ABOUT ELECTRON ENERGY INCREASE IN DISCHARGE WITH THE NON-SINUSOIDAL FIELD

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It is shown that the energy of electrons increases in the case of nonzero value of power spectral density at resonant frequencies irrespective of field strength jumps presence or absence. It is noted the inevitability of the collisional heating and its considerable contribution to the development of ionization.

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INTRODUCTION

It seems, that to have maximal electron velocity in the bounded region one has to accelerate the electron with maximum force up to the middle of the region and then change force direction with the opposite one, and the sinusoidal time dependence of force gives only somewhat less velocity, but it is much more accessible. But recently the investigations of low-pressure gas discharge in non-sinusoidal field are developed [1], in which great attention is paid to jumps of phase, and there are references to the paper [2], where a stochastic field action on plasma was considered. The paper [2] ideas were developed in the paper [3] for ionosphere heating by electromagnetic radiation. In the present work the application of the papers [2, 3] results to gas discharge is considered.

1. RESONANT FREQUENCIES ROLE

In the papers [2, 3], it was obtained electron energy increase in stochastic field. In [2] the field of waves in plasma (circularly polarized along constant magnetic field and the longitudinal waves in isotropic plasma) changes electron distribution function and dumps itself. In [3], the field of electromagnetic wave acts on probe electron. In both cases, in linear approximation, it is found perturbation of distribution function or electron velocity in the field of wave, and the rate of slow change of relevant characteristics is obtained with use of averaging over realizations of stochastic process. The main steps of derivation may be shown for the case of one probe electron in the constant magnetic field along z axis with gyrofrequency ω_0 (zero, if the field is absent) and in the time-dependent electric field along x axis with relevant electron acceleration $g(t)$ (assumed to be real function). Let the function $g_T(t)$ is defined with the equalities $g_T(t) = g(t)$ at $t \in (-T/2, T/2)$ and $g_T(t) = 0$ at $t \notin (-T/2, T/2)$, and tilde indicates the Fourier transform, $\tilde{g}_T(\omega) = (2\pi)^{-1} \int dt \exp(i\omega t)\, g_T(t)$ (here and further the limits of integration over $(-\infty,+\infty)$ are not written). Let the function $v_T(t)$ obeys the initial condition $v_T(-\infty) = 0$ and the equation

$$(d/dt)v_T(t) + i\,\omega_0\, v_T(t) = g_T(t), \qquad (1)$$

being the linear combination $v_{xT}(t) + i\, v_{yT}(t)$ of velocity components. The quantity P_T defined with the equality $P_T = T^{-1} \int dt\, g_T(t) \operatorname{Re} v_T(t)$ deals with the averaged rate of electron energy change under the action of pulse $g_T(t)$ with duration T. The solution of (1) has the form

$$v_T(t) = \int^{t} dt'\, \exp[i\omega_0(t' - t)]\, g_T(t'), \qquad (2)$$

where upper index t indicates integration over interval $(-\infty, t)$. Substituting (2) into definition of P_T, one gets

$$2T\, P_T = \int dt\, (d/dt)\{[\int^{t} dt'\, g_T(t') \cos(\omega_0 t')]^2 + [\int^{t} dt'\, g_T(t') \sin(\omega_0 t')]^2\} = |\int dt \exp(i\omega_0 t)\, g_T(t)|^2 = 4\pi^2\, |\tilde{g}_T(\omega_0)|^2. \qquad (3)$$

If the function $g(t)$ corresponds to stationary ergodic stochastic process then the functions $g(t + t_0)$ with different t_0 may be considered as realizations of the process. Denoting with angle brackets the averaging by t_0 (without writing t_0 in arguments), one can define the correlation function $.B(t) = \langle g(t' + t)\, g(t')\rangle$, the quantity $\tilde{F}(\omega) = \lim_{\{T\to\infty\}} \langle T^{-1} |\tilde{g}_T(\omega)|^2\rangle$ connected with the averaged power spectral density, and the quantity $P = \lim_{\{T\to\infty\}} \langle P_T\rangle$. Use of the equality $\tilde{B}(\omega) = 2\pi\, \tilde{F}(\omega)$ [4, p. 102] and application of averaging and limit transition to (3) give the equalities

$$P = 2\pi^2\, \tilde{F}(\omega_0) = \pi\, \tilde{B}(\omega_0), \qquad (4)$$

which show that the averaged rate of change of relevant characteristics is determined by power spectral density (and so, by correlation function) at resonant frequencies.

