QUASILINEAR KINETIC MODELLING OF RMP PENETRATION INTO A TOKAMAK PLASMA

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The linear as well as the quasilinear problem of RMP penetration in tokamaks is solved consistently with a particle and energy conserving collision operator. The new collision operator ensures the Onsager symmetry of the quasilinear transport coefficient matrix and avoids artifacts such as fake heat convection connected with simplified collision models.

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INTRODUCTION

Resonant magnetic field perturbations (RMPs) are presently used for mitigation of edge localised modes (ELMs) in tokamak H-regimes. This method is foreseen to be used in ITER. However, the basic question how well do RMPs penetrate into the plasma has not obtained a final answer yet. Linear theory [1,2] predicts that RMPs are strongly shielded at the pertinent resonant magnetic surfaces. It is known from MHD theory, there is a RMP generated torque acting on the plasma. This torque tends to slow down the electron fluid motion across the magnetic field lines and, for a certain threshold value of the RMP amplitude, causes RMPs to penetrate. In this report RMP penetration is studied within quasilinear theory in kinetic approximation. The linear problem for the RMP electromagnetic fields is solved by the code KiLCA (Kinetic Linear Cylindrical Approximation) [2,3] and this solution is selfconsistently used for the computation of the evolution of the background plasma parameters using a 1-D balance code. For this purpose, in the linear as well as in the quasilinear model a particle and energy conserving collision operator is used. Unlike the linear model, the quasilinear model is very sensitive to the details of the collision operator and full consistency with the conductivity model of the Maxwell solver has to be ensured.

1. BASIC EQUATIONS

Both, the linear plasma conductivity and the quasilinear transport coefficients are determined by the solution of the kinetic equation, $L_V f = L_{cp} f$ where L_V is the Vlasov operator, $L_{cp} = L_c + L_{cI}$,

$$\hat{L}_c = \frac{\partial}{\partial u_\parallel} D \left(\frac{\partial}{\partial u_\parallel} + \frac{u_\parallel - V_\parallel}{v_T^2(r)} \right), \qquad (1)$$

$$\hat{L}_{cI} f(v_\perp, v_\parallel) = \frac{\nu}{\sqrt{2\pi} v_T} \exp\left(-\frac{v_\parallel^2}{2v_T^2} \right) \left(\frac{v_\parallel^2}{v_T^2} - 1 \right) \times \int\limits_{-\infty}^{\infty} dv'_\parallel \left(\frac{{v'_\parallel}^2}{v_T^2} - 1 \right) f(v_\perp, v'_\parallel) \qquad (2)$$

are the Ornstein-Uhlenbeck collision operator L_c with an integral part L_{cI} to ensure energy conservation. In cylindrical geometry this equation can be solved analytically by a Green's function and the gyroaverage of the perturbed distribution function needed in the quasilinear problem is

$$f_{\mathbf{m}}(v_\perp, v_\parallel) = -\int\limits_{-\infty}^{\infty} dv'_\parallel\, G_{\mathbf{mp}}(v_\parallel, v'_\parallel) \times \left(A_1 + \frac{{v'_\parallel}^2 + v_\perp^2}{2v_T^2} A_2 \right) f_0(v_\perp, v'_\parallel)\, v^r_{\mathbf{m}}(v_\perp, v'_\parallel). \quad (3)$$

Here, $f_{\mathbf{m}}$ and $v^r_{\mathbf{m}}$ are the amplitudes of the Fourier series over toroidal and poloidal angles of the perturbed distribution function and of the radial guiding center velocity, respectively. $G_{\mathbf{mp}}$ is Green's function for L_{cp} discussed below. The thermodynamic potentials are

$$A_1 = \frac{1}{n}\frac{\partial n}{\partial r} + \frac{e}{T}\frac{\partial \Phi}{\partial r} - \frac{3}{2T}\frac{\partial T}{\partial r}, \qquad A_2 = \frac{1}{T}\frac{\partial T}{\partial r}. \qquad (4)$$

