THE LOWER HYBRID WAVES DRIVEN BY INHOMOGENEOUS ION-RING DISTRIBUTION

D.V. Chibisov, V.S. Mikhailenko
V.N. Karazin Kharkov National University,
Kharkov, Ukraine
E-mail: vmikhailenko@kipt.kharkov.ua

The electrostatic lower hybrid waves driven by the ion-ring distribution with a radially inhomogeneous density in homogeneous Maxwellian plasma are studied. It is shown that the main mechanism of excitation of instability is the ion-ring current across the magnetic field, although the effect of inhomogeneity of the ion-ring distribution on the conditions of instability also takes place. As a possible mechanism of saturation of instability the scattering of ions of ring distribution by random fluctuations of the electrostatic potential is considered.

PACS: 94.05.Lk, 94.20.wf

INTRODUCTION

Lower hybrid waves (LHWs) are commonly found in laboratory and space plasmas [1-3]. One of the possible mechanisms of LHWs excitation is the ion-ring distribution in velocity space which occurs due to injection of ions perpendicular to the magnetic field in laboratory, and precipitation of positive ions from magnetosphere into a region of increased geomagnetic field strength as well as various ion energization processes in space plasmas.

The theoretical model, used for the analysis of the stability of LHWs in plasmas with ion-ring distribution in velocity space, usually assumed plasma components as a spatially homogeneous or weakly inhomogeneous and infinite, so that the local approximation in slab model is valid [4]. This approximation is unjustified, however, in the laboratory experiments [1] as well as for small-scale ion beams in the earth's ionosphere [5], where radial spread of ions orbits are of the order of or even much less than the ion Larmor radius and strong spatial inhomogeneity of ion-ring distribution and cylindrical geometry of plasma should be taken into account. The same conclusion is applicable for the ions in the localized structures in the earth's ionosphere, such as lower hybrid solitary structures, where some of the ions can have a ring-like distribution with axis encircling orbits.

1. LINEAR THEORY

We start from the integral equation for the Fourier-Bessel transform $\Phi_m\left(K,\omega\right)$ of the perturbed electrostatic potential in the cylindrical geometry which takes into account the rotation of particles around the axis of symmetry [6, 7]

$$\Phi_m\left(K,\omega\right)+8\pi^2\sum_\alpha\frac{e_\alpha^2\left(2\Omega_\alpha+\omega_{c\alpha}\right)^2}{k^2 m_\alpha}\sum_{n=-\infty}^{\infty}\int_0^\infty R_\alpha dR_\alpha\int_0^\infty \rho_\alpha d\rho_\alpha$$
$$\times\int_{-\infty}^{\infty}dv_z\int_0^\infty dk_{1\perp}k_{1\perp}\Phi_m\left(k_{1\perp},k_z,\omega\right)J_{m+n}\left(k_\perp R_\alpha\right)J_{m+n}\left(k_{1\perp}R_\alpha\right)$$
$$\times\frac{J_n\left(k_\perp\rho_\alpha\right)J_n\left(k_{1\perp}\rho_\alpha\right)}{\omega-m\Omega_\alpha-n\left(2\Omega_\alpha+\omega_{c\alpha}\right)-k_z v_z}\left[\frac{n}{\left(2\Omega_\alpha+\omega_{c\alpha}\right)}\frac{\partial F_{0\alpha}}{\rho_\alpha\partial\rho_\alpha}\right.$$
$$\left.+\frac{m+n}{\left(2\Omega_\alpha+\omega_{c\alpha}\right)}\frac{\partial F_{0\alpha}}{R_\alpha\partial R_\alpha}+k_z\frac{\partial F_{0\alpha}}{\partial v_z}\right]=0\,, \qquad (1)$$

where α denotes electrons $(\alpha=e)$, plasma and beam ions $(\alpha=i,b)$, $K=\left(k_\perp,k_z\right)$, $k^2=k_\perp^2+k_z^2$, m is the azimuthal wave number, m_α is the mass of particles of α species, Ω_α is the angular velocity of the particles of α species, R_α and $\rho_\alpha=v_\perp/|2\Omega_\alpha+\omega_{c\alpha}|$ are the radial position of the guiding center and Larmor radius respectively, $F_{0\alpha}$ is the unperturbed distribution function of particles of α species. For plasma particles equality $\Omega_e=\Omega_i=0$ holds whereas for beam ions we assume that $\Omega_b=-\omega_{cb}$.

