

# THE RADIATION INTENSITY OF THE LYMAN ALPHA LINE AT THE IONIZATION FRONT IN THE QUASI-STEADY PLASMA ACCELERATOR

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Research of the radiation transport in the ionizing gas streams in the channel of the quasi-steady plasma accelerator is carried out. The quasi-one-dimensional model of the ionizing gas flow includes the MHD equations combined with the ionization and recombination kinetics equation within the framework of the modified diffusion approximation. Solution of the radiation transport equation is based on calculation of the plasma emissivity, the photon absorption coefficients, the line profile and use of the method of characteristics. Distribution of the radiation intensity for the Lyman alpha line is obtained, and also the integrated values of the density of the radiation energy and the radiation energy flux for all parts of the hydrogen spectrum along the accelerator channel are presented.

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## INTRODUCTION

The elementary coaxial plasma accelerator [1] schematically consists of the two coaxial electrodes connected to an electric circuit. A neutral gas being introduced between the electrodes is ionized, and ionization front is formed. Behind the front the ionized plasma is accelerated along the channel axis due to the Ampere force  $\frac{1}{c}[\mathbf{j}, \mathbf{H}]$ . Some small plasma accelerators can be used as the first step of the greater system of the quasi-steady plasma accelerator (QSPA). In the first step the ionization and preliminary acceleration of plasma is carried out. In experimental researches of QSPA (see, for example, [1-5]) the high degree of azimuthal symmetry of the low-temperature plasma streams were observed.

The theoretical researches and numerical modeling play the significant role in studying of QSPA (see, for example, [6-8]). The ionization front in the accelerator channel essentially differs from the usual ionizing shock waves of compression. The temperature and speed sharply increase at the ionization front. At the same time the density and a magnetic field sharply decrease. The narrow ionization front according to the experimental data has been obtained within the framework of the numerical model [7] where the system of the MHD-equations is combined with the ionization and recombination kinetic equations in the hydrogen plasma [9].

The given work is devoted to the solution of the radiation transport problem in the ionizing gas streams. The all basic mechanisms of radiation and absorption for the various processes are considered.

## 1. MHD-MODEL OF IONIZING GAS FLOW

The model is based on the transport equations of the three-component medium consisting of atoms, ions and electrons, and also the equation of magnetic field induction. We neglect the inertia of electrons and the displacement current. For the quasi-neutral medium  $n_i = n_e$  we use approach  $T_a = T_i = T_e = T$  and equality of velocities  $\mathbf{V}_i = \mathbf{V}_e = \mathbf{V}_a = \mathbf{V}$ . Then we have the

system of the MHD-equations added by the equation of the ionization and recombination kinetics

$$\frac{\partial n_e}{\partial t} + \text{div} (n_e \mathbf{V}) = n_a \left( n_e \beta_i^c + \beta_i^{ph} \right) - n_e n_i \left( n_e \alpha_r^c + \alpha_r^{ph} \right)$$

$$\frac{\partial \rho}{\partial t} + \text{div} \rho \mathbf{V} = 0; \quad \rho \frac{d \mathbf{V}}{d t} + \nabla P = \frac{1}{c} [\mathbf{j}, \mathbf{H}] \quad (1)$$

$$\rho \frac{d \varepsilon}{d t} + P \text{div} \mathbf{V} = \frac{\mathbf{j}^2}{\sigma} + \text{div} (\kappa \nabla T) - \text{div} \mathbf{W};$$

$$\frac{\partial \mathbf{H}}{\partial t} = \text{rot} [\mathbf{V}, \mathbf{H}] - c \text{rot} \frac{\mathbf{j}}{\sigma}; \quad \mathbf{j} = \frac{c}{4\pi} \text{rot} \mathbf{H}$$

