MAGNETIZED PLASMA IN STRONG ELECTRIC FIELD: FROM PARAMETRIC TURBULENCE TO ENHANCED CONFINEMENT

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This report is devoted to the unified kinetic approach to the theory of parametric turbulence and to the theory of plasma turbulence in strong shear flow across the confined magnetic field. The key point in that theory is the usage of the spatial and velocity variables which are co-moving with plasma particles in magnetic and strong electric field.

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INTRODUCTION

It is well known that the presence of the strong electric field in tokamaks and stellarators affects on the plasma turbulence and related phenomena. The application of the strong pumping field, used for the rf plasma heating in the ion cyclotron frequency range, leads to the relative oscillatory motion of plasma species, that resulted in the development of the parametric turbulence and effects of the anomalous absorption of the pumping wave, anomalous heating and diffusion of plasma components. The spontaneous appearance of the strong stationary radially inhomogeneous electric field in the edge layer of tokamak plasma leads to the poloidal rotation with radially inhomogeneous flow velocity. The low frequency plasma turbulence, and therefore the anomalous heat and particles diffusivity, appear suppressed by shear flow, that resulted in the formation of the improved energy-confinement regimes in tokamak plasmas (e.g. the H-mode regimes). The theory of plasma instabilities and turbulence are grounded on the application of spectral transforms on time and spatial coordinates and on the investigation of the spectral properties, stability and temporal evolution of the separate spectral harmonic. In spite of the paramount importance of the abovementioned principally different effects of the electric fields on the plasma turbulence and, as a result, on the performance of the controlled fusion, the analytical description of the stability and turbulence of the magnetized plasma in strong electric field suffer from inadequate application of the spectral transforms to plasma flows with non-stationary (as it is in the case of parametric turbulence) and inhomogeneous (as it is in the case edge shear flows) velocities. As a rule, in the kinetic theory of the magnetized plasma in electric field, the transformation to the reference frame, moving with the nonstationary or nonuniform equlibrium fluid velocity, but leaving unchanged spatial (laboratory) coordinates, is used. As a result, the nonstationarity and spatial inhomogeneity, introduced by flow velocity, remain in Vlasov equation and different dubious approximations were used for the application of the Fourier transforms to Vlasov and Maxwell equations. Here, we prove, that the application the "physical" approximations, that were used for the performance of the spectral transforms in such equation, make the results obtained senseless. We display, that the transformation both velocity and spatial coordinates in
Vlasov equation to convective reference frame is necessary for the excluding the inhomogeneities, introduced by external electric field from Vlasov equation

$$\frac{\partial F_\alpha}{\partial t}+\mathbf{v}\frac{\partial F_\alpha}{\partial \mathbf{r}} + \frac{e}{m_\alpha}\left(\mathbf{E}_0(\mathbf{r},t)+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]-\nabla\varphi(\mathbf{r},t)\right)\frac{\partial F_\alpha}{\partial \mathbf{v}}=0 \tag{1}$$

and performing the spectral transforms without additional approximations.

1. KINETIC THEORY OF PARAMETRIC PLASMA TURBULENCE

Here we compare two approaches to the theory of parametric instabilities, which were developed without the transformation to the convective spatial coordinates [1], and with transformation to convective (oscillatory) spatial coordinates [2]. The Fourier transform $\varphi(\mathbf{k},\omega)$ of the perturbed potential, performed in the laboratory frame, is governing by the equation[1]

$$\varphi(\mathbf{k},\omega)+\sum_{\alpha=i,e}\sum_{m\neq 0}\sum_{n=-\infty}^{\infty} J_n(a_\alpha)J_{n-m}(a_\alpha)e^{im\delta_\alpha}\varepsilon_\alpha(\mathbf{k},\omega+n\omega_0) \times\varphi(\mathbf{k},\omega+m\omega_0)=0. \tag{2}$$

