GRAIN CHARGING IN WEAKLY IONIZED PLASMA IN THE PRESENCE OF EXTERNAL MAGNETIC FIELD

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The problem of grain screening is solved numerically for the case of weakly ionized plasma in the presence of constant external magnetic field. The plasma dynamics is described within the drift-diffusion approach. The spatial distribution of the screened grain potential is studied and compared with the analytical estimates. It is shown that such potential has the Coulomb-like asymptotics with the effective charge dependent on the angle between the radius vector and magnetic field direction. Also, the potential can have nonmonotonic behavior in the direction parallel to magnetic field.

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INTRODUCTION

Understanding of the physical nature of various phenomena experimentally observed in dusty plasmas (dusty structure formation, excitation of dust-acoustic waves, existence of spatial domains free of dust particles (voids), etc.) requires the knowledge of the potentials of inter-grain interaction. This problem has been studied during many years both analytically and numerically (see, for example, review papers [1-6]). As a result of such studies many specific details concerning grain interactions (such as effects of grain charging [6-13], influence of the ionic bound states [14, 15], dynamical grain screening and ionic drag force [5,16-18], effect of external electric field [19] etc.) have been described. Nevertheless, many important problems still remain open. In particular, this concerns the influence of external magnetic field on the grain charging and effective potentials.

Notice that the grain charge is maintained by plasma currents and, thus, it should be calculated self-consistently using the condition of zero-value of total plasma current through the grain surface. This introduces additional complications since except the simplest case of collisionless plasma such calculations require application of numerical methods (see, for example, [10, 11]). In the case of plasma in an external magnetic field the problem becomes even more complicated in view of the extension of the problem dimensionality [20, 21].

Since grains in plasmas accumulate very large electric charge and thus the boundary-value problem for the effective grain potential in the general case is strongly nonlinear, studies of the effective potential usually also require numerical calculations. However, if the grain charge is known, there exists the possibility to perform some analytical estimates using the point sink model for the description of potential with regard to grain charging [7]. In particular, on the basis of this approximation the asymptotes of the effective grain potentials (the case of plasma in external magnetic field included) have been found [7-9]. Naturally, the accuracy of estimates can be verified only by comparison with numerical solutions.

The purpose of the present contribution is to perform numerical study of stationary charging currents, charge and effective potential of the grain embedded into weakly ionized plasma in the presence of external constant and uniform magnetic field. The main attention is paid to the effects produced by the external field on the potential distribution and its coordinate dependence. The comparison of exact numerical solutions with the asymptotes obtained analytically is also presented.

1. STATEMENT OF THE PROBLEM

Let us consider single spherical macroparticle (grain) embedded into infinite strongly collisional weakly ionized plasma in the presence of an external constant and uniform magnetic field. We assume that the grain absorbs all encountered electrons and ions. In the stationary case the grain charge is maintained by electron and ion currents which are equal to zero in sum. In the case under consideration plasma dynamics can be described by the continuity equations which have the following form:

$$\operatorname{div} \Gamma_\alpha(r) = 0, \tag{1}$$

where flux density is

$$\Gamma_\alpha(r) = -\hat{\mu}_\alpha n_\alpha(r)\nabla\Phi(r) - \hat{D}_\alpha \nabla n_\alpha(r), \tag{2}$$

$\hat{\mu}_\alpha$ is the plasma particle mobility tensor, $\hat{D}_\alpha$ is the diffusion tensor, subscript α ($\alpha = e,i$) labels plasma particle species, the rest of notation is traditional.

The electric potential $\Phi(r)$ satisfies the Poisson equation

$$\Delta\Phi(r) = -4\pi\sum_\alpha e_\alpha n_\alpha(r). \tag{3}$$

If the plasma particles velocity distribution is Maxwellian, then the Einstein relation is satisfied $\hat{\mu}_\alpha = \hat{D}_\alpha e_\alpha / T_\alpha$.

