

PHASE AND GROUP VELOCITIES OF ELECTROMAGNETIC EIGEN WAVES OF LEFT-HAND MATERIAL SLAB

N.A. Azarenkov, V.K. Galaydych, V.P. Olefir, A.E. Sporov

V.N. Karazin Kharkov National University, Kharkov, Ukraine

E-mail: viktor.galaydych@gmail.com

This work is devoted to the study of phase and group velocities of the surface eigen electromagnetic waves that propagate along the left-handed planar slab that bounded by the dielectrics with different permittivity values at the both sides of considered slab. It has been shown that the change of metamaterial slab thickness substantially influences on dispersion of the waves under investigation.

PACS: 52.35g, 52.50.Dg

INTRODUCTION

The modern composite materials, that known as metamaterials show fascination electrodynamics properties. At present time the intensive theoretical and experimental studies of metamaterials are carried out in numerous scientific laboratories all over the world. These studies are stipulated by the perspective usage in different technological applications, such as image processing, designing of subwavelength devices, and optical cloaking and superlensing [1-4]. The basic feature of these materials is the fact that the directions of the group and phase velocities of electromagnetic bulk waves in such materials are opposite directed [1].

In recent years the new artificial materials have been created with both negative effective permittivity and effective permeability over some frequency ranges [2,3]. The materials of such type are often called left-handed materials (LHM).

The existence of left-handed materials opens up the new research fields in modern science and technology. Devices, based on the waves that propagate in the left handed materials are the matters of intensive theoretical and experimental studies [2-4]. The situation, when a LHM slab divides two dielectrics with different permittivity rather than equal ones, is more frequent in the possible applications.

The aim of this work is to investigate the specific features of the electromagnetic waves that propagate along the interfaces of a left-handed planar slab that bounded by the conventional right-handed media with different permittivity.

1. TASK SETTING

The considered electromagnetic wave propagates along the planar waveguide structure that is made of left handed material slab with thickness Δ . This material is characterized by effective permittivity $\varepsilon(\omega)$ and permeability $\mu(\omega)$ that depend on the wave frequency and commonly expressed with the help of experimentally obtained expressions [4]:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \quad (1)$$

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}, \quad (2)$$

here ω_p is plasma frequency, ω_0 is the characteristic frequency of LHM. In further study it was considered LHM with $\omega_p / 2\pi = 10$ GHz and $\omega_0 / 2\pi = 4$ GHz and parameter $F = 0.56$ [4].

This left-handed media slab on both sides is bounded by semi-bounded conventional dielectrics with different constant permittivity and permeability. Dielectric with ε_1 and μ_1 is located at the one side of LHM slab and dielectric with ε_2 and μ_2 is located at another side.

Let's consider electromagnetic wave that propagates along the interface between left and right handed materials. It was assumed that wave disturbance tends to zero far away from LHM and the dependence of the wave components on time t and coordinate and z is expressed the following form:

$$E, H \propto E(x), H(x) \exp[i(k_3 z - \omega t)], \quad (3)$$

here x is coordinate rectangular to the wave propagation direction and to the LHM slab.

In this case the system of Maxwell equations split on two subsystems. One of them described the waves of H -type and another – wave of E -type.

The wave of E -type possesses the dispersion relation in the following form:

$$\frac{h_1 h_2 \varepsilon(\omega)^2 + \varepsilon_1 \varepsilon_2 \kappa^2}{\varepsilon(\omega) \kappa (h_2 \varepsilon_1 + h_1 \varepsilon_2)} \tanh(\kappa \Delta) = 0, \quad (4)$$

here $h_1 = \sqrt{k_3^2 - \varepsilon_1 \mu_1 k^2}$, $h_2 = \sqrt{k_3^2 - \varepsilon_2 \mu_2 k^2}$,

$\kappa = \sqrt{k_3^2 - \varepsilon(\omega) \mu(\omega) k^2}$, $k = \omega / c$, where c is the speed of light in vacuum.

In the region of left-handed material the wave field components, normalized on the $H_y(0)$, can be written as:

$$\begin{aligned} H_y(x) &= C_1 e^{\kappa x} + C_2 e^{-\kappa x} \\ E_x(x) &= k_3 (C_1 e^{\kappa x} + C_2 e^{-\kappa x}) / (k \varepsilon(\omega)) \\ E_z(x) &= i \kappa (C_1 e^{\kappa x} - C_2 e^{-\kappa x}) / (k \varepsilon(\omega)) \end{aligned} \quad (5)$$

here C_1 and C_2 – are wave field constants.