In Eq.(2) $a_\alpha \sim k\delta r_\alpha$ and δr_α is the displacement of the particle of species α in pumping field $\mathbf{E}_0(t)=\mathbf{E}_0\cos(\omega_0 t)$. The application of the transformtion of the Vlasov equation (1) to the oscillatory ion frame of reference [2] gives other equation for the Fourier transform $\varphi_i(\mathbf{k},\omega)$ of the perturbed potential, [2]

$$\varepsilon_i(\mathbf{k},\omega)\varphi_i(\mathbf{k},\omega) +\sum_{m\neq 0}\sum_{n=-\infty}^{\infty} J_n(a_{ie})J_{n+m}(a_{ie})e^{im\delta_{ie}}\delta\varepsilon_e(\mathbf{k},\omega-n\omega_0)=0 \times\varphi_i(\mathbf{k},\omega+m\omega_0)=0, \tag{3}$$

where $a_{ie}\sim k\delta r$ and $\delta r=\delta r_e-\delta r_i$ is the displacement of the electrons relative to ions in pumping field. It follows from Eqs. (2), (3), that the possible growth rate of the parametric instabilities will be maximal, when the arguments of the Bessel functions are of the order of unity. For Eq. (2), the analysis of the case of $a_\alpha \sim 1$

requires the solution of the infinite set of equations, but analytically only the case of small a_α for this equation was treated yet [1], that gives the negligibly small growth rate[3] and main physical processes appear overlooked. In contrary, Eq. (3) admits the treating the case of $a_{ie} \sim 1$ and receiving for the ion cyclotron kinetic parametric instability[2] rather general equations for the frequency

$$1+\varepsilon_i(\mathbf{k},\omega(\mathbf{k}))+\frac{1}{k^2\lambda_{De}^2}=0,$$

and growth rate.

$$\gamma(\mathbf{k})=-\operatorname{Im}\sum_{\nu=-\infty}^{\infty}J_\nu^2(a_{ie})\varepsilon_e(\mathbf{k},\omega(\mathbf{k})-\nu\omega_0)$$
$$\times(\partial\operatorname{Re}\varepsilon_i(\mathbf{k},\omega(\mathbf{k})/\partial\omega(\mathbf{k}))^{-1}.$$

The transformation of the Vlasov equation to oscillatory coordinates was the decisive step in the development of the weak parametric turbulence[2], which requires the knowledge of the spectral properties of the instabilities for any values of the wave numbers.

2. KINETIC THEORY OF PLASMA SHEAR FLOWS

Now, we display, how effective is the transformation to convective-sheared coordinates in the kinetic theory of plasma shear flows stability. We consider the case of plasma shear flow in linearly changing electric field, $\mathbf{E}_0(\mathbf{r})=(\partial E_0/\partial x)x\mathbf{e}_x$ with $\partial E_0/\partial x=const$. In that case $\mathbf{V}_0(\mathbf{r})=V_0(x)\mathbf{e}_y=-(c/B)(\partial E_0/\partial x)x\mathbf{e}_y=V_0'x\mathbf{e}_y$ with spatially homogeneous, $V_0'=const$, velocity shear. The transformation of the Vlasov equation to convected, $v_{xs}=v_x,\ v_{ys}=v_y+V_0'x,\ v_{zs}=v_z$, and sheared, $x_s=x,\ \ y_s=y+V_0'tx,\ \ z_s=z$, coordinates excludes the spatial inhomogeneity introduced by inhomogeneous electric field to the Vlasov equation[4,5]

$$\frac{\partial f_\alpha}{\partial t}+v_{sx}\frac{\partial f_\alpha}{\partial x_s}+(v_{sy}-v_{sx}V_0't)\frac{\partial f_\alpha}{\partial y_s}+v_{\alpha sz}\frac{\partial f_\alpha}{\partial z_s}+$$
$$\omega_{c\alpha}v_{\alpha sy}\frac{\partial f_\alpha}{\partial v_{sx}}-(\omega_{c\alpha}+V_0')v_{\alpha x}\frac{\partial f_\alpha}{\partial v_{sy}} \qquad (4)$$
$$=\frac{e_\alpha}{m_\alpha}\left(\frac{\partial\varphi}{\partial x_s}-V_0't\frac{\partial\varphi}{\partial y_s}\right)\frac{\partial F_{0\alpha}}{\partial v_{sx}}+\frac{e_\alpha}{m_\alpha}\frac{\partial\varphi}{\partial y}\frac{\partial F_{0\alpha}}{\partial v_{sy}}+\frac{e_\alpha}{m_\alpha}\frac{\partial\varphi}{\partial z_s}\frac{\partial F_{0\alpha}}{\partial v_{sz}}$$