Plasma is considered homogeneous and Maxwellian whereas the distribution function of the ion beam in laboratory frame of reference we take in form

$$F_{0b}\left(\rho_b',R_b'\right)\propto\rho_b'^2\exp\left(-\frac{\rho_b'^2}{2\rho_{Tb}'^2}-\frac{R_b'^2}{2R_{0b}'^2}-\frac{v_z^2}{2v_{Tbz}^2}\right), \qquad (2)$$

where ρ_{Tb}' is the thermal Larmor radius, R_{0b}' is the characteristic length of inhomogeneity of the guiding centers of the beam ions. It is assumed in Eq. (2) that inequality $\rho_b'>R_b'$ holds. The distribution function (2) corresponds to the ring distribution in velocity space where the dependence in radial position of the guiding centers of ions is also taken into account. In Ref. [6] was shown that in the frame of reference rotating with angular velocity $\Omega_b=-\omega_{cb}$, the Larmor radius ρ and guiding center radial coordinate R of ions related with ρ' and R' which are the Larmor radius and guiding center radial coordinate in laboratory frame of reference by relationships $\rho=R'$, $R=\rho'$. In the frame of reference rotating with angular velocity $\Omega_b=-\omega_{cb}$ around the axis of symmetry the distribution function (2) takes the form

$$F_{0b}\left(\rho_b,R_b\right)\propto R_b^2\exp\left(-\frac{\rho_b^2}{2\rho_{Tb}^2}-\frac{R_b^2}{2R_{0b}^2}-\frac{v_z^2}{2v_{Tbz}^2}\right), \qquad (3)$$

where equalities $\rho=R', R=\rho'$, as well as $\rho_{Tb}=R_{0b}', R_{0b}=\rho_{Tb}'$ are used. Unlike to Eq. (2) the distribution function of the Larmor radius (3) is the Gaussian and ring like in the radial position of the guiding centers of ions. Using distribution function (3) we integrate Eq. (1). The integration of electron and plasma ion terms is a simple and for ion beam term we use the integration procedure over the variables R and $k_{1\perp}$ in asymptotic limit $k_\perp R_{0b} \sim m \gg 1$ which is presented in Ref. [6]. This gives in the frequency range $\omega \gg \omega_{ci}, \omega_{cb}$ the following dispersion relation

$$\varepsilon(K,m,\omega) = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\sin^2\theta - \frac{\omega_{pe}^2}{\omega^2}\cos^2\theta +$$

$$+\sum_{i,b}\frac{1}{k^2\lambda_{Di,b}^2}\left(1 + i\sqrt{\pi}z_{i,b}W\left(z_{i,b}\right)\right) = 0\,, \qquad (4)$$

where $z_i = \omega/\sqrt{2}k_\perp v_{Ti}$, $z_b = \left(\omega + m\omega_{cb} - m\omega_{b*}\right)/\sqrt{2}k_\perp v_{Tb}$, $\eta_b = \left(\omega + m\omega_{cb} - m\omega_{b*}\right)/\omega_{cb}$, $\eta_i = \omega/\omega_{ci}$, $\omega_{*b} = -\omega_{cb}\rho_{Tb}^2 \times \left(d\ln n_{0b}\left(r_0\right)/r_0 dr_0\right)$ is the drift frequency of beam ions and $n_{0b}\left(r_0\right)$ is the number density of beam ions which is ring like for distribution function (3) where radial coordinate r is replaced by variable $r_0 = |m|/k_\perp$. Equation (4) in the asymptotic limit $|m| \gg 1$ determines the dispersion properties of cylindrical waves in the form of Bessel functions $J_m\left(k_\perp r\right)$. The value $r_0 = |m|/k_\perp$ in the number density dependence approximately is the radial coordinate of the first maximum of Bessel function which separates the oscillatory and aperiodic parts of this function. The dielectric permittivities for plasma and beam ions in Eq. (4) correspond to the approximation of straight orbits of ions perpendicular to the magnetic field (approximation of null magnetic field). Despite this approximation the cyclotron motion of beam ions in expressions for z_b and η_b is taken into account by the term $m\omega_{cb}$ which is the Doppler shift due to rotation around the axis of symmetry. We now assume that inequality $|z_i| \gg 1$ is met. However we suppose that Landau damping for the beam ions is significant, i.e. equality $|z_b| \sim 1$ hold. This equality is possible when the resonant condition $|\omega + m\omega_{cb} - m\omega_{b*}| \approx \sqrt{2}k_\perp v_{Tb}$ is satisfied. In this case the dispersion relation takes the form

$$\varepsilon(K,m,\omega) = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\sin^2\theta - \frac{\omega_{pe}^2}{\omega^2}\cos^2\theta - \frac{\omega_{pi}^2}{\omega^2}\sin^2\theta$$

$$+\frac{1}{k^2\lambda_{Db}^2} + i\frac{1}{k^2\lambda_{Db}^2}\sqrt{\pi}z_b\exp\left(-z_b^2\right) = 0\,. \qquad (5)$$

