UNUSUAL PHYSICS OF QUANTUM PLASMAS

Yu.O. Tyshetskiy, S.V. Vladimirov, R. Kompaneets
School of Physics, The University of Sydney, NSW2006, Australia

We discuss some of the peculiarities of collective phenomena in a quantum plasma. In particular, we consider "elementary" collective phenomena such as charge shielding, volume and surface oscillations in a degenerate quantum plasma, and discuss how they change compared to those in a classical plasma.

PACS: 05.30.-d,52.25.Dg,52.35.-g,73.20.Mf

INTRODUCTION

Before delving into the peculiarities of quantum plasmas, it is necessary to first understand what the term means. Indeed, all plasmas are in some sense quantum, as they consist of particles that obey the laws of quantum mechanics. Yet in many plasmas, which we here call classical plasmas, the quantum nature of their constituent particles does not affect, in any significant way, the macroscopic dynamics of the plasma in other words, such plasmas behave, macroscopically as though they consist of particles that obey the laws of classical mechanics (except for the close interparticle collisions which in general must be described quantum mechanically). However, as the density of a classical plasma increases, or its temperature decreases, it can enter a regime when the quantum nature of its constituent particles starts to affect its macroscopic properties and dynamics. Quite naturally, such plasmas are then called quantum. In quantum plasmas, the mean interparticle distance becomes comparable to the mean de Broglie wavelength of the lightest plasma particles, and the effects of degeneracy (e.g., quantum degeneracy of electrons due to Pauli's exclusion princinple for fermions become significant.

Examples of quantum plasmas include conductivity electrons in metals, as well as electrons and holes in the conduction zone of semiconductors. More exotically quantum plasmas appear in cores of giant planets and crusts of old stars, as well as in the interior of white dwarfs. The densities relevant to quantum plasmas may alsobe achievable in the fast ignition scenarioof inertial confinement fusion experiments, where the deuterium tritium mixture is compressed by powerful laser beams to densities exceeding that of the liquid hydrogen [1]. Recent experimental results on x-ray scattering suggest that quantummechanical effects are indeed important in dense plasmas [2].

The works on collective interactions and linear waves in quantum plasmas date back at least 60 years, probably starting with the works of Klimontovich and Silin [3, 4], Bohm, and Pines [5, 6]. Yet in the recent decade, there has been a surge of investigations of new aspects of linear and nonlinear collective effects in quantum plasmas (see [7] for a recent review), probably owing to the recent technological advances enabling a direct measurement of plasma dynamics in quantum regime via, e.g., an ultrafast x-ray Thomson scattering spectroscopic techniques [2, 8, 9].

In this paper, our aim is to highlight some of the peculiarities of quantum plasmas, which appear already in a simplest case of a nonrelativistic unmagnetized quantum plasma. To do this, we consider some of the well known concepts and phenomena familiar to all plasma physicists (this list is by no means complete, but rather is illustrative), to see how they change, often qualitatively, in a quantum plasma, as compared with their counterparts in a classical plasma. We hope that these examples will show a reader yet unfamiliar with the area of quantum plasmas how a plasma can be even more fascinatingwhen it is in a quantum regime.

1. WHAT MAKES A PLASMA QUANTUM

A plasma, which is essentially a collection of particles interacting via the Coulomb force, has two characteristic energy scales associated with it (here we are talking about electrons, which are fermions and therefore obey the Fermi-Dirac statistics): the average kinetic energy of the particles (here we consider nonrelativistic plasma, for simplicity):

$$\bar{K} = \max\left\{\frac{3}{2}k_B T,\ E_F\right\}, \tag{1}$$

where k_B is the Boltzmann constant, T is the temperature of plasma particles (electrons), and $E_F = (\hbar^2/2m_e)(3\pi^2 n)2/3$ is their Fermi energy, where n is the electron number density, and the average potential energy of interaction,

$$\bar{U} \sim 4\pi e^2 n^{1/3} \tag{2}$$

(we use CGS units). Using these energy scales, one can define two parameters that characterize the plasma. The first is the degeneracy parameter

