ANALYTICAL CALCULATIONS OF THE ANGLES OF ROTATIONAL TRANSFORM SPECIFIED BY DIFFERENT PLASMA PRESSURE PROFILES AND AN EXTERNAL TRANSVERSE MAGNETIC FIELD IN THE TORSATRON

Yu.K. Kuznetsov, I.B.Pinos, V.I.Tyupa
Institute of Plasma Physics, National Science Center "Kharkov Institute of Physics and Technology", 1 Akademicheskaya St., Kharkov 61108, Ukraine

The authors of the present work have calculated the angles specified by various plasma pressure profiles depending on the parameter α that characterizes the profile of vacuum angles of rotation angles and on the magnetic axis displacement caused by an external transverse magnetic field. The authors have considered three laws of plasma pressure distribution over vacuum magnetic surfaces: $P_1=P_0$, $P_2=P_0(1-\psi(r)/\psi(r_o))$; $P_3=P_0(1-\psi(r)/\psi(r_o))^2$. The calculations showed, that in the case of magnetic axis shifting into the inside of the torus and at low values of α, the rotational transformation angle of a torsatron decreases up to zero with plasma pressure increasing. In this case, the splitting of magnetic surfaces caused by pressure distributions $P_3=P_0(1-\psi(r)/\psi(r_o))^2$ takes place in the central region of a magnetic configuration. As plasma pressure distributed by the law $P_1=P_0$ increase the splitting of magnetic surfaces due to the helical winding perturbation only occurs.

PACS: 52.55.Hc

It is known that the correction of magnetic axis position, the rise in the limiting equilibrium pressure of plasma, and also variations in such integral characteristics as the angle of rotational transform and the average magnetic well, can be realized with the help of an external transverse magnetic field [1-3]. The authors of the present work have calculated the angles specified by various plasma pressure profiles versus the parameter α that characterizes the profile of vacuum angles of rotation, and also versus the magnetic axis displacement caused by an external transverse magnetic field. The authors have considered three laws of plasma pressure distribution over vacuum magnetic surfaces: $P=P_0$, $P=P_0(1-\psi(r)/\psi(r_o))$; $P=P_0(1-\psi(r)/\psi(r_o))^2$; where P_0 is the plasma pressure on the axis of the system: $\psi(r)$ is the averaged function of vacuum magnetic surfaces. The distribution of vacuum angle of rotational transform was calculated as $t(r)=t(r_0)[\alpha+(1-\alpha)r^2/r_o^2]$, where $-\alpha= t(0)/t(r_0)$ is the ratio of the angle of rotation of field lines on the magnetic axis to the one on the plasma boundary of radius r_o.

The equations of averaged magnetic surfaces in the cylindrical system of coordinates with due account for the external transverse magnetic field have the following forms [4].

For the pressure distribution $P=P_0$:

$$\Psi(r) = \frac{\alpha}{2}\frac{r^2}{r_0^2} + \frac{1-\alpha}{4}\frac{r^4}{r_0^4} + M_\perp\frac{r}{r_0}\cos\theta - N\frac{r}{r_0}\cos\theta \tag{1}$$

For the pressure distribution $P=P_0(1-\psi(r)/\psi(r_o))$

$$\Psi(r) = \frac{\alpha}{2}\frac{r^2}{r_0^2} + \frac{1-\alpha}{4}\frac{r^4}{r_0^4} + M_\perp\frac{r}{r_0}\cos\theta - \frac{N}{1+\alpha}\left(1-\frac{r^2}{2r_0^2}\right)\frac{r}{r_0}\cos\theta \tag{2}$$

For $P=Po(1-\psi(r)/\psi(r_o))^2$

$$\Psi(r) = \frac{\alpha}{2}\frac{r^2}{r_0^2} + \frac{1-\alpha}{4}\frac{r^4}{r_0^4} + M_\perp\frac{r}{r_0}\cos\theta - \frac{N}{1+\alpha}\left(\frac{2}{3}\frac{2+\alpha}{1+\alpha} - \frac{r^2}{r_0^2}\frac{2}{3(1+\alpha)}\left(\alpha\frac{r^4}{r_0^4} + \frac{1-\alpha}{4}\frac{r^6}{r_0^6}\right)\right)\frac{r}{r_0}\cos\theta \tag{3}$$

$$M_\perp = \frac{H_\perp A}{H_0 t(r_0)};\quad N = \frac{\beta A}{t^2(r_0)} \tag{3}$$

