regarded as some initial state for the perturbed angular motion. Than normalized quaternion Λ will define position of RCS with respect to CCS for this perturbed motion. A purely formal relation $\Lambda=\Lambda^E+x^\Lambda$, where x^Λ is some four-dimensional vector $x^\Lambda=(x_0,x_1,x_2,x_3)$ (not a quaternion), which for small x^Λ may be considered as some estimation deviation of quaternion Λ^E from Λ. Wherein the following equation, arising from the normalization condition for Λ, must be fulfilled by:

$$\lambda_0^E x_0 + \lambda_1^E x_1 + \lambda_2^E x_2 + \lambda_3^E x_3 + \frac{1}{2}\!\left(x_0^2+x_1^2+x_2^2+x_3^2\right)=0\,. \tag{17}$$

If formally substitute $\Lambda=\Lambda^E+x^\Lambda$, $\omega=x^\omega$ into (1), then equation of perturbed angular motion near $\left(\Lambda^E,0\right)$ may be written as (13).

$$\dot{x} = \begin{pmatrix} \dot{x}^{\Lambda} \\ \dot{x}^{\omega} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{2} R(\Lambda^{E}) \cdot x^{\omega} \\ O_{33} \end{pmatrix} + \begin{pmatrix} O_{43} \\ J^{-1} \cdot m_{u} \end{pmatrix} + f^{NL}(x), \qquad (18)$$

where O_{33} and O_{43} are zero matrices of 3×3 and 4×3 dimension respectfully,

$f^{NL}(x) = \begin{pmatrix} \dfrac{1}{2} R(x^{\Lambda}) \cdot x^{\omega} \\ -J^{-1}\tilde{x}^{\omega} \cdot Jx^{\omega} \end{pmatrix}$ is quadratic vector-function. Equations (18) should be

considered together with (17).

Let us choose in four-dimension set $\left\{ \Lambda^{E} : \left\| \Lambda^{E} \right\| = 1 \right\}$ a points collection $\left\{ \Lambda_{i}^{E} \right\}$ in

the neighborhood of which the normalization condition is satisfied with acceptable error. The totality of these neighborhoods gives all set of feasible Λ. It is not difficult to be done, since set of all Λ is bounded. The partition should be done in such a way, that quadratic vector-functions $f^{NL}(x)$ in the neighborhood of points

Λ_{i}^{E} are small enough allowing to carry out practical observability estimation using

matrix Γ_{0} (16) i.e. the linear approximation.

In addition to the stationary states, equations (1) allow nonstationary motion, near which it is also possible to linearize the system and to evaluate its observability.

Assume that the pair $\left(\Lambda^{E}(t), \omega^{E}(t) \right)$ corresponds to some nominal motion, in the neighborhood of which equations of perturbed motion may be written by analogy with (18). Than similar to (12) observability matrix Γ may be constructed in the neighborhood of nominal motion. Also for nominal and perturbed motion corresponding representation of observation equations are written. Then, for each time point from (12) we may estimate complete state vector observability for a selected set of measurements, i.e. determine what areas of the trajectory are observable and which are not. Such estimation will be global for the selected nominal orbital motion. Approach similar to this was used in observability analysis in [1, 2]. However, observability analysis is offered to be carried out locally on time intervals where $\Lambda^{E}(t)$ and $\omega^{E}(t)$ vary so weakly that can be considered as stationary within these intervals. If, in addition to this, assume that $\omega^{E}(t)$ is small enough and includes linear terms, which corresponds to the product of ω^{E} and x^{Λ} components, into expression f_{NL} due to their small values, we will obtain (18) with more complex representation of f_{NL}. As a result, an observability analysis may be done for all valid nominal mode subsets, i.e all the feasible orbital trajectories in general.

Definitely, taking into account terms with ω^{E} in the linear part, that can improve observability conditions, but due to its small value, the condition number will not change significantly (and in the case of it infinity can become finite, but still big). In other words, unobservable cases in the proposed approach can become

poorly observable when taking into account ω^E, and estimation algorithms will become very sensitive to errors in measurement data.