$$\varepsilon = (1 + \alpha) c_V T + \varepsilon_I;$$

$$P = P_a + P_i + P_e = (1 + \alpha) k_B \rho T / m.$$

Here  $\rho = m n_{tot}$  is the total density of heavy particles,  $n_{tot} = n_a + n_i$  is the total concentration of heavy particles,  $\alpha = n_e / n_{tot}$  is the degree of ionization,  $P$  is the total pressure,  $\mathbf{j}$  is the plasma current,  $\varepsilon_I = \zeta \alpha I / m_i$  is the ionization energy losses ( $I = 13.6 \text{ eV}$ ),  $\kappa$  is the electron-atom heat conductivity, and  $\mathbf{W}$  is the radiation energy flux. Coefficients  $\beta_i^c$  and  $\beta_i^{ph}$  answer for the ionization in the collision and radiating processes. In turn  $\alpha_r^c$  and  $\alpha_r^{ph}$  is the recombination coefficients [9].

The conductivity of medium in (1) is equal to  $\sigma = e^2 n_e / m_e \nu_e$ . Here  $\nu_e = \nu_{ea} + \nu_{ei}$  is the average frequency of collisions of electrons with other particles where  $\nu_{ea} = n_a \langle V_e \rangle S_{ea}$ ,  $\nu_{ei} = n_{ion} \langle V_e \rangle S_{ei}$ . The values  $S_{ea}$  and  $S_{ei}$  are the effective cross-sections of the electron-atom and electron-ion collisions.

The ionizing gas flow is considered in the narrow axisymmetric channel with the specified cross-section within the framework of the quasi-one-dimensional approximation [1, 6, 7]. We assume that the average radius of the channel  $r = R_0$  is a constant value. The channel cross-section square is  $f(z) = 2 \pi R_0 \Delta r(z)$  where the distance between the electrodes is  $\Delta r(z)$  and value  $z$  is the coordinate along the channel. It is possible to neglect the change of the MHD variables

across a narrow tube or the radial direction. Then the required functions satisfy to the equations which are contained only by two independent variables  $t$  and  $z$ . The used numerical methods are presented, for example, in [7]. The channel square are defined as:  $f(z) = 0.3 - 0.8z(1-z)$ , if  $z \leq 1$ ; and  $f(z) = 0.8z - 0.5$ , if  $1 \leq z \leq z_{out} = 3$ . The channel has the shape of a Laval nozzle of length equal to unity with a linearly expanding funnel added on the right. At the inlet  $z=0$  we have  $H = H_o = 2 J_p / c R_o$ ,  $n = n_o$ ,  $T = T_o$ ,  $\alpha = \alpha_{in}$ . At the accelerator outlet  $z = z_{out}$  the boundary conditions correspond to the free outflow.

The calculation case of the ionizing gas flow (see, also, [7]) is presented in Fig. 1 for the following parameters:  $J_p = 100 \text{ kA}$ ,  $n_o = 2 \cdot 10^{17} \text{ nm}^{-3}$ ,  $T_o = 500 \text{ K}$ ,

$L = 20 \text{ cm}$ ,  $R_o = L/3$ ,  $\alpha_{in} = 1.5 \cdot 10^{-7}$ , and  $V_o = H_o / \sqrt{4 \pi m_i n_o} = 1.46 \cdot 10^6 \text{ cm/s}$ .

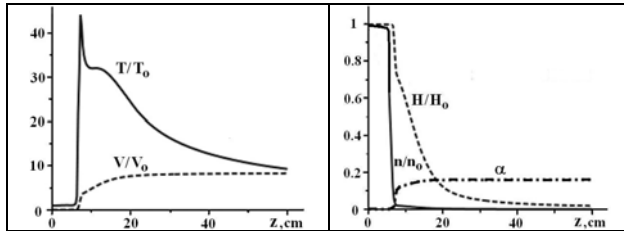


Fig. 1. Distribution of variables in the ionizing gas stream along the channel

## 2. THE RADIATION TRANSPORT EQUATION

The rate of the radiative processes is essentially above the characteristic velocities of the plasmadynamic processes. In this case the radiation field is instantly arranged under the varying parameters of a stream. Therefore we shall consider the solution of the stationary equation of the radiation transport:

$$\Omega \cdot \nabla I_\omega(\mathbf{r}, \Omega) = \eta_\omega(\mathbf{r}) - \kappa_\omega(\mathbf{r}) \cdot I_\omega(\mathbf{r}, \Omega). \quad (2)$$