Wave field components, normalized on the $H_y(0)$, in the left dielectric region possess the form:

$$\begin{aligned} H_y(x) &= e^{h_1 x} \\ E_x(x) &= k_3 e^{h_1 x} / (k \varepsilon_1) \\ E_z(x) &= i h_1 e^{h_1 x} / (k \varepsilon_1). \end{aligned} \quad (6)$$

In the right dielectric region the wave field components, normalized on the $H_y(0)$, can be written as:

$$\begin{aligned} H_y(x) &= B e^{-h_2 x} \\ E_x(x) &= B k_3 e^{-h_2 x} / (k \varepsilon_2) \\ E_z(x) &= -i B h_2 e^{-h_2 x} / (k \varepsilon_2), \end{aligned} \quad (7)$$

here B is wave field constant. Such constants are of the following form:

$$\begin{aligned} B &= -2 h_1 \varepsilon_2 \varepsilon(\omega) e^{(h_2 + \kappa)\Delta} / (\varepsilon_1 \Psi) \\ C_1 &= h_1 \varepsilon(\omega) (h_2 \varepsilon(\omega) - \varepsilon_2 \kappa) / (\varepsilon_1 \kappa \Psi) \\ C_2 &= -h_1 \varepsilon(\omega) (h_2 \varepsilon(\omega) + \varepsilon_2 \kappa) e^{2\kappa\Delta} / (\varepsilon_1 \kappa \Psi), \end{aligned} \quad (8)$$

here $\Psi = (e^{2\kappa\Delta} + 1) \varepsilon(\omega) h_2 + (e^{2\kappa\Delta} - 1) \varepsilon_2 \kappa$.

Similarly wave of H -type possesses the dispersion relation in the following form

$$\frac{h_1 h_2 \mu(\omega)^2 + \mu_1 \mu_2 \kappa^2}{\mu(\omega) \kappa (h_2 \mu_1 + h_1 \mu_2)} \tanh(\kappa \Delta) = 0. \quad (9)$$

In the region of left-handed material the wave field components, normalized on the $E_y(0)$, can be written as

$$\begin{aligned} E_y(x) &= C_1 e^{\kappa x} + C_2 e^{-\kappa x} \\ H_x(x) &= -k_3 (C_1 e^{\kappa x} + C_2 e^{-\kappa x}) / (k \mu(\omega)) \\ H_z(x) &= -i \kappa (C_1 e^{\kappa x} - C_2 e^{-\kappa x}) / (k \mu(\omega)), \end{aligned} \quad (10)$$

here C_1 and C_2 – are wave field constants.

Wave field components, normalized on the $E_y(0)$, in the left dielectric region

$$\begin{aligned} E_y(x) &= e^{h_1 x} \\ H_x(x) &= -k_3 e^{h_1 x} / (k \mu_1) \\ H_z(x) &= -i h_1 e^{h_1 x} / (k \mu_1). \end{aligned} \quad (11)$$

In the right dielectric region the wave field components, normalized on the $E_y(0)$, can be written as

$$\begin{aligned} E_y(x) &= B e^{-h_2 x} \\ H_x(x) &= -B k_3 e^{-h_2 x} / (k \mu_2) \\ H_z(x) &= i B h_2 e^{-h_2 x} / (k \mu_2), \end{aligned} \quad (12)$$

here B is wave field constant. Such constants are of a following form

$$\begin{aligned} B &= -2 h_1 \mu_2 \mu(\omega) e^{(h_2 + \kappa)\Delta} / (\mu_1 \Psi) \\ C_1 &= h_1 \mu(\omega) (h_2 \mu(\omega) - \mu_2 \kappa) / (\mu_1 \kappa \Psi) \\ C_2 &= -h_1 \mu(\omega) (h_2 \mu(\omega) + \mu_2 \kappa) e^{2\kappa\Delta} / (\mu_1 \kappa \Psi), \end{aligned} \quad (13)$$

here $\Psi = (e^{2\kappa\Delta} + 1) \mu(\omega) h_2 + (e^{2\kappa\Delta} - 1) \mu_2 \kappa$.