In that case, the Fourier transformation of the Vlasov equation over spatial sheared coordinates is performed exactly without application of the dubious approximations such as the "slow" spatial variation of the flow velocity[6,7]. With leading center coordinates X,Y, determined with convective-shearing coordinates by the relations

$$x_s=X-(v_\perp/\omega_c)\sin\phi,\quad z_1=z_s-v_zt,$$
$$y_s=Y+(v_\perp/\omega_c)\cos\phi+V_0't\,(X-x_s),$$

the Vlasov equation (1) obtains the canonical form

$$\frac{\partial F}{\partial t}+\frac{e}{m\omega_c}\left(\frac{\partial\varphi}{\partial X}\frac{\partial F}{\partial Y}-\frac{\partial\varphi}{\partial Y}\frac{\partial F}{\partial X}\right)$$
$$+\frac{e}{m}\frac{\omega_c}{v_\perp}\left(\frac{\partial\varphi}{\partial\phi_1}\frac{\partial F}{\partial v_\perp}-\frac{\partial\varphi}{\partial v_\perp}\frac{\partial F}{\partial\phi_1}\right)-\frac{e}{m}\frac{\partial\varphi}{\partial z_1}\frac{\partial F}{\partial v_z}=0, \qquad (5)$$

in which electrostatic potential is determined by the Fourier transform over the sheared coordinates x_s, y_s, z_s, determined through the leading center coordinates,

$$\varphi(x_s,y_s,z_s,t)=\int\varphi(k_{xs},k_{ys},k_{zs},t)$$
$$\times e^{ik_{xs}x_s+ik_{ys}y_s+ik_{zs}z_s}dk_{xs}dk_{ys}dk_{zs}$$
$$=\int\varphi(k_{xs},k_{ys},k_{zs},t)\exp\Big[ik_{xs}X_i+ik_{ys}Y_i+ik_{zs}z_s \qquad (6)$$
$$-i\frac{k_\perp(t)v_{\perp s}}{\omega_{ci}}\sin(\phi_1-\omega_{ci}t-\theta(t))\Big]dk_{xs}dk_{ys}dk_{zs},$$

where $k_\perp^2(t)=\left(k_{xs}-V_0'tk_{ys}\right)^2+k_{ys}^2$, and $\tan\theta=k_{ys}/(k_{xs}-V_0'tk_{ys})$. It follows from Eq. (6) that finite Larmor radius effect of the interaction of the perturbation with time independent wave numbers k_{xs},k_{ys},k_{zs} with ion, Larmor orbit of which is observed in sheared coordinates as a spiral continuously stretched with time, appears identical analytically to the interaction of the perturbation with wave numbers $k_{xs}-V_0'tk_{ys}$, k_{ys}, k_{zs} with ion, which rotates on the elliptical orbit that is observed in the laboratory frame. The time dependence of the finite Larmor radius effect is the basic linear mechanism of the action of the velocity shear on waves and instabilities in plasma shear flow.

One can come to these conclusions even without the transformation to the sheared coordinates, performing only the transformation to convective coordinates, as it is done usually (see, for example Refs. [6, 7]). In that case the Vlasov equation becomes

$$\frac{\partial f_\alpha}{\partial t}-ik_yV_0(x)f_\alpha-i(\mathbf{kv})f_\alpha+\omega_{c\alpha}v_{sy}\frac{\partial f_\alpha}{\partial v_{sx}}$$
$$-(\omega_{c\alpha}+V_\alpha')v_{sx}\frac{\partial f_\alpha}{\partial v_{sy}}=-\frac{e_\alpha}{m_\alpha}\mathbf{k}\varphi(\mathbf{k},t)\frac{\partial F_{0\alpha}}{\partial\mathbf{v}}. \qquad (7)$$

Usually (see, for example Refs.[6,7]), the approximation of the "slow" spatial variation of the flow velocity is applied. Then, velocity is absorbed into the identical for both plasma species Doppler shifted frequency [6,7] $\omega_D=\omega-k_yV_0(x)$, and, in fact, the velocity shear is excluded from the subsequent analysis. If we apply, however, the Fourier transform directly to (7) over the spatial coordinates in the laboratory frame (wave numbers are denoted as k_y,k_y,k_z) without the application of the approximation of the "slow" spatial variation of the flow velocity, i.e.