Here  $I_\omega(\mathbf{r}, \Omega)$  is the radiation intensity with frequency  $\omega$  in a point with the coordinate  $\mathbf{r}$  which extends in the direction of the spatial angle  $\Omega$ . The equation (2) is written down in the assumption of the isotropic scattering. Knowing the radiation intensity in channel  $I_\omega(\mathbf{r}, \Omega)$  it is possible to find the density of the radiation energy flux

$$\mathbf{W}(\mathbf{r}) = \int_0^\infty \int_0^{4\pi} I_\omega(\mathbf{r}, \Omega) \Omega d\Omega d\omega.$$

The emissivity  $\eta_\omega(\mathbf{r})$  and the absorption coefficient  $\kappa_\omega(\mathbf{r})$  (see, for example, [9-11]) consist of three parts corresponding to the bound-bound, bound-free and free-free processes caused by transitions of electron from one state to another. The total absorption coefficient corrected for the stimulated radiation is equal to

$$\begin{aligned} \kappa_\omega = n_{tot} \sum_{\substack{k < j \\ k, j=1}}^{N-1} x_k \frac{\pi e^2}{m_e c} f_{kj} \phi_{kj}(\omega) \left( 1 - \frac{n_j g_k}{n_k g_j} \right) + \\ + n_{tot} \sigma_K^{ff}(\omega) (1 - e^{-\hbar\omega/k_B T}) + n_{tot} \sum_{k=1}^{N-1} x_k \sigma_k^{PI}(\omega) * \end{aligned}$$

$$* \left( 1 - \frac{1}{2} \left( \frac{2\pi \hbar^2}{m_e k_B T} \right)^{3/2} \frac{n_i g_k}{n_{tot} x_k \Sigma_i} \exp \left( \frac{\hbar\omega_k - \hbar\omega}{k_B T} \right) \right).$$

Here all variables have usual sense, in particular,  $x_k$  is the relative concentration of  $k$ -th state of atom,  $f_{kj}$  is the oscillator strength for  $k \rightarrow j$  transition. The emissivity of medium is calculated similarly. The line profile  $\phi_{kj}(\omega)$  taking into account the different broadening mechanisms is defined by the Voigt formula [11]. The graph of the absorption coefficient over the radiation energy for hydrogen plasma with accounting of 10 energy levels in the assumption of their equilibrium populations is presented in Fig. 2 for  $T = 0.8 \text{ eV}$  and  $n_{tot} = 1.4 \cdot 10^{15} \text{ cm}^{-3}$ .

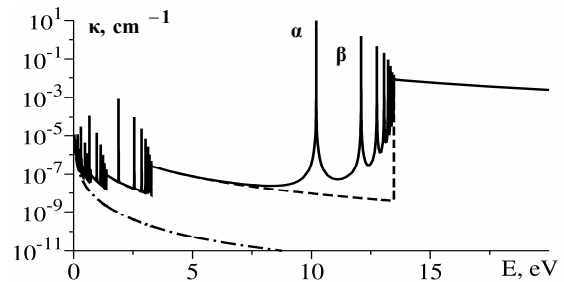


Fig. 2. Values of the absorption coefficient including the Lyman alpha and beta lines, the ionization spectrum (dotted curve) and scattering (dash-dotted line)

For solution of the radiation transport equation the method of characteristics [10] was used, in particular. In this case the beams are issued from the any point of the accelerator channel in the direction set by the uniform angular grid. In a place of the beam falling on the plasma-electrode boundary the intensity was necessary equal to zero. Distribution of the radiation intensity with the energy  $\hbar\omega = 10.2 \text{ eV}$  corresponding to the center of Lyman alpha line in the radial direction is presented in Fig. 3. The peak of distribution located at the ionization front corresponds to the available experimental data according to which the front has enough narrow formation.