$A=R/r_o$ is the aspect ratio, R is the major radius of the torus, $\frac{H_\perp}{H_0}$ is the ratio of the external homogeneous transverse magnetic field $H_\perp$ to the longitudinal field H_0.

The analytical formulas of the angles of rotational transform for different plasma pressure profiles were calculated by formulas of averaging over magnetic surfaces [5,6,7]. For the distribution $P=P_0$ and $Po=P(1-\psi(r)/\psi(r_o))$, the angle of rotational transform is given by the following expression:

$$\frac{< t(r_1) >}{t(r_0)} = a\sqrt{(1+c)^2 - b^2}\sqrt{1-\frac{4c(1+c)}{b^2}\left(1-\sqrt{1-b^2/(1+c)^2}\right)}, \tag{4}$$

a, b, c are expressed for the pressure distribution of $P=P_0$

$$a = \alpha + (1-\alpha)\frac{r_1^2}{r_o^2} + 2(1-\alpha)\frac{r_c^2}{r_o^2},\quad b = \frac{3(1-\alpha)}{a}\frac{r_c r_1}{r_o^2},\quad c = \frac{1-\alpha}{a}\frac{r_c^2}{r_o^2} \tag{5}$$

For the distribution $Po=P(1-\psi(r)/\psi(r_o))$ these factors have the following forms

$$a = \alpha + (1-\alpha)\frac{r_1^2}{r_o^2} + 2(1-\alpha)\frac{r_1^2}{r_o^2} - 2\frac{r_c}{r_o}\frac{(1-\alpha)\left(\frac{r_{c\perp}^3}{r_o^3}-\frac{r_c^3}{r_o^3}\right)+\alpha\left(\frac{r_{c\perp}}{r_o}-\frac{r_c}{r_o}\right)}{1-3r_c^2/2r_o^2};$$

$$b = \frac{1}{a}\left(-3(1-\alpha)\frac{r_c}{r_o}\frac{r_1}{r_o} + \frac{3}{2}\frac{r_1}{r_o}\frac{(1-\alpha)\left(\frac{r_{c\perp}^3}{r_o^3}-\frac{r_c^3}{r_o^3}\right)+\alpha\left(\frac{r_{c\perp}}{r_o}-\frac{r_c}{r_o}\right)}{1-3r_c^2/2r_o^2}\right)$$

$$c = \frac{1}{a}\left((1-\alpha)\frac{r_c^2}{r_o^2} - \frac{r_c}{r_o}\frac{(1-\alpha)\left(\frac{r_{c\perp}^3}{r_o^3}-\frac{r_c^3}{r_o^3}\right)+\alpha\left(\frac{r_{c\perp}}{r_o}-\frac{r_c}{r_o}\right)}{1-3r_c^2/2r_o^2}\right) \tag{6}$$

where $r_{c\perp}$ is the initial displacement caused by the external transverse magnetic field.

For the plasma pressure distribution $P=P_0\,(1-\psi(r)/\psi_{(r0)})^2$, the angle of rotational transform was calculated as

$$\frac{<t(r_1)>}{t(r_o)}=\frac{aa_1}{\frac{1}{\sqrt{p+2\sqrt{q}}}\left(A+\frac{B}{\sqrt{q}}\right)+\frac{1}{\sqrt{p_1+2\sqrt{q_1}}}\left(C+\frac{D}{\sqrt{q_1}}\right)},\qquad(7)$$

where

$$D=\frac{q_1+\frac{p_1}{q_1}+\frac{q-q_1}{p-p_1}\left(3-p_1-\frac{1}{q_1}\right)-3}{\frac{q-q_1}{p-p_1}\left(1-\frac{q}{q_1}\right)-p+q\frac{p_1}{q_1}},$$