The diffusion tensor is [20] ($\mathbf{B} = B\mathbf{e}_z$, z-axis directed along the magnetic field)

$$\hat{D}_\alpha = D_{\alpha\parallel} \times \begin{pmatrix} \left(1+\gamma_\alpha^2\right)^{-1} & \gamma_\alpha\left(1+\gamma_\alpha^2\right)^{-1} & 0 \\ \gamma_\alpha\left(1+\gamma_\alpha^2\right)^{-1} & \left(1+\gamma_\alpha^2\right)^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

where $\gamma_\alpha = \Omega_\alpha / \nu_\alpha$, $\Omega_\alpha = e_\alpha B / m_\alpha c$ is the cyclotron frequency, ν_α is the collision frequency, $D_{\alpha\parallel} = v_{T\alpha}^2 / \nu_\alpha$.

The above stated problem was solved numerically using the FlexPDE program, in which the finite element method is realized.

As the problem has the cylindrical symmetry the unknown functions depends on two variables $n_\alpha(r) = n_\alpha(r_\perp, z)$, $\Phi(r) = \Phi(r_\perp, z)$. The calculations were performed for the domain $z > 0$, $r_\perp > 0$ bounded by the circles $r = a$ and $r = b$.

Eqs. (1) – (3) should be supplemented by the proper boundary conditions. To describe the absorbtion of electrons and ions the condition $n_\alpha(r)|_{r_\perp^2+z^2=a^2} = 0$ is used. The potential on the grain surface satisfies the Gauss law $\mathbf{n}\nabla\Phi(r)|_{r_\perp^2+z^2=a^2} = -q/a^2$, where a is the grain radius, q is the stationary grain charge which is determined by the equation

$$e_i \int_S \Gamma_i d\mathbf{S} + e_e \int_S \Gamma_e d\mathbf{S} = 0, \quad (5)$$

S is any spherical surface concentric with grain. On the outer boundary of the domain the conditions

$$n_\alpha(r)|_{r_\perp^2+z^2=b^2} = n_0, \quad \Phi(r)|_{r_\perp^2+z^2=b^2} = 0 \quad (6)$$

are satisfied, where n_0 is the unperturbed density of particles. On the other boundaries: $\mathbf{n}\nabla\Phi(r)|_{z=0} = 0$, $\mathbf{n}\nabla\Phi(r)|_{r_\perp=0} = 0$, $\mathbf{n}\nabla n(r)|_{z=0} = 0$, $\mathbf{n}\nabla n(r)|_{r_\perp=0} = 0$.

The charge is a parameter in one of the boundary conditions. Its value was adjusted until the condition (5) was satisfied with some tolerance. The following parameters were used in this computation $D_{eP}/D_{iP} = 1000$, $T_e = T_i$. Results, which concerning charging currents and the stationary grain charge were obtained for $b = 300r_D$, where r_D is Debye length. Increase of b influence the result slightly.

In order to test the computational program we start the calculations with the limiting case of plasma without external magnetic field. The reliable results for the case of the absence of external magnetic field can be taken from [10]. Since the calculations of the grain charge are consistent with the boundary condition (5) and thus obtained values are in good agreement with those from [10].

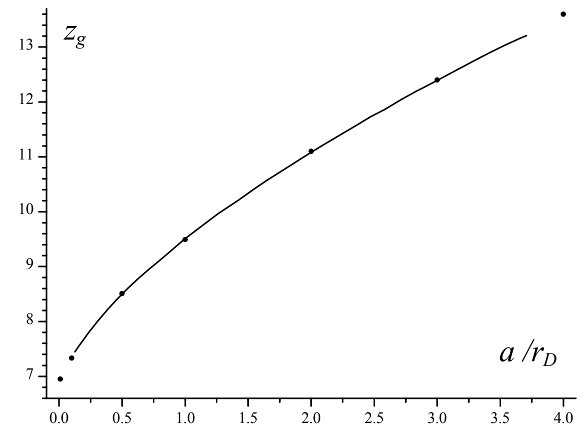

Fig. 1. Dimensionless grain charge $z_g = -eq/aT_e$ vs. grain radius. Points are our calculations, solid line is taken from [10]

requires the knowledge of the electric potential, it is sufficient to perform comparison of the results for the grain charge only. Below we present the dependence of the grain charge on grain radius (Fig. 1). As is seen, the

2. GRAIN IN PLASMA WITH MAGNETIZED ELECTRONS AND UNMAGNETIZED IONS

Now, let us consider the results of calculations of the effective grain potential in the presence of external magnetic field. The strength of magnetic field induction B is determined by ion cyclotron to collision frequency ratio Ω_i/ν_i. The same ratio for electrons is $\Omega_e/\nu_e = (\Omega_i/\nu_i)(D_{eP}/D_{iP})(T_i/T_e) = 1000\Omega_i/\nu_i$.