In the right hand sides of the equalities (8, 10, 12, 21, 22) in [2], for the averaged rate of different quantities change, there are the products of the form $|\bar{E}|^2\, \tilde{V}$, where $\bar{E}$ is wave amplitude, the factor at $\exp(-i\omega_1 t)$, ω_1 is wave frequency, $|\bar{E}|$ changes slowly, and $|\bar{E}|^2\, \tilde{V}$ corresponds to Fourier transform of correlation function of the function, which remains of field strength after removing the factor $\exp(-i\omega_1 t)$. In consequence of its removing, the argument of $\tilde{V}$ is not the frequency resonant for electrons with the given velocity, but the difference between it and the frequency ω_1. And so, the product $|\bar{E}|^2\, \tilde{V}$ corresponds to power spectral density. Also, in [3], in right hand side of the equality for the averaged rate of change of square of velocity, there is Fourier transform of the correlation function of field strength at resonant (there, zero) frequency.

So, in view of (4), the relationships obtained in [2, 3] show, that the averaged rates of change of relevant characteristics are determined with power spectral density at resonant frequencies. And if the correlation factor is taken just as in [2, 3], namely,

$$V(t) = \exp(-|t/\tau|),\ \tilde{V}(\omega) = [\pi\, (1 + \omega^2 \tau^2)]^{-1}\, \tau, \qquad (5)$$

then at any frequency (and, in particular, at resonant ones) there is nonzero power, and just it is the cause of electron energy increase.

There are different processes with the correlation factor (5). One of them is Poisson process with average frequency τ^{-1}, consisting of piecewise constant values, in which the probability of absence of changes during time t after previous change is equal to $\exp(-t/\tau)$, and the probabilities to get the opposite values after new change are equal. In particular case of piecewise constant values $\exp(i\phi)$ with $\phi\in(0, 2\pi)$ the process is associated with jumps of phase. As the second example of process with the correlation factor (5) it may be taken the process, each realization of which is given by the sum of pulses $\pm f_1(t - t_n)$, where the signs $\pm$ are random, independent and equiprobable, time instants t_n correspond to Poisson process with average frequency ν (they are successive, $t_{n+1} > t_n$, random and independent), and the function $f_1(t)$ is defined with the equalities $f_1(t) = \exp(-t/\tau)$ at $t > 0$ and $f_1(t) = 0$ at $t < 0$.

Here, it is noteworthy to point out that passing through linear media the signals with the identical amplitude-frequency characteristic give equal values of transited and reflected energy (in connection with the possibility to apply Fourier transform and with independence of different harmonics propagation).

Returning to the relevant pulse stochastic process with the function $f_1(t)$ and replacing it with the function $f_2(t)$ defined by the equality $f_2(t) = \exp(-\tau^{-2} t^2)$ one can obtain the equalities

$$V(t) = \exp(-2^{-1} \tau^{-2} t^2),\ \tilde{V}(\omega) = (2\pi)^{1/2} \tau \exp(-2^{-1} \omega^2 \tau^2),$$

also corresponding to gradual systematic electron energy increase, in absence of jumps in pulse.