$$A=\frac{D\left(1-\frac{q}{q_1}\right)-3+p+\frac{1}{q_1}}{p-p_1},\quad B=\frac{1-qD}{q_1},$$

$$C=\frac{3-p_1-\frac{1}{q_1}-D\left(1-\frac{q}{q_1}\right)}{p-p_1},$$

where

$$p=\frac{b_1+\sqrt{8y+b_1^2-4c_1}}{2},\quad q=y+\frac{b_1y-d_1}{\sqrt{8y+b_1^2-4c_1}},$$

$$p_1=\frac{b_1-\sqrt{8y+b_1^2-4c_1}}{2},\quad q_1=y-\frac{b_1y-d_1}{\sqrt{8y+b_1^2-4c_1}},$$

$$y=\sqrt[3]{-q_o+\sqrt{E_o}}+\sqrt[3]{-q_o-\sqrt{E_o}}+\frac{c_1}{6},\quad E_o=q_o^2+p_o^3,$$

where

$$p_o=\frac{b_1d_1-4e_1}{12}-\frac{c_1^2}{36},$$

$$q_o=-\frac{c_1^3}{216}+\frac{c_1b_1d_1}{48}-\frac{e_1b_1^2+d_1^2}{16}+\frac{e_1c_1}{6},$$

$$a_1=1-b+c-d+e,$$

$$b_1=\frac{4-2b-4c+14d-28e}{a_1},$$

$$c_1=\frac{6-10c+70e}{a_1}$$

$$d_1=\frac{4+2b-4c-14d-28e}{a_1}$$

$$e_1=\frac{1+b+c+d+e}{a_1},$$

$$\begin{aligned}a&=\alpha+(1-\alpha)r_1^2/r_o^2+2(1-\alpha)r_c^2/r_o^2+\\&2Nr_c/r_o\{-2+6M\alpha\,r_c^2/r_o^2+3M(1-\alpha)r_c^4/r_o^4+\\&3M[2\alpha+3(1-\alpha)r_c^2/r_o^2]r_1^2/r_o^2+3M(1-\alpha)r_1^4/r_o^4\},\\b&=r_1/ar_o\{3(1-\alpha)r_c/r_o-N[3(-1+9M\alpha\,r_c^2/r_o^2\\&+15/2M(1-\alpha)r_c^4/r_o^4)+\\&5M[\alpha+9/2(1-\alpha)r_c^2/r_o^2]r_1^2/r_o^2+7/4(1-\alpha)Mr_1^4/r_o^4]\},\\c&=r_c/ar_o\{(1-\alpha)r_c/r_o+N[-2+8\alpha\,Mr_c^2/r_o^2+\\&9/2M(1-\alpha)r_c^4/r_o^4+\\&4M[2\alpha+4(1-\alpha)r_c^2r_o^2]r_1^2/r_o^2+9/2(1-\alpha)Mr_1^4/r_o^4]\},\\d&=-\frac{3MNr_c^2r_1}{ar_o^3}\left[\alpha+\frac{5}{4}(1-\alpha)\frac{r_c^2}{r_o^2}+\frac{5}{4}(1-\alpha)\frac{r_1^2}{r_o^2}\right],\\e&=+\frac{MN(1-\alpha)}{a}\frac{r_c^3r_1^2}{r_o^5},\end{aligned}$$

$$N=\frac{\alpha(1+\alpha)\left(\frac{r_{c\perp}}{r_o}-\frac{r_c}{r_o}\right)+(1+\alpha^2)\left(\frac{r_{c\perp}^3}{r_o^3}-\frac{r_c^3}{r_o^3}\right)}{\frac{2}{3}\frac{2+\alpha}{1+\alpha}-3\frac{r_c^2}{r_o^2}+M\left[5\alpha\frac{r_c^4}{r_o^4}+\frac{7}{4}(1-\alpha)\frac{r_c^6}{r_o^6}\right]}=\qquad(8)$$

$$=\frac{\beta A_o}{t^2(r_o)(1+\alpha)},\quad M=\frac{2}{3(1+\alpha)},$$

In calculations it was supposed that the magnetic axis displacement r_c/r_o occurs on the beam $\theta=\pi$ directed inward the torus.