Therefore, observability analysis using (18) for the given measurement system will be sufficient if on all feasible set $\left\{\Lambda_i^E\right\}$ full observability conditions are fulfilled. If subsets $\left\{\Lambda_i^E\right\}$ with no full observability or with big condition number exist, then trajectories which go through this subset can be unobservable with such values of parameters and corresponding time intervals. Though more rigorous evaluation of observability carried out with taking into account the non-stationarity may have a significant impact on the observability. However, spacecraft control system usually includes slow motion as well. Therefore, the proposed observability analysis is more general with respect to the different control system operation modes.

From the foregoing it follows that the observability estimations obtained using proposed approach are more suitable from the practical point of view, than those obtained on the basis of dynamic filtering.

Using the considered approach, the matrix will have the following form:

$$A = \begin{bmatrix} O_{44} & A_{43} \\ O_{34} & O_{33} \end{bmatrix},$$

where O_{33}, O_{34}, O_{44} are zero matrices of 3×3, 3×4, 4×4 dimensions respectfully, matrix A_{43} has the following representation

$$A_{43} = \begin{bmatrix} -\lambda_1^E/2 & -\lambda_2^E/2 & -\lambda_3^E/2 \\ \lambda_0^E/2 & -\lambda_3^E/2 & \lambda_2^E/2 \\ \lambda_3^E/2 & \lambda_0^E/2 & -\lambda_1^E/2 \\ -\lambda_2^E/2 & \lambda_1^E/2 & \lambda_0^E/2 \end{bmatrix}.$$

It is necessary to include the normalization condition (17) into the observation equation at any configuration of measuring system. For example, for magnetometer taking into account normalization condition in decomposition (12) matrix H is following

$$H = \begin{bmatrix} H_{44} & O_{43} \end{bmatrix}, \tag{19}$$

$$H_{44} = \begin{bmatrix} \lambda_3^E n_2 - \lambda_2^E n_3 & \lambda_2^E n_2 + \lambda_3^E n_3 & -2\lambda_2^E n_1 + \lambda_1^E n_2 - \lambda_0^E n_3 & -2\lambda_3^E n_1 + \lambda_0^E n_2 + \lambda_1^E n_3 \\ -\lambda_3^E n1 + \lambda_1^E n_3 & \lambda_2^E n_1 - 2\lambda_1^E n_2 + \lambda_0^E n_3 & \lambda_1^E n1 + \lambda_3^E n_3 & -\lambda_0^E n_1 - 2\lambda_3^E n_2 + \lambda_2^E n_3 \\ \lambda_2^E n1 - \lambda_1^E n_2 & \lambda_3^E n_1 - \lambda_0^E n_2 - 2\lambda_1^E n_3 & \lambda_0^E n_1 + \lambda_3^E n_2 - 2\lambda_2^E n_3 & \lambda_1^E n1 + \lambda_2^E n_2 \\ \lambda_0^E & \lambda_1^E & \lambda_2^E & \lambda_3^E \end{bmatrix},$$

$n_1 = \cos\alpha$, $n_2 = \cos\beta$, $n_3 = \cos\gamma$, and expression h_{NL}

$$h_{NL} = \begin{pmatrix} 2(-x_0\breve{x} + \breve{x}x)b_0 \\ \dfrac{1}{2}(x_0^2 + x^T x) \end{pmatrix},$$

where $x^{\Lambda} = \left(x_0, x^T\right)$, $x^T = \left(x_1, x_2, x_3\right)$ and $x^{\omega} = \left(x_4, x_5, x_6\right)^T$.

It should be noted that in fact we have $\alpha = \alpha(t)$, $\beta = \beta(t)$, $\gamma = \gamma(t)$. However, on the observation interval, where $\Lambda^E(t)$ and $\omega^E(t)$ are almost constant, direction of the magnetic field also varies a little and therefore may be considered under the problem solution as stationary related to some small part of the spacecraft trajectory. Taking into account an existing error between calculated magnetic field of Earth and its real value this is quite acceptable.