Equation (6) under assumptions $k_z/k < m_e/m_i$, as well as $k^2\lambda_{Db}^2 > 1$ yields the real frequency $Re\,\omega_m(K)$ and growth rate $\gamma_m(K)$ for lower hybrid waves

$$Re\,\omega_m\left(k_\perp\right) = \omega_{LH} = \frac{\omega_{pi}}{\sqrt{1+\omega_{pe}^2/\omega_{ce}^2}}\,, \qquad (6)$$

$$\gamma_m\left(k_\perp\right) \approx -\sqrt{\pi}\omega_{LH}\frac{\omega_{pb}^2\left(r_0\right)}{\omega_{pi}^2}\frac{\omega_{LH}^2}{2k_\perp^2 v_{Tb}^2}$$

$$\times\frac{\left(\omega_{LH} + m\omega_{cb} - m\omega_{b*}\right)}{\sqrt{2}k_\perp v_{Tb}}\exp\left(-z_b^2\right). \qquad (7)$$

From Eq. (7) follows, that the growth rate γ is positive and lower hybrid waves become unstable when the azimuthal wavenumber $m<0$ and satisfied inequality $|m| > \omega_{LH}/\left(\omega_{cb} - \omega_{b*}\right) \gg 1$. It follows that in the region of beam, where $\nabla n_{0b}(r) > 0$ unstable waves have a greater azimuthal wave numbers, than where $\nabla n_{0b}(r) < 0$. The instability develops due to inverse Landau damping caused by thermal motion of beam ions perpendicular to the magnetic field when resonant condition $|\omega_{LH} + m\omega_{cb} - m\omega_{b*}| \approx \sqrt{2}k_\perp v_{Tb}$ for ion beam propagating perpendicular to the magnetic field is met. If the beam and plasma ions are the same species, i. e. $m_i = m_b$, then in Eq. (7) $\omega_{pb}^2\left(r_0\right)/\omega_{pi}^2 = n_b\left(r_0\right)/n_i$. The approximation of null magnetic field is valid, when inequality $\gamma > \omega_{cb}/2\pi$ holds. This gives the condition for the ratio of the number densities of the beam and plasma ions $n_b/n_i > \left(\omega_{cb}/\omega_{LH}\right)\left(\rho_{Tb}^2/R_{0b}^2\right)$, where the right-hand side is much less than unity. Thus LHWs may be excited by a low-density ion-ring beam.

2. NONLINEAR SATURATION OF THE LOWER HYBRID INSTABILITY

As a possible mechanism of saturation of the lower hybrid instability driven by ion-ring beam we consider the effect of scattering of beam ions by the random fluctuations of the electrostatic turbulence. This effect is taken in consideration by the renormalized theory [9]. The renormalization of the dispersion relation (5) consists in change of the oscillation frequency ω by their modified values $\omega + i\nu_{i,b}$ where $\nu_{i,b}$ are the effective frequencies of collisions between plasma and beam ions and fluctuations of electrostatic potential which for cylindrical geometry are given in Ref. [10]. In the asymptotic limit $\omega_m\left(k_\perp\right) \gg \omega_{ci,b}$ the values for the effective frequency of collisions with electrostatic high frequency turbulence in the saturated state are

$$\nu_i\left(k_\perp\right) \sim \frac{k_\perp^2 e_i^2}{m_i^2\omega_{ci}^2}\int dk'_\perp k'_\perp \frac{k'^2_\perp I_m\left(k'_\perp\right)\left(\nu_i\left(k'_\perp\right) + \gamma_{NL}\right)}{\omega'^2\left(k'_\perp\right) + \left(\nu_i\left(k'_\perp\right) + \gamma_{NL}\right)^2}$$

$$\times\frac{1}{2\pi}\int_0^{2\pi} J_m^2\left(k'_\perp\sqrt{R_i^2 + \rho_i^2 - 2R_i\rho_i\cos\phi}\right)d\phi\,, \qquad (8)$$

$$\nu_b\left(k_\perp\right) \sim \frac{k_\perp^2 e_b^2}{m_b^2\omega_{cb}^2}\int dk'_\perp k'_\perp \frac{k'^2_\perp I_m\left(k'_\perp\right)\left(\nu_b\left(k'_\perp\right) + \gamma_{NL}\right)}{k'^2_\perp v_{Tb}^2 + \left(\nu_b\left(k'_\perp\right) + \gamma_{NL}\right)^2}$$

$$\times\frac{1}{2\pi}\int_0^{2\pi} J_m^2\left(k'_\perp\sqrt{R_b^2 + \rho_b^2 - 2R_b\rho_b\cos\phi}\right)d\phi \qquad (9)$$