$$\chi = E_F/k_B T, \tag{3}$$

which characterizes the importance of quantum statistical effects due to Pauli blocking of available electron states. Its value determines whether a plasma is classical ($\chi \geq 1$) or quantum ($\chi \geq 1$). The second is the plasma coupling parameter:

$$\Gamma = \bar{U}/\bar{K} \sim \begin{cases} e^2 n^{1/3}/k_B T, & \text{for } \chi \ll 1, \\ e^2 n^{1/3}/E_F, & \text{for } \chi \gg 1, \end{cases} \tag{4}$$

which characterizes the strength of interparticle interaction. Its determines whether a plasma (being either classical or quantum, depending on its degeneracy parameter χ) is weakly coupled ($\Gamma \geq 1$) or strongly coupled ($\Gamma \geq 1$). In a weakly coupled plasma, the correlation between particles is weak, and such plasma can be described in terms of the single-particle distribution function [10, 11] within the mean field approximation (also called Hartree's mean-field approximation). In a moderately or strongly correlated plasma, particle correlations are significant, and a much more complex description in terms of a hyerarchy of many-particle distribution functions is required in general [10]. It is interesting to note that an electron gas in the quantum regime ($\chi \geq 1$) becomes less coupled as its density increases, unlike an electron gas in the classical regime. Indeed, for $\chi \geq 1$ (classical regime), the coupling parameter $\Gamma \geq n^{1/3}$ increases with density, while for $\chi \geq 1$ (deeply quantum regime) the coupling parameter $\Gamma \geq n^{1/3}/EF(n) \geq n-1/3$ decreases with density.

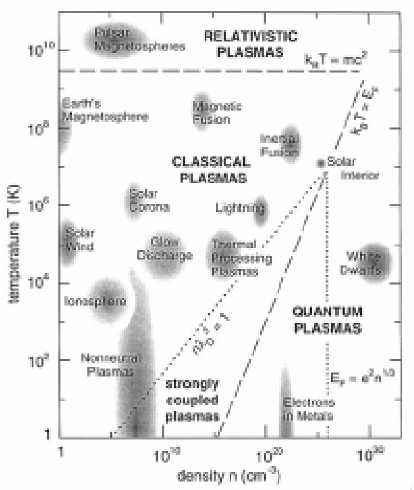

Fig. 1. The density-temperature map of common plasmas, indicating different plasma regimes, depending on the degeneracy and coupling parameters Reprinted with permission fromRef. [12], courtesy of the National Academies Press, Washington, D.C.

Fig. 1 shows the regimes of common plasmas on the density-temperature map. In this paper, we will mostly discuss non-relativistic weakly coupled quantum plasmas. However, we note that the qualitative picture of collective effects in a weakly coupled quantum plasma remains the same in amoderately coupled plasma (e.g., electrons in metals for which $\Gamma \geq 1$), despite the non-weak correlation between electrons.

2. COLLECTIVE EFFECTS IN A QUANTUM PLASMA

A complete statistical description of a nonrelativistic plasma can be done in terms of density matrix, which allows to obtain the mean values and probability distributions of macroscopical physical parameters of the plasma. In particular, it is convenient to describe a plasma in terms of the quantum distribution function, suggestedbyWigner [13] and thus sometimes called the *Wigner function*, which is the density matrix in mixed coordinate-momentum representation. An *N*-particle Wigner function is defined as

$$f_N(\mathbf{r}_N, \mathbf{p}_N, t) = (2\pi)^{-3N} \int d\mathbf{k} e^{-i\mathbf{k}\cdot\mathbf{p}_N} \times \rho_N (\mathbf{r}_N - \hbar\mathbf{k}/2, \mathbf{r}_N + \hbar\mathbf{k}/2, t),$$

where ρ_N (r_N,r'_N, t) is the plasma density matrix in coordinate representation, N is the number of particles in the system, and r_N and p_N are 3*N*-dimensional vectors containing coordinates and canonical momenta all particles in the system. The properties of $f_N(r_N,p_N, t)$ are discussed in detail in Ref. [14]. In the classical limit $\hbar \rightarrow 0$, f_N becomes the classical *N*-particle distribution function, hence the description of a plasma in terms of the Wigner function covers both quantum and classical plasma regimes.