Matrix H and vector function h_{NL} are written analogously with other combinations of measuring devices.

In the observability analysis using the described approach, stationary points for Λ_i^E were determined based on the values of Krylov angles (γ, ψ, θ), which varied between

$$\gamma \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \quad \psi, \theta \in \left[-\pi, \pi\right].$$

Then according to [11] quaternion values in the corresponding points were defined by formulas

$$\lambda_0^E = \cos\frac{\gamma^E}{2}\cos\frac{\psi^E}{2}\cos\frac{\theta^E}{2} - \sin\frac{\gamma^E}{2}\sin\frac{\psi^E}{2}\sin\frac{\theta^E}{2},$$
$$\lambda_1^E = \sin\frac{\gamma^E}{2}\cos\frac{\psi^E}{2}\cos\frac{\theta^E}{2} - \cos\frac{\gamma^E}{2}\sin\frac{\psi^E}{2}\sin\frac{\theta^E}{2},$$
$$\lambda_2^E = \sin\frac{\gamma^E}{2}\cos\frac{\psi^E}{2}\sin\frac{\theta^E}{2} + \cos\frac{\gamma^E}{2}\sin\frac{\psi^E}{2}\cos\frac{\theta^E}{2}, \tag{20}$$
$$\lambda_3^E = \cos\frac{\gamma^E}{2}\cos\frac{\psi^E}{2}\sin\frac{\theta^E}{2} + \sin\frac{\gamma^E}{2}\sin\frac{\psi^E}{2}\cos\frac{\theta^E}{2},$$

Normalization conditions in this case are fulfilled automatically.

When the measurement complex include magnetometer, properties of matrix H in (19) are additionally analyzed depending on the orientation of the Earth's magnetic field with respect to the ICS. Computational experiments were conducted in order to determine such angles α, β, γ for which matrix condition number is the best and the worst. It should be noted that matrix H in (19) is either singular (rank less than 3) or very ill-conditioned. In the latter case the approximate normalization condition took effect under the linearization. This was confirmed by verifying matrix H singularity, which was built when the positional orientation parameters were Krylov angles (γ, ψ, θ). In this case, the rank of the matrix H for all α, β, γ was less than three, and the system was unobservable at steady approximation. While using magnetometer in combination with other measurements the worst and the best orientation of the magnetic field with respect to an inertial coordinate system were determined based on the condition number.

Moreover, for parameter (γ, ψ, θ) without singularity in kinematic equations based on Krylov angles, comparison of observability results for two models was

conducted: using Krylov angles and using (17), (18). Results turned out to be similar, which confirmed the validity of the formalism (17), (18).

For different combinations of measurement equipment matrices Γ_0 were formed. Using the SVD-transformation, matrix Γ_0^* was separated from matrix Γ_0 and condition number $\chi_2\left(\Gamma_0^*\right)$ was calculated. Including astro-sensor into the system in addition to other measurement equipment led to Γ_0^* be mainly formed from corresponding to astro-sensor columns. Wherein the condition number of matrix Γ_0^* for astro-sensor was equal to 2 and was the best in relation to all other possible combinations. Therefore, astro-measurement system gave the best observability result due to conditionality stability of the inverse problem.

From all other measurement sensors, different configurations of measurement complex were formed and presented in table 1. There are 31 such possible combinations. "1" denotes that corresponding sensor is included into the configuration, «-» denotes that the sensor is absent. Observability results for all shown in table 1 configurations are presented in table 2 for different equilibrium values Λ_i^E in points of set $\left\|\Lambda^E\right\| = 1$. In table 2 "+" and "-" denote that the system is observable or not observable respectively, "+/-" denotes that such combination of measurement equipment is not always observable.