Before to discuss the results of calculations notice, that analytical estimates predict the existence of the Coulomb-like asymptotics of the potential with the effective charge dependent on the angle between the radius vector and external magnetic field [8].

$$\Phi(r) = -\frac{1}{r}\sum_\alpha \frac{I_\alpha}{k_D^2 D_{\alpha\perp}\sqrt{1+(D_{\alpha\parallel}/D_{\alpha\perp}-1)\sin^2\theta}}, \quad (7)$$

where $I_i = I$, $I_e = -I$.

The typical distribution of normalized potential $-e\Phi/T_e$ lines for two different values of the magnetic field is shown on Figs. 2, 3. The grain radius is $a = 0.5r_D$, $\Omega_i/\nu_i = 0.02$, $\Omega_i/\nu_i = 0.05$. Dashed lines are plotted by asymptotic formula (7) with current value taken from the calculations. To achieve a good agreement of calculated data with asymptotic formula it was necessary to increase the computation domain, namely $b = 10000r_D$ for $\Omega_i/\nu_i = 0, 0.02$ and $b = 5000r_D$ for $\Omega_i/\nu_i = 0.05$.

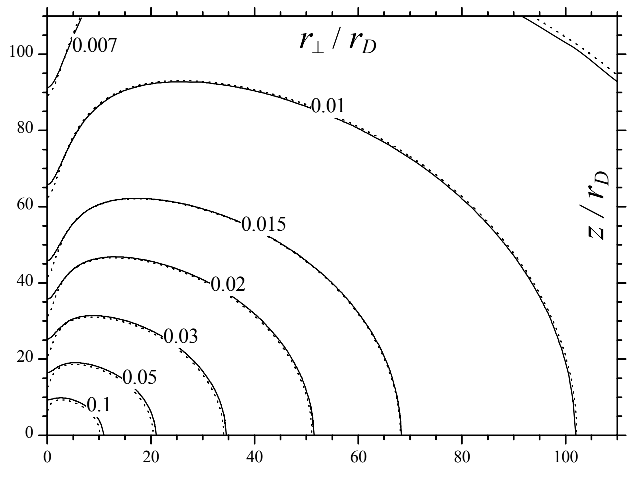

Fig. 2. Isolines of the dimensionless potential $-e\Phi/T_e$, $\Omega_i/\nu_i = 0.02$, $a = 0.5r_D$. Solid lines are calculated, dotted lines correspond to (7)

As is seen in the case under consideration the potential lines manifest specific behavior, namely, the potential decreases more slowly in the perpendicular direction then in the parallel one at distances larger than the Debye length. This effect becomes more pronounced with the

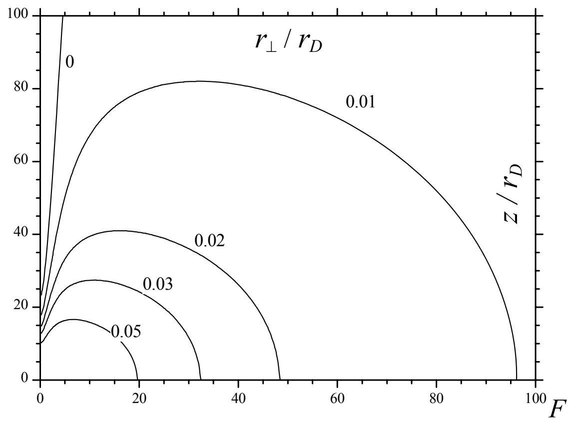

Fig. 3. Isolines of the dimensionless potential $-e\Phi / T_e$, $\Omega_i / \nu_i = 0.05$, $a = 0.5 r_D$

magnetic field growth. The potential is the symmetric one near the grain surface only. It is interesting to note that the asymptotic formula (7) gives satisfactory agreement with the numerical calculations.

The coordinate dependencies of the potential perpendicularly and along magnetic field, i.e. along z and $r_\perp$ axis are presented on Figs. 4, 5.