These potentials determine particle and energy fluxes

$$\Gamma^{(\mathrm{EM})}_{(\mathrm{e,i})} = -n_{\mathrm{e,i}} \left(D_{11} A_1 + D_{12} A_2 \right), \qquad (5)$$

$$Q^{(\mathrm{EM})}_{(\mathrm{e,i})} = -n_{\mathrm{e,i}} T_{\mathrm{e,i}} \left(D_{21} A_1 + D_{22} A_2 \right), \qquad (6)$$

through quasilinear diffusion coefficients. Retaining in $v^r_{\mathbf{m}}$ only parallel motion along the perturbed magnetic field and the ExB-drift (these are the dominant processes for electrons), these coefficients are

$$D_{kl} = \frac{1}{\sqrt{8\pi} v_{\mathrm{T}} B_0^2} \mathrm{Re} \sum_{m} \int_{\mathbb{R}} dv_\parallel \int_{\mathbb{R}} dv'_\parallel\, G_{\mathbf{m}p}(v_\parallel, v'_\parallel) \times \exp\left(-\frac{{v'_\parallel}^2}{2v_T^2} \right) \times (v_\parallel B^r_{\mathbf{m}} + cE_{\mathbf{m}\perp})^* (v'_\parallel B^r_{\mathbf{m}} + cE_{\mathbf{m}\perp}) a_{kl}(v_\parallel, v'_\parallel), \quad (7)$$

$$(a_{kl}) = \begin{pmatrix} 1 & 1 + \frac{{v'_\parallel}^2}{2v_T^2} \\ 1 + \frac{v_\parallel^2}{2v_T^2} & 2 + \frac{v_\parallel^2 + {v'_\parallel}^2}{2v_T^2} + \frac{v_\parallel^2 {v'_\parallel}^2}{4v_T^4} \end{pmatrix}. \quad (8)$$

The Fourier amplitude of parallel current density responsible for RMP shielding can be expressed in terms of the radial component of the magnetic perturbation field $B^r_{\mathbf{m}}$ and the electrostatic field component $E_{\mathbf{m}\perp}$ tangential to the unperturbed flux surface and perpendicular to the main magnetic field.

$$j_{\mathbf{m}\parallel} = -\frac{nev_T}{\nu B_0} \left[\left((A_1 + A_2) I^{10}_{\mathrm{p}} + \frac{1}{2} A_2 I^{21}_{\mathrm{p}} \right) cE_{\mathbf{m}\perp} + \left((A_1 + A_2) I^{11}_{\mathrm{p}} + \frac{1}{2} A_2 I^{31}_{\mathrm{p}} \right) v_T B^r_{\mathbf{m}} \right]. \qquad (9)$$

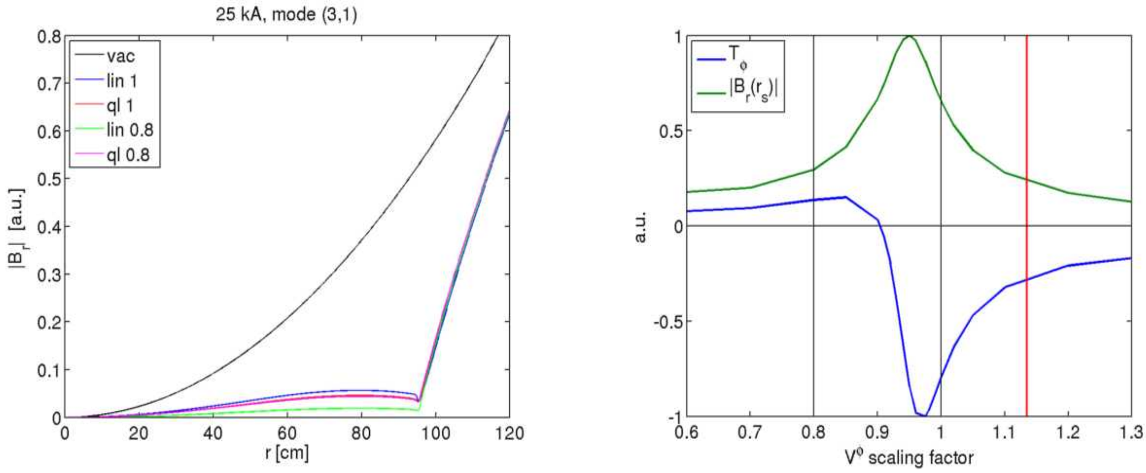

Fig. 1. Left: Radial profiles of $|B^r_{\mathbf{m}}|$ before and after quasilinear relaxation. Right: Toroidal torque and $|B^r_{\mathbf{m}}|$ at the resonant surface as functions of the toroidal velocity scaling factor. Starting scaling factor values used for the computations shown on the left (see legend) and the value corresponding to zero electron fluid velocity at the resonant surface are indicated by the black lines and the red line respectively on the right