2. MAIN RESULTS

The results of numerical calculation of dispersion equations for E - and H -waves for the case $\varepsilon_1 \neq \varepsilon_2$, $\mu_1 = \mu_2$ and are shown at Fig. 1. This case was analyzed in detail in our previous work [5]. The dispersion equations (4, 9) possesses six solutions. Curves marked by the numbers 1, 2, 3, 4 correspond to waves of E -type and curves marked by the numbers 5, 6 correspond to waves of H -type.

For the chosen set of parameters the region when central material demonstrates left-handed properties (simultaneously $\varepsilon(\omega) < 0$ and $\mu(\omega) < 0$) lies in the region where $1 < \Omega < 1.5$. The lines (a, d) corresponds with $\xi = \Omega \sqrt{\varepsilon_{1,2}}$, the line (c) corresponds to $\Omega = \omega / \omega_0 = 1$ and the line (b) - $\xi = \Omega \sqrt{\varepsilon(\omega) \mu(\omega)}$.

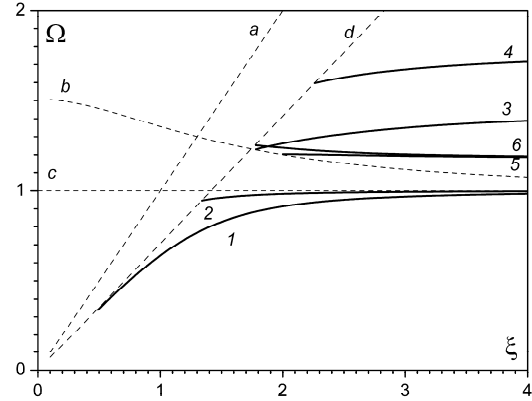


Fig. 1. The dependence of the normalized frequency $\Omega = \omega / \omega_0$ on dimensionless wave number $\xi = k_3 c / \omega_0$ for left-handed material slab thickness $\omega_0 \Delta / c = 2$, ($\varepsilon_1 = 1$, $\varepsilon_2 = 2$, $\mu_1 = \mu_2 = 1$ for all Figs)

Let's consider the phase $V_{ph} = \omega / k_3$ and group velocity V_{gr} for the surface E - and H -waves that corresponds to line 3, 5, and 6 on Fig.1. The dependence of the normalized phase velocity on normalized frequency for fixed value of slab thickness Δ is shown at Fig. 2.

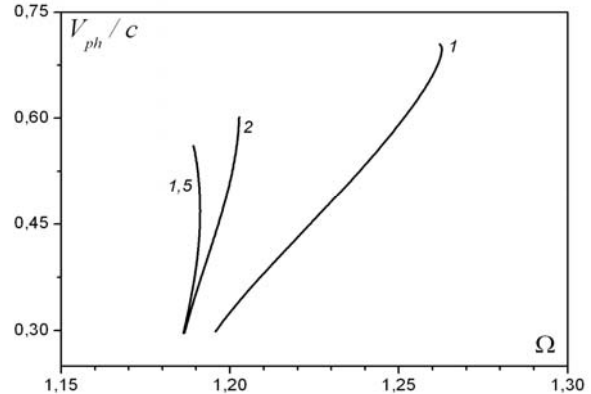


Fig. 2. The dependence of the normalized phase velocity V_{ph} / c of H -wave (line 6 on Fig. 1) on of the normalized frequency $\Omega = \omega / \omega_0$ for different left-handed material slab thickness $\omega_0 \Delta / c = 1; 1,5; 2$

It is shown that the surface waves are slow and strong frequency-dependent in that frequency region when central slab material demonstrates left-handed behavior. It is clear that this dependence is different for different values of slab thickness. So, the phase velocity of considered waves at fixed frequency is changed with changing value of slab thickness Δ . This so-called a geometric dispersion is shown on Fig. 3. At Fig. 3 the lines 1, 2 correspond to H -waves (see lines 5, 6 on Fig. 1) at fixed frequency ($\Omega = 1,198$ and $\Omega = 1,215$ accordingly) and the line 3 correspond to E -wave. (see line 3 on Fig. 1) at fixed frequency $\Omega = 1,315$.