Since in a plasma $N \geq 1$, the description in terms of f_N is prohibitive. Luckily, it is also unnecessary: in physical applications only the knowledge of single-particle distribution and perhaps a few next higher-order distributions is required. Using a quantum analogue of the BBGKY (Bogoliubov-Born-Green-Kirkwood-Yvon) approach [10], a hierarchy of coupled equations for ever higher-order distribution functions (starting with an equation for the single particle distribution f_1) is derived from the equation for f_N, with an equation for fn containing a correlation term with f_{n+1} (n = 1,2, ... , *N*−1). For a weakly coupled plasma, this chain of equations can be truncated at the very first equation for the 1-particle Wigner function:

$$f_1(\mathbf{r}_1, \mathbf{p}_1, t) \equiv \int f_N(\mathbf{r}_N, \mathbf{p}_N, t) d\mathbf{r}_2 \ldots d\mathbf{r}_N d\mathbf{p}_2 \ldots d\mathbf{p}_N,$$

which reads, for a weakly coupled plasma with electromagnetic interaction of particles:

$$\frac{\partial f_1(\mathbf{r}, \mathbf{P}, t)}{\partial t} = \frac{1}{(2\pi)^6} \frac{i}{\hbar} \int d\vec{\lambda} d\mathbf{k} d\vec{\eta} d\mathbf{q}\, e^{i[\vec{\lambda}\cdot(\vec{\eta}-\mathbf{P})+\mathbf{k}\cdot(\mathbf{q}-\mathbf{r})]} \times f_1(\mathbf{q}, \vec{\eta}, t) \left[\mathcal{H}\left(\mathbf{q} - \frac{1}{2}\hbar\vec{\lambda}, \vec{\eta} + \frac{1}{2}\hbar\mathbf{k}, t\right) - \mathcal{H}\left(\mathbf{q} + \frac{1}{2}\hbar\vec{\lambda}, \vec{\eta} - \frac{1}{2}\hbar\mathbf{k}, t\right)\right], \quad (5)$$

where $P = p + (e/c)\, A(r, t)$ is the electron canonical momentum, $p = m_e v$ is its kinetic momentum, and the Hamiltonian H (r, P, t) contains the mean (averaged over the ensemble of particles) electromagnetic field potentials φ andA:

$$\mathcal{H}(\mathbf{r}, \mathbf{P}, t) = \frac{1}{2m_e}\left(\mathbf{P} - \frac{e}{c}\mathbf{A}(\mathbf{r}, t)\right)^2 + e\phi(\mathbf{r}, t). \quad (6)$$

Eq. (5) is coupled with the Maxwell's equations for φ and A, in which the source terms (charge and current densities) are defined in terms of f_1 as

$$en_e(\mathbf{r}, t) = e \int f_1(\mathbf{r}, \mathbf{P}, t) d\mathbf{P}, \quad (7)$$

$$\mathbf{j}_e(\mathbf{r}, t) = \frac{e}{m_e} \int \mathbf{p} f_1(\mathbf{r}, \mathbf{P}, t) d\mathbf{P}. \quad (8)$$

We note that the quantum kinetic equation (5) can be cast in a form of the Boltzmann equation with the effects of quantum interference due to overlapping of electron wave functions contained in the right-hand side in the "quantum interference integral" [15].

2.1. LINEAR RESPONSE OF AN ISOTROPIC QUANTUM PLASMA

The linear response of a medium is characterized by its dielectric permittivity tensor $\varepsilon_{ij}(\omega, k)$. For an isotropic quantum plasma with mobile electrons and immobile ion background (we are not concerned about ions here, as they are much heavier than electrons and thus behave classically), $\varepsilon_{ij}(\omega,k)$ is obtained by linearizing (5) on a small perturbation δf1 of the equilibrium isotropic distribution f0(|p|). The result is [15]

$$\varepsilon_{ij}(\omega,\mathbf{k}) = \varepsilon^{l}(\omega,\mathbf{k})\frac{k_i k_j}{k^2} + \varepsilon^{tr}(\omega,\mathbf{k})\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right),$$