But if the definition of stochastic process with the pulses $\pm f(t - t_n)$ is been modified so that the signs $\pm$ become altered by turn, then, denoting $b(t) = \int dt' f(t') f(t' + t)$, for the correlation function one can get the equalities

$$B(t) = \nu\,[b(t) - \nu \int dt' \exp(-2\nu|t' - t|)b(t')],$$

$$\tilde{B}(\omega) = (\omega^2 + 4\nu^2)^{-1} \omega^2 \nu\, \tilde{b}(\omega),$$

in derivation of which it was taken into account that for the considered Poisson process, at $k\geq 0$, $\xi>0$, the difference $t_{\pm(k+1)} - t_0$ belongs to small interval $(\pm\xi,\pm\xi+d\xi)$ with the probability $d\xi\, \nu\, (k!)^{-1} (\nu\, \xi)^k \exp(-\nu\xi)$. The correlation function with $\tilde{B}(0) = 0$ corresponds to absence of gradual systematic electron energy increase (in this case, because of compensation of alternating pulses contribution).

In absence of the constant magnetic field, the electron energy change is determined with value of Fourier transform of electric field strength at zero frequency, which plays the role of resonant one. From the definition of the averaged power spectral density and its connection with the Fourier transform of correlation function, it is followed that the value $\tilde{B}(0)$ may be finite and nonzero only in the case when, with increase of time interval T, in which $g_T(t) \neq 0$, the value of $\int dt\, g_T(t)$ increases, in average, as $T^{1/2}$. And then, in particular, the direct field (proportional to $g(t)$), in average, is absent, $\lim_{\{T\to\infty\}}\{T^{-1} \int dt\, g_T(t)\} = 0$. The integral $\int dt\, g_T(t)$ deals with electron velocity obtained during time T, so, finite and nonzero $\tilde{B}(0)$ corresponds to electron energy increase linear in time.

2. PHASE AND FIELD JUMPS

In the paper [2], the jumps of phase are mentioned only in description of one model, to which the results are applicable, and in drawing an analogy with collisional heating. Namely, in introduction, it is mentioned the model of plasma consisting of regions, the relevant waves in which have equal amplitudes but mutually non-correlated phases, and electron passing the boundary transits into the field of another wave. It should be underlined that there are taken in mind not the phase jumps in the fixed point of space, but the phase jumps corresponding to electron movement relative to waves. As for quick field change in the given point of space, to realize it, one has to transform quickly different kinds of energy (inherent to relevant wave), and invariability of amplitudes may be violated. However, the equations (8, 12, 21) in [2] remains valid if one replaces the products of the form $|\bar{E}|^2\, \tilde{V}$ with Fourier transforms of correlation functions. In any case, the results of [2, 3] are identical for the signals with identical spectral distribution of energy, so, there are no reasons to connect these results with jumps of somehow defined phase.

Moreover, there are some problems arising with attempt to give definition of phase for an arbitrary time dependence of a quantity. If one demands that the phase value should not depend on the option of time origin and of units of time and of relevant varying quantity then it should be given the same phase value (may be, differing on multiple of 2π) to the different intervals of positive value linear increase (in connection with a possibility to transform linearly any such interval to the part of main diagonal of Cartesian plane with the end in the point (1, 1)), and then, in the case, when any number of points disposed at the first quarter of period of sinusoidal time depemdence are linked with lines, one has to set the mentioned phase value for all these intervals of linear dependence, which makes it impossible the limit transition to usual phase of sinusoid. But if one defines phase in other way, by setting the values multiple of $\pi/2$ to extremes and zeroes supplemented with any interpolation rules, then to determine the present phase value one needs the information about time of achievement of the nearest zero or extreme in future, and such phase (to determine it, one needs to know future) cannot be used as cause of the present value of any quantity.

Also, as for field strength jumps, the nonzero instant change of the value of electric field strength component normal to some surface requires (according to Maxwell equations) the infinite tangential values of magnetic field components or current densities.