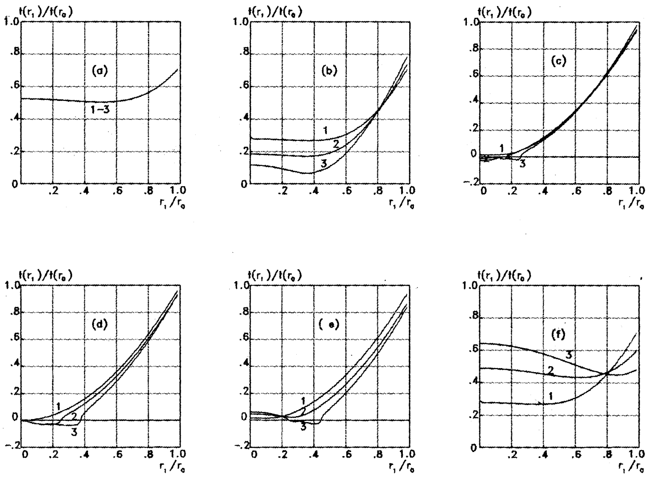
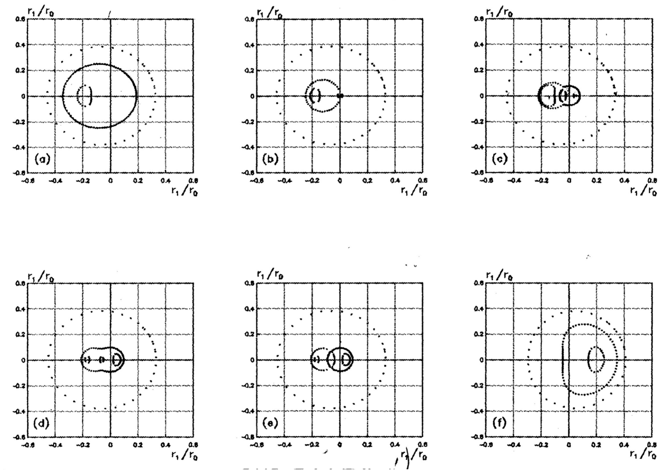


Fig. 1. Distribution of angles of rotational transform versus different plasma pressure profiles R/ro=8.9

1 - $P=P_0$; 2 - $P=P_0(1-\psi(r)/\psi(r_0))$;



3 - $P=P_0(1-\psi(r)/\psi(r_o)^2)$;(a- $\frac{r_c}{r_0}=0.5$; b- $\frac{r_c}{r_0}=0.2$; c- $\frac{r_c}{r_0}=0$; d- $\frac{r_c}{r_0}=-0.1$; e- $\frac{r_c}{r_0}=-0.2$; f- $\frac{r_c}{r_0}=-0.4$)

Fig.2. The shapes of magnetic surfaces specified by the pressure distribution $P=P_0(1-\psi(r)/\psi(r_o))$ $R/r_0=8.9$

(a- $\frac{r_c}{r_0}=0.2$; b- $\frac{r_c}{r_0}=0.05$; c- $\frac{r_c}{r_0}=0$; d- $\frac{r_c}{r_0}=-0.07$; e- $\frac{r_c}{r_0}=-0.08$; f- $\frac{r_c}{r_0}=-0.2$)

Fig. 1 shows the angles of rotational transform due to different pressure profiles as functions of the magnetic axis displacement. It can be seen from the figure that for small magnetic-axis displacements and small α values, on certain radii r_1/r_o, the angle of rotational transform turns to zero (Figs.