The mismatch between perturbed magnetic flux surfaces and perturbed equipotential surfaces leads to quasilinear transport. Lowest order Larmor radius approximation used in (7) and in (9) is sufficient for the electrons. Moments I^{mn} of $G_{\mathbf{m}}$ (without L_{cl}) and moments I_p^{mn} of $G_{\mathbf{m}\mathrm{p}}$ (with L_{cl})

$$I^{mn}_{(p)} = \frac{\nu}{\sqrt{2\pi} v_T^{m+n+1}} \int\limits_{-\infty}^{\infty} \mathrm{d}v_\parallel \int\limits_{-\infty}^{\infty} \mathrm{d}v'_\parallel\, G_{\mathbf{m}(\mathrm{p})}(v_\parallel, v'_\parallel) \times \exp\left(-\frac{{v'_\parallel}^2}{2v_T^2}\right) v_\parallel^m {v'_\parallel}^n, \qquad (10)$$

are related by

$$I_{\mathrm{p}}^{mn} = I^{mn} + \frac{\left(I^{m0} - I^{m2}\right)\left(I^{n0} - I^{n2}\right)}{1 - I^{00} + 2I^{20} - I^{22}}. \qquad (11)$$

Using the representation for I^{mn} by parameter differentiation discussed in [3], the following recursion formula is obtained

$$I^{mn} = \frac{\nu}{ik_\parallel v_T}\left[\left(i(\omega - \omega_E)/\nu + 1 - n\right) I^{mn-1} + (n-1)(n-2) I^{mn-3} + H^{m+n-1}\right], \qquad (12)$$

$$H^m = \begin{cases} (m-1)!! & m \quad \text{even} \\ 0 & m \quad \text{odd}. \end{cases} \qquad (13)$$

With this recursion it is straightforward to show that Onsager symmetry is valid, in particular

$$D_{12}^a = \frac{c v_T}{4\nu B_0^2} \mathrm{Im}\left(E^*_{\mathbf{m}\perp} B^r_{\mathbf{m}}\right) \mathrm{Im}\left(I_p^{21} - I_p^{30}\right) = 0 \qquad (14)$$

which follows immediately from $I_p^{21} - I_p^{30} = 0$ because

$$I^{21} - I^{30} = \frac{2\nu}{ik_\parallel v_T}\left(I^{20} - I^{00}\right), \qquad (15)$$

$$I^{10} - I^{12} = \frac{\omega - \omega_E}{k_\parallel v_T}\left(I^{00} - I^{02}\right), \qquad (16)$$

$$I^{30} - I^{32} = \frac{\nu}{ik_\parallel v_T}\left\{\left[i(\omega - \omega_E)/\nu - 2\right]\left(I^{20} - I^{22}\right) - 2 + 2\left(I^{00} - I^{02}\right)\right\}. \qquad (17)$$

2. APPLICATION

Using (5) and (6), balance equations for plasma density n_e, toroidal ion rotation velocity V_i^φ, and electron and ion temperatures $T_{e,i}$ presented in [4] were solved for JET like parameters in experiments with ELM mitigation by C-coil. Only the 3/1 mode of the coil spectrum has been retained. Modelled are two variants of starting equilibria obtained by scaling the toroidal rotation velocity V_i^φ by factors 0.8 and 1.0 as shown in Fig. 1 for two values of the anomalous diffusion coefficient, 10^4 and $5 \cdot 10^3$ cm^2/s indicated in Fig. 2. It is found that thresholds of RMP bifurcation obtained in the modelling are in the range of RMP amplitudes used in current experiments. The results also show that quasilinear effects do not necessarily lead to a significant increase in field penetration but may lead to even stronger shielding despite the fact that the parallel electron current in the resonant zone is reduced (see Fig. 2). In contrast to earlier MHD theories [3], the main quantity changed by RMPs is the electron temperature and not so much the toroidal rotation velocity. The change is such that the perpendicular electron fluid velocity becomes zero around the resonant surface and RMP shielding is modified but not removed. The electron diamagnetic velocity as the most affected quantity agrees with a feature observed in recent quasilinear modelling based on Drift-MHD theory [5]. In all cases, the perpendicular electron fluid velocity is evolving to zero in the resonant zone. In MHD theory, this would lead to field penetration. In kinetic theory, the radii of maximum radial magnetic field and zero toroidal torque are not the same (see Fig. 1) and, as a consequence, shielding is not necessarily reduced. The results presented are not sensitive to the chosen value of the anomalous diffusion coefficient.

