STABILIZATION OF CLASSIC AND QUANTUM SYSTEMS

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It is shown that the mechanism of quantum whirligig can be successfully used for stabilization of classical systems. In particular, the conditions for stabilization of charged particles and radiation fluxes in plasma are found.

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INTRODUCTION

The quantum Zeno effect has been described in theoretical work [1]. Later this effect has been verified by experiments. The similar effect – the effect of quantum whirligig has been suggested in work [2-3]. Originally it has been formulated for quantum systems but later it appeared that the effect can be efficiently used for stabilization of classical systems too. The mechanism is very simple. Let's formulate the basic requirements which are necessary for realization of this effect. Assuming that we have a system state we want to stabilize and know characteristic life time of this state (T_L), it is necessary to involve the system in a fast changing process (e.g. a periodic process), with characteristic time (period) Δt essentially smaller than T_L. This process can stabilize the system's state. An elementary example of such stabilization is a children's toy whirligig stabilized in the vertical position. The rotation period of the whirligig is much smaller than the time necessary for it falling from the vertical position in absence of rotation. $\left(T_L >> \Delta t\right)$.

Below we show that this mechanism can be effectively used in plasma physics to suppress various instabilities. The conditions for stabilization of charged particles flow in plasma are formulated in section 2, when section 3 derives the conditions to suppress the decay and the explosive instability.

1. CHARGED PARTICLE BEAM STABILIZATION IN PLASMA

In this part we consider the following most simple model. Let we have a plasma cylinder ($0 < r < R_p$). It is placed in a metal cylinder of the same radius. All system is subjected to a strong external longitudinal magnetic field. The beam passes along the axis of the plasma cylinder. The beam radius coincides with the plasma radius. The cylinder has gaps for connection with an external electrodynamic structure (waveguide). A spiral with radius R_H can be an example of such structure. Frequency and wave vector of the eigenwave of this structure should coincide with the frequency and the wave vector of a wave excited by the beam in the plasma. The system of the equations which describes this system consists of the hydrodynamic equations for particles of the plasma and the beam and Maxwell equations for fields. One can look for solutions in the form $n_k \sim v_k \sim \exp(-ik_z z + i\omega t)$, $E_j \sim \exp(-ik_z z + i\omega t)H(r)$ which then lead to the following dispersion equation:

$$\left[1-\frac{\omega_p^2}{\omega^2}-\frac{\omega_b^2}{\left(\omega^*\right)^2}\right]-\frac{\mu\mu_1}{\left(\omega^2+k_\perp^2c^2\right)}\left(\frac{\omega_p^2}{\omega^2}+\frac{\omega_b^2}{\left(\omega^*\right)^2}\right)=0, \quad (1)$$

where μ_k – is the connectivity factor of the fields $\omega^* = \omega - k_z V$ and $k_\perp^2 = \left(\lambda_n^2 / R_s^2 - k_z^2\right)$ is the transversal wave vector of the wave in the external structure.

One can see that in absence of the connection between the plasma fields and the external structure fields ($\mu_j = 0$) one gets the usual dispersion equation of a plasma-beam system. On the contrary, if the beam is absent ($\omega_b = 0$) we get the dispersion equation which describes energy exchanging between plasma waves and the waves of the external structure. Frequency of such exchanging is equal to $\Omega = \sqrt{\mu\mu_1 / 4}$. Let's assume that when this frequency is lager then the increment of the beam-plasma instability, such instability can be suppressed. To check this assumption we consider the values of the parameters close to the limit of beam-plasma instability: $\omega \sim \omega_p \sim k_z V$,

$\left(\omega-\omega_p\right)=\left(\omega-k_zV\right)=\left(\omega-\left|k_\perp\right|c\right)=\delta$.

Thus, for the deviation δ one gets the following third order algebraic equation:

$$\delta^3 - \mu\mu_1\delta/4 - \omega_b^2\omega_p/2 = 0 .$$

This equation has three roots. The instability does not develop if all these roots are real. The condition for only real roots to appear (a condition to suppress the instability) is:

$$\sqrt{\mu\mu_1/4} > \left(\omega_b^2\omega_p/2\right)^{1/3} . \quad (2)$$

The decrease of the connectivity factor (or the increase of the beam density) leads to two real roots disappearing. This leaves only one real root when two complex conjugated roots appear and describe the instabilities increments.