1c, d, e). The magnetic axis was originally displaced by the transverse magnetic field to the inside of the torus by $\frac{r_{c\perp}}{r_0} = 0.5$.
The sign "minus" at r_c/r_o means that the magnetic axis displacement is on the beam θ =0 directed to the outside of the torus. Fig. 2 shows the magnetic surface shapes determined by the pressure profile Po=P $(1-\psi(r)/\psi(r_o))$ versus the magnetic axis displacement. It is obvious from the figure that for magnetic axis displacement r_c/r_o=0 the splitting of magnetic surfaces takes place.
A flat distribution of plasma pressure P = P_0 does not cause the magnetic surfaces to split (Fig. 3). The magnetic surfaces in this case will be split only due to the perturbation caused by a helical winding [7]. A similar splitting of magnetic surfaces was observed by Shishkin [8] when investigating the plasma equilibrium of the l=3 stellarator at correction by an external transverse magnetic field.
The condition of magnetic surface splitting for the pressure distribution Po=P $(1-\psi(r)/\psi(r_o))$ is

$$N > \frac{2}{3}\sqrt{\alpha(1+\alpha)(1-\alpha^2)},\ N = \frac{\beta A}{\iota^2(r_0)} \qquad (9)$$

In the case of P=P_0 $(1-\psi(r)/\psi(r_o)^2)$, the condition of splitting is the following expression:

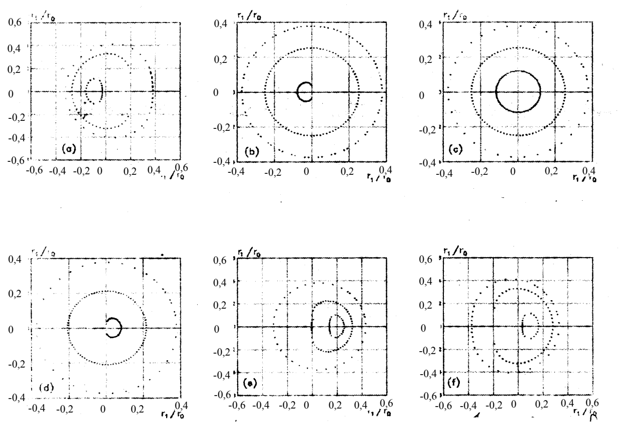


Fig. 3. The shapes of magnetic surfaces specified by the pressure distribution P=P_0 R/r_0 =8.9

(a- $\frac{r_c}{r_0} = 0.3$; b- $\frac{r_c}{r_0} = 0.05$; c- $\frac{r_c}{r_0} = 0$; d- $\frac{r_c}{r_0} = -0.05$; e- $\frac{r_c}{r_0} = -0.2$; f- $\frac{r_c}{r_0} = -0.3$)

$$N > \sqrt{\frac{1+\alpha}{1920\alpha}}\sqrt{\left(13\alpha^4+414\alpha^2-27\right)+\sqrt{\left(13\alpha^4+414\alpha^2-27\right)^2+34560\alpha^2\left(1-\alpha^2\right)^3}} \qquad (10)$$

In the case of magnetic axis displacement to the outside of the torus, there occurs an increase in the angle of rotational transform, caused by both the external transverse magnetic field and the pressure of plasma.
Thus, it has been demonstrated here that in the case of magnetic axis displacement to the inside of the torus and small α values, the angle of rotational transform decreases down to zero on certain radii of magnetic surfaces as the plasma pressure increases. In this case, in the central region of the magnetic configuration there occurs magnetic surface splitting determined by the distributions P=P_0 $(1-\psi(r)/\psi(r_o))$ and P=P_0 $(1-\psi(r)/\psi(r_o)^2)$. The conditions of magnetic surfaces splitting specified by plasma pressure distributions in the radius have been found here.