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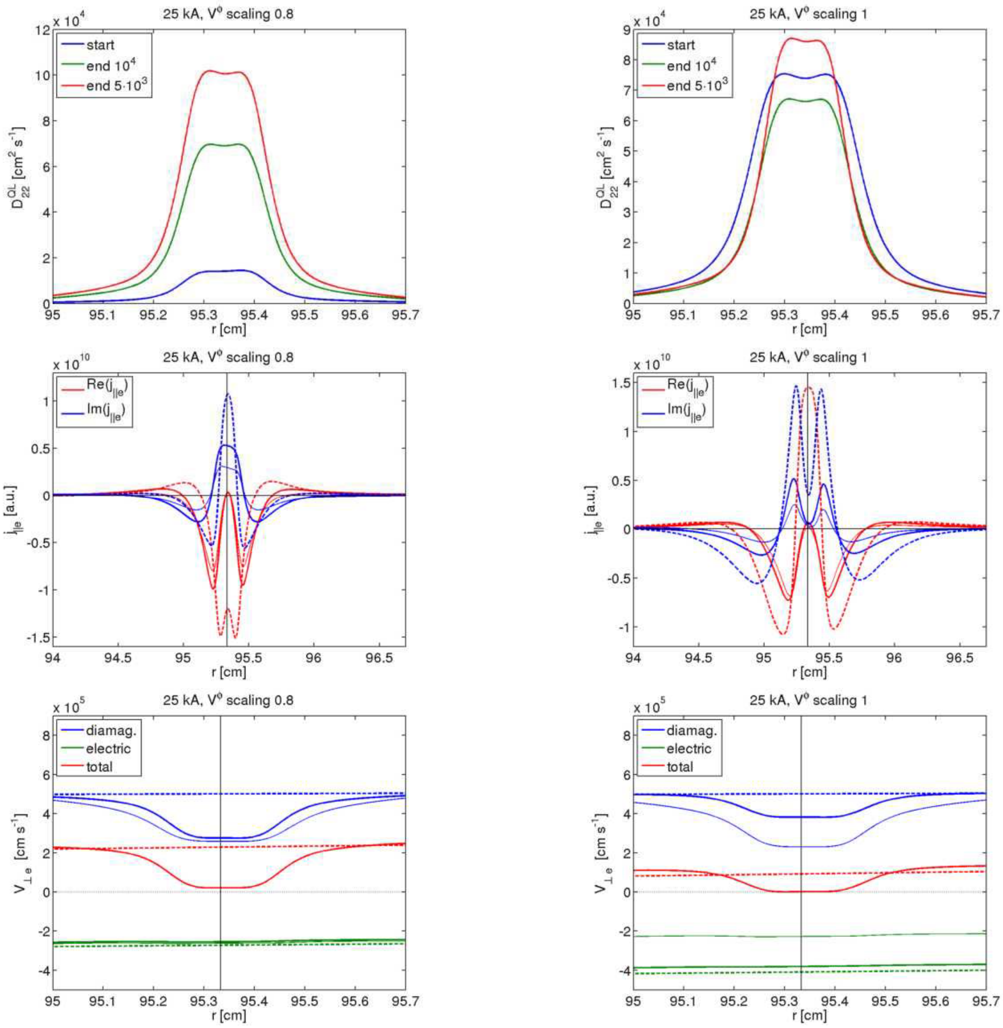

Fig. 2. Quasilinear heat conductivity coefficient (top), parallel electron current (middle), and perpendicular components of electron fluid velocity (bottom). Dashed lines for the currents and rotation velocity components show initial values. Thin lines correspond to the evolution with the anomalous diffusion coefficient reduced

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КВАЗИЛИНЕЙНОЕ КИНЕТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПРОНИКНОВЕНИЯ РМВ В ПЛАЗМУ ТОКАМАКА

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Рассмотрено самосогласованное решение линейной и квазилинейной задач о проникновении РМВ в плазму токамака с учетом сохранения частиц и энергии при кулоновских столкновениях. Новый оператор столкновений обеспечивает симметрию Онзагера для матрицы квазилинейных коэффициентов переноса, а также отсутствие артефактов в виде ложной конвекции тепла, свойственной упрощенным моделям столкновений.

КВАЗІЛІНІЙНЕ КІНЕТИЧНЕ МОДЕЛЮВАННЯ ПРОНИКНЕННЯ РМЗ У ПЛАЗМУ ТОКАМАКА

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Розглянуто самоузгоджене вирішення лінійної та квазілінійної задач про проникнення РМЗ у плазму токамака з урахуванням збереження частинок та енергії при кулонівських зіткненнях. Новий оператор зіткнень забезпечує симетрію Онзагера для матриці квазілінійних коефіцієнтів переносу, а також відсутність артефактів у вигляді підробної конвекції тепла, що властива до спрощених моделей зіткнень.