2. STABILIZATION OF RADIATION FLOWS IN PLASMA

In this section we show that not only the flow of the charged particles in plasma but also the flow of radiation can be also stabilized. We illustrate this result by the three-wave interaction example. The processes of three-wave interaction play a fundamental role in the plasma physics. Below we show that if one of the waves participating in a three-wave interaction participates in some additional periodic process (stabilization process) the instabilities can be suppressed. The elementary system of the equations which describes such processes can be presented in the following form:

$$\frac{dA_0}{dt} = -VA_1A_2 + \frac{\mu}{2i}A_3, \quad \frac{dA_3}{dt} = \frac{\mu}{2i}A_0 \quad , \qquad (3)$$
$$\frac{dA_1}{dt} = VA_0A_2^*, \quad \frac{dA_2}{dt} = VA_1^*A_0 .$$

This system describes interaction of four waves. Two of them (zero and the third in our notation) are connected with each other through the linear connection factor μ. There is the periodic exchange of energy between the basic wave and the stabilization wave if other waves are absent. Frequency of such exchange is equal to $\Omega = \mu/2$. The system (3) at $\mu = 0$ describes ordinary three-wave interaction of the waves with the increment of the decay instability $\delta = V|A_0(0)|$. Let's notice that if we change the sign before the first term in the right side of the first equation in (3), then such system describes the explosive instability.

Below we show that the addition of the third wave (A_3) can suppress both the decay processes, and the process of explosive instability. It is necessary to notice that system (3) considers the connection between the zero-index wave and the stabilization wave (third) only. The same result can be obtained when any other wave (first or second wave) is involved in the process of the stabilizing interaction.

According to general ideology we assume that decay instability is suppressed as soon as the condition $\mu/2V > |A_0(0)|$ is satisfied. The left side of this inequality is the frequency of the energy exchange between the stabilizing wave and one of the waves participating in the three-wave interaction. The right side is the increment of the decay instability. We analyze system (3) numerically. For that it is convenient to introduce the following parameters and new real variables:

$$A_0 = x_0 + ix_1,\ A_1 = x_2 + ix_3,\ A_2 = x_4 + ix_5,\ A_3 = x_6 + ix_7$$
$$\varepsilon \equiv \mu/2V \quad , \quad \tau \equiv Vt .$$

The usual decay process is observed if the stabilizing wave is absent ($\varepsilon = 0$). The decay instability is stabilized in all cases when the wave A_3 (stabilization wave) is present and the condition $\mu/2V > |A_0(0)|$ is satisfied.

Stabilization of explosive instability. It is worth noticing that stabilization can be achieved for the explosive instability case too. Really, in Figs. 1-3 the dynamics of wave amplitudes is presented ($x_0(0) = 0.1,\ x_2(0) = 0.001,\ x_6(0) = 0.01$) at explosive instability in absence stabilization wave (see Fig. 1), and also dynamics of these amplitudes in the presence of a stabilization wave (see Fig. 2-3). It is visible from these figures that already at values of parameter $\varepsilon = 0.09$ full stabilization of explosive instability have been observed.

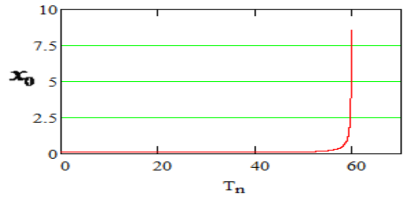

Fig. 1. Explosive instability at $\varepsilon = 0$

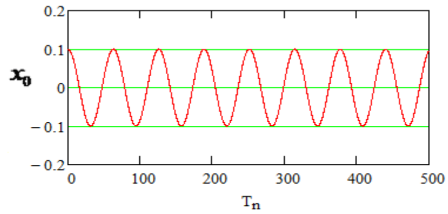

Fig. 2. Suppression of explosive at $\varepsilon = 0.09$

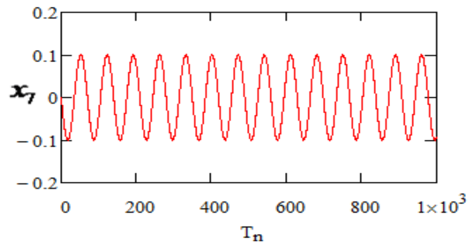

Fig. 3. Suppression of explosive at $\varepsilon = 0.09$

Only the basic wave (A_0) and the stabilization wave (A_3) show periodic oscillations and exchange energy between each other. Other waves almost not change. However at $\varepsilon = 0.08$ the explosion still appears but its occurrence is substantially delayed (to times of more then 400).