with the longitudinal and transverse permittivities given by

$$\varepsilon^{l}(\omega,\mathbf{k}) = 1 + \frac{4\pi e^2}{k^2}\int d\mathbf{p}\frac{D_{\mathbf{p},\mathbf{k}}(f_0)}{\omega - \mathbf{k}\cdot\mathbf{p}/m_e + i0}, \quad (9)$$

$$\varepsilon^{tr}(\omega,\mathbf{k}) = 1 - \frac{\omega_{pe}^2}{\omega^2} + \frac{2\pi e^2}{m_e^2\omega^2}\int d\mathbf{p}\frac{p_\perp^2 \hat{D}_{\mathbf{p},\mathbf{k}}(f_0)}{\omega - \mathbf{k}\cdot\mathbf{p}/m_e + i0}, \quad (10)$$

where $\omega_{pe} = (4\pi e^2 n/m_e)^{1/2}$ is the electron plasma frequency, $p_\square$ is the absolute value of the component of p perpendicular to k, $D?_{p,k}(f_0) = [f_0\ (p+\hbar k/2)-f_0\ (p-\hbar k/2)]/\hbar$, and +i0 specifies the direction of bypassing the singularity at $p_\perp = m\omega/k$ when integrating over $p_\parallel = (k\cdot p)/k$ (Landau's rule [16]). In the classical limit (formally $\hbar\to 0$), the quantum distribution function f_0 reduces to the classical distribution, and (9)–(10) reduce to the well-known permittivities of an isotropic classical plasma.

In what follows, we will consider electrostatic collective effects only, defined by the longitudinal permit tivity $\varepsilon^l(\omega,k)$. In a quantum plasma with completely degenerate electrons ($k_BT < E_F$), Eq. (9) yields [17]

$$\varepsilon^{l}(\omega,\mathbf{k}) = 1 + \frac{3\omega_{pe}^2}{2k^2v_F^2}\left[1 - g(\omega_+) + g(\omega_-)\right], \quad (11)$$

where $\omega_\pm=\omega\pm\hbar k^2/2m_e$, and

$$g(\omega_\pm) = \frac{m_e\left(\omega_\pm^2 - k^2v_F^2\right)}{2\hbar k^3 v_F}\log\left(\frac{\omega_\pm + kv_F}{\omega_\pm - kv_F}\right), \quad (12)$$

in which log(u)= log(|u|)−iπ if u<0.

2.2. CHARGE SCREENING

Consider a test point charge qt at rest in a completely degenerate plasma. The Fourier component of the electrostatic potential of q_t in the plasma is $\varphi_k = 4\pi q_t/k^2\varepsilon^l(0,k)$, with $\varepsilon^l(0,k)$ being the static limit $\omega\to 0$ of (11). Thus the potential $\varphi(r)$ of the charge q_t is

$$\phi(r) = \int \phi_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\frac{d\mathbf{k}}{(2\pi)^3} = \frac{1}{2\pi^2 r}\Im\int_0^\infty \phi_{\mathbf{k}} e^{ikr}k\,dk. \quad (13)$$

The asymptotic behavior of $\varphi(r)$ at $r \to \infty$ is defined by the contribution into the integral in (13) of the so-called Kohn singularity of the function φ_k at $\hbar k = 2m_ev_F$, where an argument of one of the logarithms in $\varepsilon^l(0,k)$ becomes zero [17]. Near this singularity, φ_k is approximated as

$$\phi_{\mathbf{k}} = \frac{q_t\pi\hbar^2}{\beta m_e^2 v_F^2}\left[1 + \frac{\alpha}{\beta}\xi\log\frac{1}{|\xi|}\right], \quad (14)$$

where $\xi = (\hbar k-2m_ev_F)/2m_ev_F$, $\alpha = e^2/2\pi\hbar v_F$ and β is a constant. The resulting contribution of this singularity to $\varphi(r)$ at $r\to\infty$ is [17]

$$\phi_{\mathrm{Kohn}}(r) = \frac{q_t\alpha\hbar^2}{2\beta^2 m_e^2 v_F^2}\frac{\cos(2m_e v_F r/\hbar)}{r^3}. \quad (15)$$