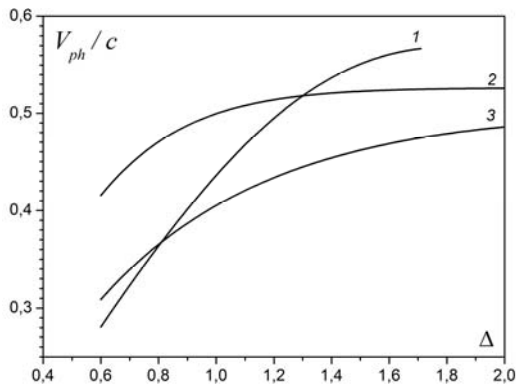


Fig. 3. The dependence of the normalized phase velocity V_{ph}/c on left-handed material slab thickness $\omega_0\Delta/c = 2$, for the surface E - and H -waves

Dependence of V_{ph} and V_{gr} on the slab thickness is most strong at the values of metamaterial thickness, which, as we know, while it is not succeeded to create.

Practical possibility of control on velocities of waves will be possible in such structures on condition of creation of so thin metamaterials.

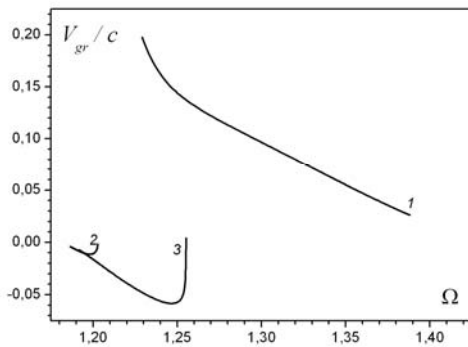


Fig. 4. The dependence of the normalized group velocity V_{gr}/c on of the normalized frequency $\Omega = \omega/\omega_0$ for left-handed material slab thickness $\omega_0\Delta/c = 2$

Group velocity of the wave is defined in such manner

$$V_{gr} = \partial\omega/\partial k_3. \quad (14)$$

The dependences of the normalized group velocity on normalized frequency for fixed value of slab thickness Δ are shown at Fig. 4.

ФАЗОВАЯ И ГРУППОВАЯ СКОРОСТИ СОБСТВЕННЫХ ЭЛЕКТРОМАГНИТНЫХ ВОЛН В СЛОЕ ЛЕВОСТОРОННЕГО МАТЕРИАЛА

Н.А. Азаренков, В.К. Галайдыч, В.П. Олефир, А.Е. Споров

Работа посвящена изучению фазовой и групповой скоростей собственных поверхностных электромагнитных волн, распространяющихся вдоль плоского слоя левостороннего материала, ограниченного диэлектриками с различными значениями диэлектрической проницаемости. Показано, что изменение толщины слоя метаматериала существенно влияет на фазовую и групповую скорости исследуемых волн.

ФАЗОВА ТА ГРУПОВА ШВИДКОСТІ ВЛАСНИХ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ У ШАРІ ЛІВОСТОРОННЬОГО МАТЕРІАЛУ

М.О. Азаренков, В.К. Галайдыч, В.П. Олефір, О.Є. Споров

Роботу присвячено вивченню фазової і групової швидкостей власних поверхневих електромагнітних хвиль, що поширюються уздовж плоского шару лівостороннього матеріалу, що обмежений діелектриками з різними значеннями діелектричної проникності. Показано, що зміна товщини шару метаматеріалу суттєво впливає на фазову і групову швидкості хвиль, що вивчаються.

At Fig. 4 the line 1 corresponds to E -wave (see line 3 on Fig. 1) and the lines 2, 3 correspond to H -waves (see lines 5, 6 on Fig. 1). It's clear, that the surface E -wave is a forward, and the surface H -waves are the backward waves.

Thus the group velocity of E -wave on an order is higher than one for H -wave.

All modes on Fig. 4 show the strong group velocity dependence on wave frequency. Moreover, there are the linear dependences in some frequency bands.

These features may be very useful for practical application both in linear and nonlinear regimes.

CONCLUSIONS

We have shown that the surface electromagnetic waves in structures which involved a LHM slab, may be both forward waves and backward waves. In the backward waves the directions of the group and phase velocities are opposite directed.

The considered waves are slow, have the clearly mutually spaced frequency bands and the different wave polarizations.

It was obtained that the phase and group velocity of considered waves at fixed frequency essentially depends on the value of slab thickness. This fact allows us to signal control more effectively.

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Article received 20.06.12