Table 1

Possible Configurations of Measurement Systems

№	Magnetometer	Angular velocity sensor 1	Angular velocity sensor 2	Angular velocity sensor 3	Local vertical builder	Astromeasurement system	№	Magnetometer	Angular velocity sensor 1	Angular velocity sensor 2	Angular velocity sensor 3	Local vertical builder	Astromeasurement system
0	-	-	-	-	-	1	16	1	-	-	-	-	-
1	-	-	-	-	1	-	17	1	-	-	-	1	-
2	-	-	-	1	-	-	18	1	-	-	1	-	-
3	-	-	-	1	1	-	19	1	-	-	1	1	-
4	-	-	1	-	-	-	20	1	-	1	-	-	-
5	-	-	1	-	1	-	21	1	-	1	-	1	-
6	-	-	1	1	-	-	22	1	-	1	1	-	-
7	-	-	1	1	1	-	23	1	-	1	1	1	-
8	-	1	-	-	-	-	24	1	1	-	-	-	-
9	-	1	-	-	1	-	25	1	1	-	-	1	-
10	-	1	-	1	-	-	26	1	1	-	1	-	-
11	-	1	-	1	1	-	27	1	1	-	1	1	-
12	-	1	1	-	-	-	28	1	1	1	-	-	-
13	-	1	1	-	1	-	29	1	1	1	-	1	-
14	-	1	1	1	-	-	30	1	1	1	1	-	-
15	-	1	1	1	1	-	31	1	1	1	1	1	-

Rank of the matrix Γ_0^* as well as minimum and maximum values of the condition number are reported. The value $\inf$ corresponds to an infinite condition number, i.e. unobservable state. Configuration of measurement sensors that ensure full observability, have rank 7. This holds for numbers 17, 19, 21, 23, 25–31. There is no full observability in all other cases.

Table 2

Observability Analysis of the Measurement Systems Configurations

№	Rank of matrix Γ_0	Observability of the system	Condition number	№	Rank of matrix Γ_0	Observability of the system	Condition number	№	Rank of matrix Γ_0	Observability of the system	Condition number
0	7	Yes	2	11	4-7	Not always	10^{11}- inf	22	6-7	Not always	10^{11}- 10^{17}
1	3-6	No	inf	12	2-3	No	inf	23	7	Yes	2.32-67
2	1-2	No	inf	13	5-7	Not always	10^{11}- 10^{39}	24	6-7	Not always	10^{11}- 10^{17}
3	4-6	No	inf	14	3	No	inf	25	7	Yes	3.05-111
4	1-2	No	inf	15	5-7	Not always	10^{11}- 10^{27}	26	7	Yes	10^{11}- 10^{13}
5	4-7	Not always	10^{12}- inf	16	6-7	Not always	10^{11}- 10^{17}	27	7	Yes	2.42-67
6	3	No	Inf	17	7	Yes	3.72-122	28	7	Yes	10^{11}- 10^{13}
7	4-7	Not always	10^{11}- inf	18	6-7	Not always	10^{11}- 10^{17}	29	7	Yes	2.41-73
8	2	No	inf	19	7	Yes	3.10-67	30	7	Yes	10^{11}- 10^{12}
9	4-6	No	inf	20	6-7	Not always	10^{11}- 10^{17}	31	7	Yes	2.24-67
10	2-3	No	inf	21	7	Yes	2.61-79				

STATE ESTIMATOR USING ASTRO-SENSORS

In the previous section, it was found out that the best observability conditions has astro-measurement system even without involvement of other equipment. The output of this system are quaternion components with discrete time step, therefore, it is possible to correctly estimate their derivative, using regularizing operator. In the simplest case regularizing operator for the derivative calculation of a function given approximately, has the following form

$$R(y') = \frac{y_k - y_{k-1}}{\Delta},\qquad(21)$$

where Δ is mesh width of function y discretization, error which is selected in accord with the function y error. If Δ is more than astro-sensor quantization step, than regularizing operator can be composed based on more than two points k and $k-1$. When Δ is smaller than the pitch of the signal entering astro-measurement system, it is advisable to improve the accuracy of estimation by using two or more astro-measurement systems.