At large distance from the charge q_t, the power-law attenuated contribution $\varphi_{Kohn}(r)$ due to the Kohn singularity dominates over the exponentially attenuated Debye-like contribution $\varphi_D(r) \propto (q_t/r)\exp(-r/\lambda_F)$ due to the integration in (1√3) away from the Kohn singularity (here $\lambda_F = v_F / 3\omega_{pe}$ is the Thomas-Fermi length analogous to the Debye length in a classical plasma). Thus the shielding of a stationary test charge in a degenerate quantum plasma is qualitatively different from the Debye shielding in a classical plasma. A similar effect also exists for a moving charge in a degenerate quantum plasma [18].

2.3. LANGMUIR OSCILLATIONS

Like a classical plasma, a quantum plasma also supports Langmuir oscillations, whose dispersion and damping are defined by the dispersion equation $\varepsilon^l(\omega, \mathbf{k}) = 0$, with ε^l defined by Eq. (11) for a completely degenerate plasma. In the semiclassical range of k, $hk < m_ev_F$, Eq. (11) becomes

$$\varepsilon^{l}(\omega,k) = 1 + \frac{3\omega_{pe}^2}{k^2v_F^2}\left[1 - \frac{\omega}{2kv_F}\log\left(\frac{\omega + kv_F}{\omega - kv_F}\right)\right] + i3\pi\omega_{pe}^2\omega/(kv_F)^3\sigma\left(kv_F - |\omega|\right), \quad (16)$$

where $\sigma(x)$ is the Heaviside step function. The corresponding dispersion of Langmuir waves for $kv_F/\omega_{pe} < 1$, $\hbar k < m_ev_F$ is then [19]

$$\omega \approx \omega_{pe}\left[1 + \frac{3}{10}\left(\frac{kv_F}{\omega_{pe}}\right)^2\right], \quad (17)$$

which is analogous to the dispersion of Langmuir waves in a classical plasma with Maxwellian electrons, for $kv_{Te}/\omega_{pe} < 1$. For $kv_F/\omega_{pe} > 1$ (but still $\hbar k < m_ev_F$), however, the spectrum of Langmuir oscillations approaches an intrinsically quantum "zero sound" mode
$\omega = kv_F$ of a Fermi gas [17, 20]:

$$\omega = kv_F\left[1 + 2\exp\left(-\frac{2k^2v_F^2}{3\omega_{pe}^2} - 2\right)\right]. \quad (18)$$

The Landau damping of Langmuir oscillations in a degenerate quantum plasma also differs significantly from that in a classical Maxwellian plasma. In the semiclassical range of k, $\hbar k < m_ev_F$, the phase velocity of Langmuir oscillations exceeds the maximum velocity of plasma electrons, $\omega/k > v_F$ [as seen from (17) and (18)], so that the Cherenkov resonance condition $\omega = \mathbf{k}\,\mathbf{v}$ is not met for any plasma electrons, and hence the Landau damping rate is *exactly* zero. For $\hbar k \geq m_ev_F$, however, the Cherenkov condition of particle-wave resonance starts to be significantly modified by the quantum recoil effect, and becomes, for non-relativistic electrons [11, 21]

$$\omega = \mathbf{k}\cdot\mathbf{v} \pm \frac{\hbar}{2m_e}\left(k^2 - \frac{\omega^2}{c^2}\right) \approx \mathbf{k}\cdot\mathbf{v} \pm \frac{\hbar k^2}{2m_e} \quad (19)$$

(the latter approximation takes into account that for Langmuir oscillations $\omega/k \approx v_F < c$ at $hk > m_e v_F$). At large k, the phase velocity of Langmuir oscillations differs from v_F only by an exponentially small term [see Eq. (18)], and thus the quantum-modified resonance condition (19) becomes satisfied for electrons on the Fermi sphere (with $|\mathbf{v}| = v_F$) for

$$k > k_c \approx \frac{\omega_{pe}}{v_F}\sqrt{\frac{3}{2}\left(\left|\log\frac{\hbar\omega_{pe}}{4E_F}\right| - 1\right)}. \quad (20)$$

For $k > k_c$, the number of electrons participating in Landau damping of Langmuir oscillations quickly increases with k, and the oscillations become strongly damped. Thus, unlike in a classical plasma with Maxwellian electrons, the Langmuir oscillations in a completely degenerate weakly coupled quantum plasma are undamped for $k < k_c$, and become strongly Landaudamped
for $k > k_c$.