3. PARADOX OF TWO SOLUTIONS

The possibility of the stabilization of a quantum mechanics system has strict justification for an induced processes only (see [1]). It is reasonable to expect that spontaneous transitions can be stabilized under the same conditions. However, here one faces a contradiction. The solution of the quantum mechanics equations using usual perturbation theory does not provide such stabilization. There is a paradox. To illustrate this situation we consider the particular case of the synchrotron radiation stabilization. The system of the equations, which describe the dynamics of the amplitudes of the synchrotron radiation wave functions with the presence of the stabilizing perturbation, can be written as follows:

$$i\hbar\dot{A}_0 = V^+A_1\exp(-i\cdot\Delta E\cdot t/\hbar) \qquad i\hbar\dot{A}_2 = V_{21}\cdot A_1$$
$$i\hbar\dot{A}_1 = V^-A_0\exp(i\cdot\Delta E\cdot t/\hbar) + V_{12}\cdot A_2 \qquad (4)$$

Here $V^+ = \int\psi_0^+U^+\psi_1 d^3x$; $V^- = \int\psi_1^+U^-\psi_0 d^3x$;

$V_{12} = \int\psi_1^+U\psi_2 d^3x$; $V_{12} = V_{21}$; A_1 – amplitude of wave function of the excited state; A_0 – amplitude of wave function of the ground state; A_2 – amplitude of wave function of an additional state were the transitions as a result of the induced transition caused by stabilizing perturbation occur.

The system of equations (4) differs from the one presented in [3] only by presence of the additional equation for A_2, and also by the additional term in the right part in equation for A_1. This additional equation and the additional term describe the dynamics of the induced transitions between the basic (first) level and additional (second) level. The operators $U^{\pm}$ are defined in [4]. The operator U describes the potential of external periodic perturbation.

The system (4) allows the existence of two essentially different solutions. ***Let's consider the first***, strict solution. The solution in the spirit of accepted in the quantum mechanics. We have the induced process caused by presence of external perturbation. This process is much faster than the processes connected with the spontaneous transitions. Therefore in zero order approximation we can obtain such solution:

$$A_1 = \cos(\Omega \cdot t) \;\; A_2 = -i\sin(\Omega \cdot t), \tag{5}$$

where $\Omega = V_{12} / \hbar$.

According to the perturbation theory, one must substitute these solutions to the system (4). There is no stabilization if one does this. The spectra of spontaneous radiation slightly changes and there is the splitting of the spectra. In addition, there is some periodic modulation of the probability density. These changes do not significantly affect the life time of the levels. ***Let's consider the second solution***. For this purpose we introduce the following characteristic times: $\Delta t = \pi / 2\Omega$ the characteristic time of the transition from main stare to new state under the influence of the stabilizing perturbation and T_L the life time of the excited state in the absence of the perturbation ($T_L = \hbar \cdot R / r_0 \cdot mc^2 \cdot \gamma$, where r_0 – electron's classical radius; R – the radius of electron's orbit in synchrotron). Let's assume $\Delta t / T_L << 1$. Then the lifetime of the excited state is Δt and the probability of the transition from the excited state to the ground level under the influence of zero fluctuations is proportional to the square of the ration $\Delta t / T_L$. Thus the probability to stay on the excited level can be estimated by the expression:

$$w \sim \left[1 - \left(\Delta t / T_L\right)^2\right]. \tag{6}$$

The system returns to the initial excited level after the time $2\Delta t$. It occurs as the result of the induced transitions. The probability to stay on this excited level during next interval Δt can be estimated by the same formula (6). All these processes of transition are independent. Therefore the probability to remain at the excited level is proportional to the product of the probabilities to remain at the excited level in each of these intervals Δt. After time T_L the probability to remain at the excited level can be estimated by the formula:

$$w_n \sim exp(\Delta t / 2T_L).$$

One can see that at $\Delta t / T_L \to 0$ the probability to remain at the excited level converge to unit.

Therefore, we see that two ways of solving the equations (4) lead to two solutions which contradict each other. We choice second solution because only it corresponds to quantum Zeno effect.

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СТАБИЛИЗАЦИЯ КЛАССИЧЕСКИХ И КВАНТОВЫХ СОСТОЯНИЙ

В.А. Буц

Показано, что механизм квантовой юлы может быть эффективно использован для стабилизации классических систем. В частности, найдены условия стабилизации потоков заряженных частиц и потоков излучения в плазме.

СТАБІЛІЗАЦІЯ КЛАСИЧНИХ ТА КВАНТОВИХ СТАНІВ

В.О. Буц

Доведено, що механізм квантової дзиґи може бути ефективно використовуватися для стабілізації класичних систем. Зокрема, знайдені умови стабілізації потоків заряджених часток та потоків випромінювання в плазмі.