for plasma and beam ions respectively. In Eqs. (8), (9) $I_m\left(k_\perp\right)$ is the spectral intensity of cylindrical waves [10], γ_{NL} is the nonlinear growth rate of fluctuations. The level of saturation of the instability is determined by taking the lowest of these values obtained from the equations of balance of instability growth with linear growth rate $\gamma(k)$ and nonlinear damping resulted from plasma and beam ions scattering by lower hybrid turbulence, $\nu_{i,b}(k) = \nu_{i,b}(k') = \gamma(k)$. It turns out that the lowest level of saturation gives the beam ions. The saturation of instability occurs when the value ν_b reaches the magnitude of the order of $k_\perp v_{Tb}$ so that z_b in Eq. (6)

exceeds unity and energy from Landau damping in the excitation of instability is reduced. The level of saturation of instability estimated from (9) approximately equals

$$\frac{e_b \langle \Phi(r) \rangle}{T_b} \approx \frac{\gamma(r)}{\omega_{LH}} \frac{\omega_{cb}}{\omega_{LH}} \frac{R_{0b}^2}{\rho_{Tb}^2},$$

where $\langle \Phi(r) \rangle = \left(\sum_{m'} \int I_{m'}(k'_\perp, r) J_m^2(k'_\perp r) k'_\perp dk'_\perp \right)^{1/2}$ is a root-mean-square amplitude of the perturbed electrostatic potential. In Eq. (10) a weak dependence on the radius is taken into account. This dependence is determined by corresponding dependence of the number density of beam ions, which is in the expression for growth rate (7). Note that the value of the perturbed electrostatic potential can reach values of order T_b when the ratio of the number densities of the beam and plasma ions is approximately ρ_{Tb}^2 / R_{0b}^2.

CONCLUSIONS

1. The excitation of LHWs in a homogeneous plasma with inhomogeneous ion-ring distribution is caused by an azimuthal ion current across the magnetic field.

2. The resonance condition $|\omega_{LH} + m\omega_{cb} - m\omega_{b*}| \approx \sqrt{2} k_\perp v_{Tb}$ for excitation of LHWs must be satisfied.

3. The waves growth occurs due to the inverse ion-beam Landau damping when the azimuthal wave numbers satisfied inequality $|m| > \sqrt{2} k_\perp v_{Tb} / (\omega_{cb} - \omega_{b*})$ under condition $m < 0$.

4. The ratio of the number density of the ring-ion beam to plasma density must be exceed the value $n_b / n_i > (\omega_{cb} / \omega_{LH})(\rho_{Tb}^2 / R_{0b}^2)$.

5. In the region of the inhomogeneous ion-ring distribution with a positive density gradient, the unstable waves have the greater absolute values of the azimuthal wave numbers than in the region where the density gradient is negative.

6. The saturation of instability occurs due to scattering of the beam ions in the random fluctuations of the electrostatic turbulence.

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Article received 19.09.12

НИЖНЕГИБРИДНЫЕ ВОЛНЫ, ВОЗБУЖДАЕМЫЕ НЕОДНОРОДНЫМ ИОННЫМ КОЛЬЦЕВЫМ РАСПРЕДЕЛЕНИЕМ

Д.В. Чибисов, В.С. Михайленко

Исследуются электростатические нижнегибридные волны, обусловленные радиально-неоднородным ионным кольцевым распределением в пространстве скоростей в однородной максвелловской плазме. Показано, что основным механизмом возбуждения неустойчивости является ионный кольцевой ток поперек магнитного поля, хотя на условия неустойчивости также влияет неоднородность ионного кольцевого распределения. В качестве возможного механизма насыщения неустойчивости рассматривается рассеяние ионов кольцевого распределения случайными флуктуациями электростатического потенциала.

НИЖНЬОГІБРИДНІ ХВИЛІ, ЗБУДЖУВАНІ НЕОДНОРІДНИМ ІОННИМ КІЛЬЦЕВИМ РОЗПОДІЛОМ

Д.В. Чібісов, В.С. Михайленко

Досліджуються електростатичні нижньогібридні хвилі, обумовлені радіально-неоднорідним іонним кільцевим розподілом у просторі швидкостей в однорідній максвеллівській плазмі. Показано, що основним механізмом збудження нестійкості є іонний кільцевий струм поперек магнітного поля, хоча на умови нестійкості також впливає неоднорідність іонного кільцевого розподілу. В якості можливого механізму насичення нестійкості розглядається розсіювання іонів кільцевого розподілу випадковими флуктуаціями електростатичного потенціалу.