2.4. ELECTROSTATIC SURFACE OSCILLATIONS

Finally, let us consider electrostatic surface oscillations of a semi-bounded degenerate plasma with a sharp boundary at which the electrons are perfectly reflected, bounded by a vacuum. An initial-value problem for the charge density in such system has a solution in the form [22, 23]

$$\rho(\mathbf{k}, t) = \frac{1}{2\pi}\int_{i\sigma-\infty}^{i\sigma+\infty} \rho(\omega, \mathbf{k}) e^{-i\omega t} d\omega, \quad \sigma > 0, \quad (21)$$

where $\rho(\omega, \mathbf{k})$ is found to b

$$\rho(\omega, \mathbf{k}) = \frac{iI(\omega, \mathbf{k})}{\varepsilon^l(\omega, \mathbf{k})} + \frac{ik_\parallel}{2\pi\zeta(\omega, k_\parallel)}\left[1 - \frac{1}{\varepsilon^l(\omega, \mathbf{k})}\right] \times \int_{-\infty}^{+\infty} \frac{dk'_x}{k'^2 \varepsilon^l(\omega, \mathbf{k}')} I(\omega, \mathbf{k}'), \quad (22)$$

$$\zeta(\omega, k_\parallel) = \frac{1}{2}\left(1 + \frac{k_\parallel}{\pi}\int_{-\infty}^{+\infty} \frac{dk_x}{k^2 \varepsilon^l(\omega, \mathbf{k})}\right), \quad (23)$$

with $\mathbf{k}' = (k'_x , \mathbf{k}_\parallel)$, k_x and $\mathbf{k}_\parallel$ are the components of $\mathbf{k}$ perpendicular and parallel to the plasma boundary, respectively, and

$$I(\omega, \mathbf{k}) = e\int d\mathbf{v} \frac{G(\mathbf{v}, \mathbf{k})}{\omega - \mathbf{k}\cdot\mathbf{v}},$$

where $G(\mathbf{v}, \mathbf{k})$ is the Fourier transform in x direction (normal to the plasma boundary)of the initial perturbation $\delta f\,(t = 0)$of electron distribution function, $G(\mathbf{v}, \mathbf{k}) = \int_{+\infty} dx \exp(-ik_x x)\delta f(x, v_x, \mathbf{k}_\parallel, \mathbf{v}_\parallel, t = 0)$.

To obtain the asymptotic behavior of $\rho(\mathbf{k}, t)$ at $t \to \infty$, one can perform the integration in (21) be shifting the integration contour from the upper semiplane in complex ω down into the lower semiplane, $\Im(\omega) < 0$, deforming it in such a way as to avoid any singularities of the integrand (analytically continued into $\Im(\omega) < 0$) in the lower semiplane. The least damped contributions of such singularities into the integral in (21) give rise to observable oscillations in the system. The first term in (22) gives rise to the Langmuir oscillations discussed above, due to the poles at $\varepsilon^l(\omega, \mathbf{k}) = 0$ [with some general assumptions about the initial perturbation, contribution of singularities of $I(\omega, \mathbf{k})$ into the inverse Laplace transform (21) quickly decay with time, and are not considered]. The second term in (22) appears entirely due to the boundary, and leads to the surface oscillations. Beside the singularities of $I(\omega, \mathbf{k})$ and $\varepsilon^l(\omega, \mathbf{k})$, its contribution into (21) are due to the singularities of the function $\zeta(\omega, k_\parallel)$ analytically continued into the lower semiplane $\Im(\omega) < 0$ of complex ω. For $\zeta(\omega, k_\parallel)$ defined by (23) for $\Im(\omega) > 0$, the analytical continuation into $\Im(\omega) < 0$ leads to two kinds of singularities of the continued $\zeta(\omega, k_\parallel)$ in the lower semiplane of complex ω [23]: (i) poles of $1/\zeta(\omega, k_\parallel)$ at $\zeta(\omega, k_\parallel) = 0$, and (ii) branch cuts of $1/\zeta(\omega, k_\parallel)$. Below we consider the time evolution of their respective contributions into the integral (21) of $\rho(\mathbf{k}, t)$, along the contour shown in Fig. 2.