REFERENCES

1. V.N. Pyatov, V.P. Sebko, V.I. Tyupa// *Preprint KFTI 76-25, Kharkov*, 1976 (in Russian).
2. Yu. K. Kuznetsov, I.B .Pinos, V.I. Tyupa//*IAEA Techn. Comm. Meeting 8th Stellarator Workshop,* Kharkov, USSR, IAEA, Vienna 317 (1991).
3. Yu.K. Kuznetsov, I.B. Pinos, V.I. Tyupa// *Preprint KFTI 92-31*, Kharkov, 1992 (in Russian).
4. I.S. Danilkin, L.M. Kovrizhnykh, S.V. Shchepetov *Preprint FIAN 75,* Moscow, 1981(in Russian).
5. Yu.K. Kuznetsov, I.B. Pinos, V.I. Tyupa// *Problems of Atomic Science and Technology, Series: Plasma Physics*. 1999,ISSUES 1 (1), 2 (2), p. 52-54.
6. L.S. Solovyov, V.D. Shafranov// *Vopr. Teor. Plazmy*, Moscow, Gosatomizdat, 1967, v.5, pp. 3-208.
7. V.V. Nemov//*Preprint KFTI 75-28*, Kharkov, 1975 (in Russian).
8. V.N. Pyatov, A.A. Shishkin// *Nucl. Fusion.* (16). 1976, №6, p. 937-941.

АНАЛІТІЧНІ РОЗРАХУНКИ КУТІВ ОБЕРТАЛЬНОГО ПЕРЕТВОРЕННЯ, ЗУМОВЛЕНІ РІЗНИМИ РОЗПОДІЛАМИ ТИСКУ ПЛАЗМИ ТА ЗОВНІШНІМ ПОПЕРЕЧНИМ МАГНІТНИМ ПОЛЕМ У ТОРСА ТРОНІ

Ю.К. Кузнєцов, І.Б. Пінос, В.І. Тюпа

Авторами роботи виконані розрахунки кутів обертального перетворення, зумовлених різними розподілами тиску плазми в залежності від параметра α, характеризую чого розподіл вакуумних кутів повороту силових ліній, а також від зміщення магнітної осі, викликаного зовнішнім поперечним магнітним полем. Авторами роботи розглянуто три закони розподілу тиску плазми по вакуумним магнітним поверхням P_1=P_o; P_2=P$(1-\psi(r)/\psi(r_o))$; P_3=$P_o$$(1-\psi(r)/\psi(r_o))^2$. Розрахунки показали, що у випадку зміщення магнітної осі у середину тора та малих α кути обертального перетворення можуть обертатися в нуль на певних радіусах та викликати розщеплення магнітних поверхонь. Розподіл тиску плазми P_1=P_o не викликає розщеплення магнітних поверхонь. Магнітні поверхні у цьому випадку розщепляються тільки за рахунок збурень, зумовлених гвинтовою обмоткою.

АНАЛИТИЧЕСКИЕ РАСЧЕТЫ УГЛОВ ВРАЩАТЕЛЬНОГО ПРЕОБРАЗОВАНИЯ, ОБУСЛОВЛЕННЫХ РАЗЛИЧНЫМИ ПРОФИЛЯМИ ДАВЛЕНИЯ ПЛАЗМЫ И ВНЕШНИМ ПОПЕРЕЧНЫМ МАГНИТНЫМ ПОЛЕМ В ТОРСАТРОНЕ

Ю.К. Кузнецов, И.Б. Пинос, В.И. Тюпа

Авторами работы выполнены расчеты углов, обусловленных различными профилями давления плазмы в зависимости от параметра α, характеризующего профиль вакуумных углов поворота, а также от смещения магнитной оси, вызванного внешним поперечным магнитным полем. Рассмотрены три закона распределения давления плазмы по вакуумным

магнитным поверхностям: $P_1=P_o$; $P_2=P(1-\psi(r)/\psi(r_o))$; $P_3=P_o(1-\psi(r)/\psi(r_o))^2$. Показано, что распределение давления плазмы $P_2=P(1-\psi(r)/\psi(r_o))$ и $P_3=P_o(1-\psi(r)/\psi(r_o))^2$ вызывают расщепление магнитных поверхностей при $r_c/r_o=0$. Пологое распределение давления плазмы $P_1=P_o$ не вызывает расщепления магнитных поверхностей. Магнитные поверхности в этом случае будут расщепляться только за счет возмущения, вызванного винтовой обмоткой.