$$\frac{\partial f_\alpha}{\partial t}-V_0'k_y\frac{\partial f_\alpha}{\partial k_x}-i(\mathbf{kv})f_\alpha+\omega_{c\alpha}v_{sy}\frac{\partial f_\alpha}{\partial v_{sx}}$$
$$-(\omega_{c\alpha}+V_\alpha')v_{\alpha x}\frac{\partial f_\alpha}{\partial v_{sy}}=-\frac{e_\alpha}{m_\alpha}\mathbf{k}\varphi(\mathbf{k},t)\frac{\partial F_{0\alpha}}{\partial\mathbf{v}_s}, \qquad (8)$$

we obtain for f_α the solution $f_\alpha=f_\alpha(k_x+V_0'tk_y,k_y,k_z,t)$, where $k_x+V_0'tk_y=K_x$, where K_x as the integral of Eq. (8) is time independent. It reveals that the wave number components k_x and k_y

have to be changed in such a way that $k_x + V_0' t k_y$ leaves unchanged with time. Such solution to Eq. (8) for f_α can't be presented in the laboratory coordinates in a form, in which the time and spatial dependences are separable, as it is for the normal mode solutions of Eq. (7) obtained with assumption of the "slow" spatial variation of the flow velocity[6, 7]. If we use, however, $k_x = K_x - V_0' t k_y$ in Eqs. (4) and (5), we obtain for the electrostatic potential the presentation (5), and we obtain Eq.(4) for f_α, with time independent $K_x = k_{xs}$, $k_y = k_{ys}$, $k_z = k_{zs}$.

The obtained results prove, that the solution of the Vlasov equation in the form of the separate Fourier harmonic with time independent wave numbers may be obtained only in convected-sheared coordinates. That solution reveals in the laboratory frame as a shearing mode with time dependent x-component of the wave number, $k_x = k_{xs} + V_0' t k_{ys}$

The oversimplification of the problem, which resulted from the application of the assumption of "slow" spatial variation of $V_0(x)$, leads to the overlooking of that principal effect of shear flow. It is obvious, that the time dependence in K_x may be neglected only in the case of negligible velocity shear, or when the very short evolutionary time is considered. Really, for $k_y \sim k_x$ and $V_0' \sim \gamma$ for time $t > \gamma^{-1}$ the we have $k_y V_0' t > k_x$ in the integral $K_x = k_x + V_0' k_y t$. Therefore the assumption of "slow spatial variation of flow velocity" is not valid for the investigations of the effects of shear flow in real experiments, where observed velocity shearing rate may be of the order or above of the growth rate of the instability and the time of the observations is of the order of the inverse growth rate or longer.

CONCLUSIONS

The joint transformation of the velocity and spatial coordinates in Vlasov equation from laboratory to oscillatory set in the case of the parametric turbulence in the field of the strong pumping field, as well as the transformation from laboratory to convected-sheared coordinates in the case of strong shear flow in the crossed magnetic and inhomogeneous electric field, is the unavoidable for the proper treating the spectral properties of plasma in inhomogeneous non-stationary electric field.

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ЗАМАГНИЧЕННАЯ ПЛАЗМА В СИЛЬНОМ ЭЛЕКТРИЧЕСКОМ ПОЛЕ: ОТ ПАРАМЕТРИЧЕСКОЙ ТУРБУЛЕНТНОСТИ ДО УЛУЧШЕННОГО УДЕРЖАНИЯ

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Статья посвящена единому подходу к кинетической теории параметрической турбулентности и к теории турбулентности сильного сдвигового течения плазмы поперек удерживаемого магнитного поля. Ключевым моментом в этой теории является использование как скоростей, так и пространственных координат, определенных в системе координат, движущейся вместе с частицами в магнитном и электрическом полях.

ЗАМАГНІЧЕНА ПЛАЗМА В ПОТУЖНОМУ ЕЛЕКТРИЧНОМУ ПОЛІ: ВІД ПАРАМЕТРИЧНОЇ ТУРБУЛЕНТНОСТІ ДО ПОКРАЩЕНОГО УТРИМАННЯ

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Стаття присвячена єдиному підходу до кінетичної теорії параметричної турбулентності та до теорії турбулентності потужної зсувної течії плазми поперек утримуваного магнітного поля. Ключовим моментом у цій теорії є використання як швидкостей, так і просторових координат, визначених у системі координат, що рухається разом з частинками у магнітному та електричному полях.