3. CONTRIBUTION OF POLES OF $1/\zeta(\omega, k_\parallel)$

The contribution of poles of $1/\zeta(\omega, k_\parallel)$ into (21) give rise to exponentially damped surface oscillations

$$\rho(\mathbf{k}, t) \propto e^{-|\gamma_s|t}\cos(\omega_s t), \quad (24)$$

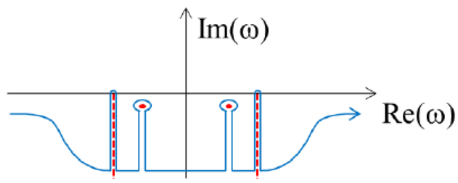

Fig. 2. The singularities of $1/\zeta(\omega, k_\parallel)$ at $\Im(\omega) < 0$ (poles, shown with circles, and branch cuts, shown with dashed lines), and the integration contour in (21) (solid blue line)

whose frequency $\omega_s(k_\parallel)$ and damping rate $\gamma_s(k_\parallel) < 0$ are the real and imaginary parts of the complex solution of the dispersion equation $\zeta(\omega, k_\parallel) = 0$, for $k_\parallel \in$ R [23]. The dispersion of such oscillations at small $k_\parallel\lambda_F$ (λ_F being the Thomas-Fermi length) is similar to that in a semi-bounded classical plasma with Maxwellian electrons: at $k_\parallel \to 0$ $\omega_s \to \omega_{pe}/\sqrt{2}$, from which it increases linearly with $k_\parallel\lambda_F$ (only the slope is different from that in the Maxwellian plasma). However, at $k_\parallel\lambda_F > 1$, the frequency of these surface oscillations tends to that of the intrinsically quantum zero sound mode,

$$\omega_s = k_\parallel v_F\left[1 + 2\exp\left(-\frac{4k_\parallel^2 v_F^2}{3\omega_{pe}^2} - 2\right)\right],$$

very similar to the spectrum of the Langmuir oscillations [cf. Eq. (18)]. Moreover, unlike in a Maxwellian plasma, in a degenerate quantum plasma the surface oscillations due to the poles of $1/\zeta(\omega, k_\parallel)$ remain weakly damped, $|\gamma_s/\omega_s| < 1$, for *all* values of $k_\parallel\lambda_F$, with a preferential (maximum) damping occuring at the wavelength $\lambda \approx 5\pi\lambda_F$ [23]. We note, however, that this result is valid only for a degenerate plasma with a sharp boundary; it remains to be seen how this result will change when the non-sharpness of the electron densityat the boundary is taken into account.

4. CONTRIBUTION OF BRANCH CUTS OF $1/\zeta(\omega, \mathbf{k}_{\parallel})$

The function $\zeta(\omega, k_{\parallel})$, analytically continued from its definition (23) at $\mathfrak{I}(\omega) > 0$ to $\mathfrak{I}(\omega) < 0$, also has two branching points at $\omega = \pm\omega_v(k_{\parallel}) \in$ R defined by the equation $\varepsilon^l(\omega, k_{\parallel}) = 0$ [with ε^l given by (16)], and two branch cuts shown in Fig. 2. The integration along these branch cuts yields another, intrinsically quantum, type of surface oscillation, with

$$\rho(\mathbf{k}, t) \propto \begin{cases} \cos(\omega_v t)/t^{1/2} + O(t^{-3/2}), \; k_x \to 0, \\ \cos(\omega_v t)/t^{3/2} + O(t^{-5/2}), \; k_x \neq 0 \end{cases} \tag{25}$$

which has a differrent frequency, and qualitatively different attenuation compared to the surface oscillations (24) due to the poles of $1/\zeta$. Thus we see that the electrostatic surface oscillations of a semi-bounded degenerate plasma are quite different from those in a classical Maxwellian plasma: there are two types of oscillations, one of them is an exponentially attenuated oscillation (24) (weakly damped at all wavelengths, unlike its counterpart in a Maxwellian plasma), the other is an intrinsically quantum, powerlaw attenuated oscillation (25) with a different frequency $\omega_v > \omega_s$ (for the same $k_{\parallel}$).