Then astro-sensors are querying with some time offset, it is possible to obtain data with a suitable for (22) value of Δ and get stable value of the derivative. For definiteness we assume that (21) gives acceptable approximation of the derivative. Then the first equation in (1) may be used to estimate the vector ω, in which $\dot{\Lambda}$ is replaced by a regularizing operator, evaluated in accordance to (21). We get the overdetermined system for calculation of ω_{k-1}. To solve it we use Least Square Method and get

$$\omega_{1,k-1} = \lambda_{0,k-1} \cdot R_1 - \lambda_{1,k-1} \cdot R_0 - \lambda_{2,k-1} \cdot R_3 + \lambda_{3,k-1} \cdot R_2 \,,$$
$$\omega_{2,k-1} = \lambda_{0,k-1} \cdot R_2 + \lambda_{1,k-1} \cdot R_3 - \lambda_{2,k-1} \cdot R_0 - \lambda_{3,k-1} \cdot R_1 \,, \qquad (22)$$
$$\omega_{3,k-1} = \lambda_{0,k-1} \cdot R_3 - \lambda_{1,k-1} \cdot R_2 + \lambda_{2,k-1} \cdot R_1 - \lambda_{3,k-1} \cdot R_0 \,,$$

where R_0 is a regularizing operator for calculating derivative of λ_0 component, and R_1, R_2, R_3 are regularizing operators of component-vector λ.

Considering (22) as initial condition for vector ω at point $k-1$ and using the second equation in (1) it is possible to predict its value in the required for the control point (e.g., point k or $k+1$). It is better to do this from a discrete predictor, which is obtained from the second equation of (1) in the form

$$\omega_k = \omega_{k-1} + \Delta J^{-1}\big(m_{u,k-1} - \breve{\omega}_{k-1} J \omega_{k-1}\big). \qquad (23)$$

We note here that measuring quaternion with less error allows to reduce the parameter Δ and regularizing operator (21) more accurately approximates the derivative. Reducing measurement errors may be achieved by applying various filtering or averaging procedures when a large number of measurement data is used.

CONCLUSIONS

As it was expected, magnetometer with three orthogonal magnetically sensitive probes does not ensure practical observability. Adding local vertical builder solves the observability problem with acceptable condition number. According to the condition number analysis, there is no significant improvement in the properties of estimator as a filter, if angular velocity sensors are added to the system.

The most effective observability is ensured by astro-measurement system. It is characterized by close to an absolute minimum (one) condition number. Moreover, the simplest state estimator, which is described in this article, may be build using astro-sensors.

According to the research carried out in this article, spacecraft attitude parameters estimator should be constructed using only position measurements — Krylov angles or quaternions.

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OBSERVABILITY ANALYSIS OF SPACECRAFTS' ATTITUDE MEASUREMENT SYSTEMS

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Introduction. One of the important tasks for small spacecrafts is the optimization of onboard measurement equipment, which on the one hand is not excessive and on the other — allows to estimate all attitude parameters with required accuracy.

The purpose of the article is to conduct observability analysis of the most commonly used measurement systems, such as magnetometer, star and angular

velocity sensors, local vertical builder in order to identify the minimum required set of onboard measurement equipment, which ensures observability of the spacecraft.

Approach and Methods. Measurement systems observability assessment utilizes existing methods of dynamic systems observability theory and is based on observation and spacecraft's angular motion equations. Model of the spacecraft's motion is described using quaternion components as positional parameters. Since the models are essentially nonlinear, obtaining the overall global observability conditions for such system is a complex problem. Therefore, linearization procedure is applied and local observability conditions are assessed based on the rank and condition numbers of observability matrices of the linear approximation.

Results. Astro-measurement system ensures the most effective observability and may be used as the simplest measurement system. Magnetometer with three orthogonal magnetically sensitive probes does not ensure practical observability of the system, unless local vertical builder is added.

Keywords: State estimation, observability, quarternion, spacecraft, magnetometer, star sensor, local vertical builder.

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