3. COLLISIONS ROLE

Ionization is impossible without collisions, and the characteristic electron free path between elastic collision λ is usually much less than electron path between ionization acts λ_i, $\lambda << \lambda_i$, so, electrons, in average, take part in a large amount of elastic collisions, before ionization act. Combination of collisions with sinusoidal time dependence of field strength causes the increase of averaged electron energy. If a possibility existed to control the elastic collisions and to put neutrals for collisions at the instants when field direction is altered to eliminate the electron motion against field direction then electron velocity would increase with time linearly and electron energy would increase as square of time. But in the case of chaotic collisions the electron energy increases linearly (collisional heating), in average, at each collision, on the quantity $2\bar{W}$, where $\bar{W}$ is electron oscillatory energy, averaged by period, $\bar{W} = 4^{-1} m u^2$, u is amplitude of oscillatory velocity, m is mass of electron [5, p. 98].

For electron avalanche multiplication in the gap with characteristic dimension L it is necessary to keep the relationship $\lambda_i \le L$ resulted in $\lambda << L$, in view of $\lambda << \lambda_i$. So, electron motion in the gap is diffusion with coefficient $D \sim v^2 \tau_e$, where v is electron thermal velocity, τ_e is time interval between elastic collisions, $\tau_e = v^{-1} \lambda$. For the averaged rates of increase of mid-square displacement z from an initial point and for velocity v, there are relationships $(d/dt)\, z^2 \sim v^2 \tau_e$ (diffusion) and $(d/dt)\, v^2 \sim (u^2/\tau_e)$ (collisional heating), respectively. Having taken the ratio of the written relationships, one can get that energy W obtained by electron with collisional heating in varying field is connected with diffusion displacement z by the relationship

$$z^{-2}\, W \sim \lambda^{-2}\, \bar{W}. \tag{6}$$

In sinusoidal oscillations without collisions with the amplitude of oscillatory displacement A and the corresponding amplitude of energy W_A, for the fixed frequency, the value $A^{-2} W_A$ is independent on A. Replacing $\bar{W}$ and z in (6) with W_A and L, for the energy obtained by electron with aid of collisions during displacement on distance L, one can get the relationships $W \sim (L/\lambda)^2 W_A = W_{AL/\lambda}$ (the fixed frequency is assumes). So, the collisional oscillations give the same order of energy of electron before its outcome on the boundary as it is given by non-collisional oscillations with the same frequency, but with the strength amplitude L/λ times greater. In other case, when field strength amplitude is fixed and frequency is diminishing the electron oscillatory energy increases with the displacement amplitude increase still slower, and maximal attainable energy value is still less. As a result, when electron motion is limited in space and field strength amplitude is limited the electron energy increase without collisions, at least, cannot be much greater than one with collisions, for any time dependence of field.

CONCLUSIONS

So, heating of plasma by random field is determined by power at resonant frequencies irrespective of field jumps presence or absence. As the cross-section of elastic collisions is much greater than one of ionization, an ionization development means intensive collisions and collisional heating of electrons, which makes considerable contribution to the ionization development.

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ОБ УВЕЛИЧЕНИИ ЭНЕРГИИ ЭЛЕКТРОНОВ В РАЗРЯДЕ С НЕСИНУСОИДАЛЬНЫМ ПОЛЕМ

В. Остроушко

Показано, что энергия электронов увеличивается при ненулевом значении спектральной плотности мощности на резонансных частотах, независимо от присутствия или отсутствия скачков напряженности поля. Указано на неизбежность столкновительного нагрева и его значительный вклад в развитие ионизации.

ПРО ЗБІЛЬШЕННЯ ЕНЕРГІЇ ЕЛЕКТРОНІВ У РОЗРЯДІ З НЕСИНУСОЇДАЛЬНИМ ПОЛЕМ

В. Остроушко

Показано, що енергія електронів збільшується при ненульовому значенні спектральної густини потужності на резонансних частотах, незалежно від присутності або відсутності стрибків напруженості поля. Вказано на неминучість зіткненного нагрівання та його значний внесок у розвиток іонізації.