This work was supported by the Australian Research Council.

REFERENCES

1. S. Son, N.J. Fisch. // *Phys. Rev. Lett.* 2005, v. 95, p. 225002.
2. S.H. Glenzer et al. // *Phys. Rev. Lett.* 2007, v. 98, p. 065002.
3. Y.M. Klimontovich and V.P. Silin, Zh. Eksp. // *Teor. Fiz*. 1952, v. 23, p. 151.
4. Y.L. Klimontovich and V.P. Silin // *Sov. Phys. Usp*. 1960, v. 3, p. 84.
5. D. Bohm and D. Pines // *Phys. Rev*. 1953, v. 92, p. 609.
6. D. Bohm // *Phys. Rev*. 1953, v. 92, p. 626.
7. P.K. Shukl, B. Eliasson. // *Rev. Mod. Phys*. 2011, v. 83, p. 885.
8. A.L. Kritcher et al. // *Science*. 2008, v. 322, p. 69.
9. H.J. Lee et al. // *Phys. Rev. Lett*. 2009, v. 102, p. 115001.
10. Y.L. Klimontovich. *Statistical Physics.* Harwood Academic Publishers, 1986.
11. S.V. Vladimirov, Y.O. Tyshetskiy // *Phys. Usp*. 2011, v. 54, p. 1243.
12. N.R. Council. *Plasma Science: From Fundamental Research to Technological Applications.* The National Academies Press, 1995.
13. E. Wigner // *Phys. Rev*. 1932, v. 40, p. 749.
14. V.I. Tatarskii // *Sov. Phys. Usp.* 1983, v. 26, p. 311.
15. Y. Tyshetskiy, S.V. Vladimirov, R. Kompaneets // *Phys. Plasmas*. 2011, v. 18, p. 112104.
16. L.D. Landau // *J. Phys. USSR*. 1946, v. 10, p. 25.
17. E. M. Lifshitz, L.P. Pitaevskii. *Physical Kinetics.* Pergamon Press, 1981.
18. D. Else, R. Kompaneets, S.V.Vladimirov // *Phys. Rev*. 2010, v. E 82, p. 026410.
19. A.A. Vlasov, Zh. Eksp. // *Teor. Fiz.* 1938, v. 8, p. 291.
20. I.I. Gol'dman, Zh. Eksp. // *Teor. Fiz*. 1947, v. 17, p. 681.
21. V.S. Krivitskii, S.V. Vladimirov, Zh. Eksp. // *Teor. Fiz.* 1991, v. 100, p. 1483.
22. R.L. Guernsey // *Phys. Fluids*. 1969, v. 12, p. 1852.
23. Y. Tyshetskiy, D.J. Williamson, R. Kompaneets, S.V. Vladimirov // *Phys. Plasmas*. 2012, v. 19, p. 032102.

Article received 25.12.12

НЕОБЫЧНАЯ ФИЗИКА КВАНТОВОЙ ПЛАЗМЫ

Ю.О. Тышецкий, С.В. Владимиров, Р. Компанеец

Обсуждаются некоторые особенности коллективных эффектов в квантовой плазме. В частности, рассматриваются такие "элементарные" коллективные явления, как экранирование заряда, объемные и поверхностные колебания в вырожденной квантовой плазме, и обсуждаются их отличия от аналогичных явлений в классической плазме.

НЕЗВИЧАЙНА ФІЗИКА КВАНТОВОЇ ПЛАЗМИ

Ю.О. Тишецький, С.В. Владіміров, Р. Компанєєц

Обговорюються деякі особливості колективних ефектів у квантовій плазмі. Зокрема, розглядаються такі "елементарні" колективні явища, як екранування заряду, об'ємні та поверхневі коливання в виродженій квантовій плазмі, та обговорюються їх відмінності від аналогічних явищ у класичній плазмі