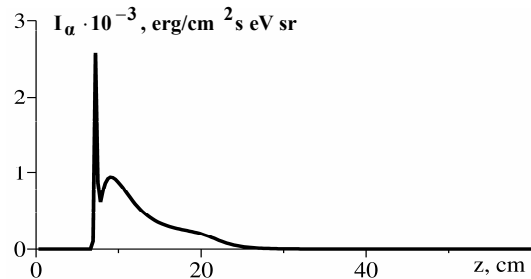


Fig. 3. Distribution of the radiation intensity of the Lyman alpha line along the accelerator channel

The total contribution of the various portion of the spectrum defines the integrated values of density of the radiation energy flux  $\mathbf{W}$ . The dependency of the longitudinal component  $W_z$  is presented in Fig. 4. Here the dotted line corresponds to the separate portion of the Lyman alpha line. We can see that this line brings the essential contribution to the radiation energy flux and transfers about half of all radiation energy. Calculations have shown that the contribution of the recombination

radiation is less significant behind front but its penetration into the low-ionized gas before the front is appreciable.

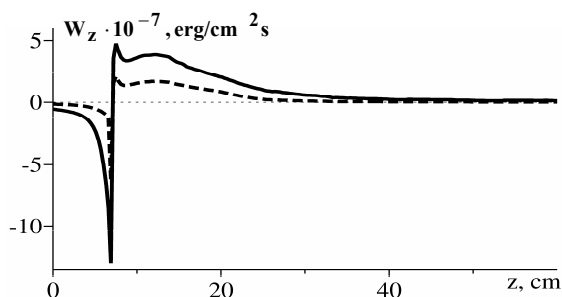


Fig. 4. Distributions of density of radiation energy flux for all spectrum and for Lyman alpha line (dotted curve)

### CONCLUSIONS

Thus numerical research of the radiative transport in the ionizing gas streams is carried out. Radiation of the ionization front plays a significant role in formation of the radiation field. The Lyman alpha line brings the essential contribution to the density of the radiation energy and the radiation energy flux. The developed models allow to define parameters of the line emission outspreading and intensity of its radiation in any point of the accelerator channel and simultaneously to calculate the corresponding values of the density, temperature and the ionization stage of medium. The given research is directed to the further comparison of the calculated data with experiment.

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### ИНТЕНСИВНОСТЬ ИЗЛУЧЕНИЯ АЛЬФА-ЛИНИИ ЛАЙМАНА НА ФРОНТЕ ИОНИЗАЦИИ В КВАЗИСТАЦИОНАРНОМ ПЛАЗМЕННОМ УСКОРИТЕЛЕ

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Проведено исследование переноса излучения в потоках ионизирующегося газа в канале квазистационарного плазменного ускорителя. Квазидвумерная модель течения ионизирующегося газа основана на МГД-уравнениях с учетом кинетики ионизации и рекомбинации в рамках модифицированного диффузионного приближения. Моделирование переноса излучения основано на вычислении излучательной способности и коэффициента поглощения фотонов, определении профиля и использовании метода характеристик. Получено распределение интенсивности излучения центра альфа-линии Лаймана, а также распределения плотности энергии и плотности потока энергии излучения для всего спектра вдоль канала ускорителя.

### ИНТЕНСИВНІСТЬ ВИПРОМІНЮВАННЯ АЛЬФА-ЛІНІЇ ЛАЙМАНА НА ФРОНТІ ІОНІЗАЦІЇ В КВАЗИСТАЦІОНАРНОМУ ПЛАЗМОВОМУ ПРИСКОРЮВАЧІ

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Проведено дослідження перенесення випромінювання в потоках газу, що іонізується, в каналі квазистационарного плазмового прискорювача. Квазидвумірною моделлю течії газу, що іонізується, заснованою на МГД-рівняннях з урахуванням кінетики іонізації і рекомбінації в рамках модифікованого дифузійного наближення. Моделювання перенесення випромінювання засноване на обчисленні випромінювальної здатності і коефіцієнта поглинання фотонів, визначенні профілю і використанні методу характеристик. Отримано розподіл інтенсивності випромінювання центра альфа-лінії Лаймана, а також розподіли густини енергії і густини потоку енергії випромінювання для всього спектра уздовж каналу прискорювача.