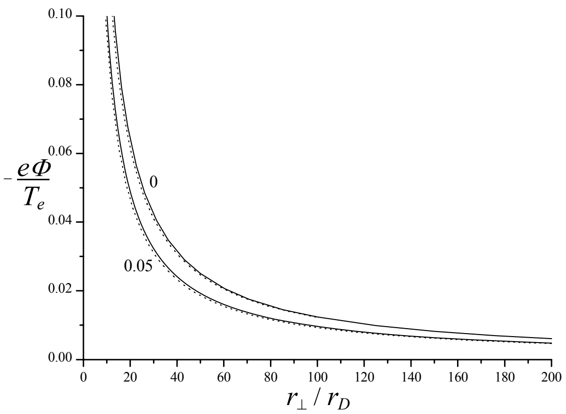

Fig. 4. Dimensionless potential $-e\Phi / T_e$ distribution perpendicularly the magnetic field, $a = 0.5 r_D$, $\Omega_i / \nu_i = 0, 0.05$. Solid lines are calculated, dotted lines correspond to (7)

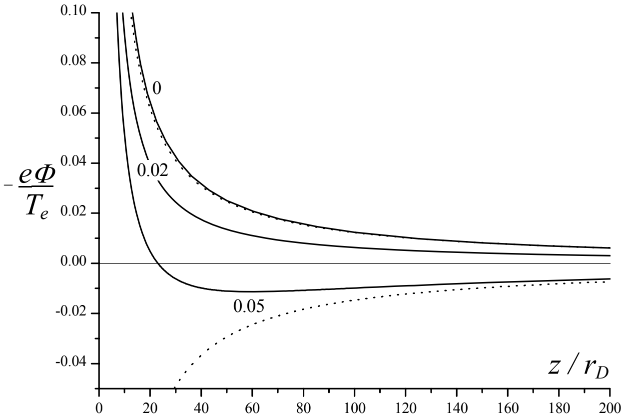

Fig. 5. Dimensionless potential $-e\Phi / T_e$ distribution along the magnetic field, $a = 0.5 r_D$, $\Omega_i / \nu_i = 0$, 0.02, 0.05. Solid lines are calculated, dotted lines correspond to (7)

Figures show, that the potential distribution near the z-axis become negative in the strong magnetic field ($\Omega_i / \nu_i = 0.05$). The asymptotic formula (7) describes the potential distribution better perpendicularly the magnetic field than along and for weak magnetic field.

The modification of potential distribution near the grain due to the magnetic field is described by Fig. 6. One can see the decrease of surface potential with the increase of ion cyclotron to collision frequencies ratio. It is worth to note, that potential distribution near the grain is sensitive to weak magnetic field. Even small value of $\Omega_i / \nu_i = 0.001$ leads to noticeable deviation of surface potential. This corresponds to Fig. 7, on which the grain charge z_g vs. Ω_i / ν_i is plotted. The increase of magnetic field leads to rapid decrease of the grain charge in the domain $\Omega_i / \nu_i :\ 0.001$ and more slow decrease in the

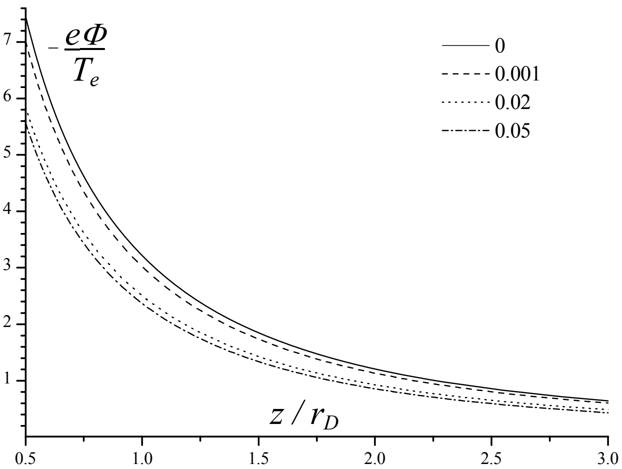

Fig. 6. Calculated dimensionless potential $-e\Phi / T_e$ distribution along the magnetic field, $a = 0.5 r_D$, $\Omega_i / \nu_i = 0, 0.001, 0.02, 0.05$

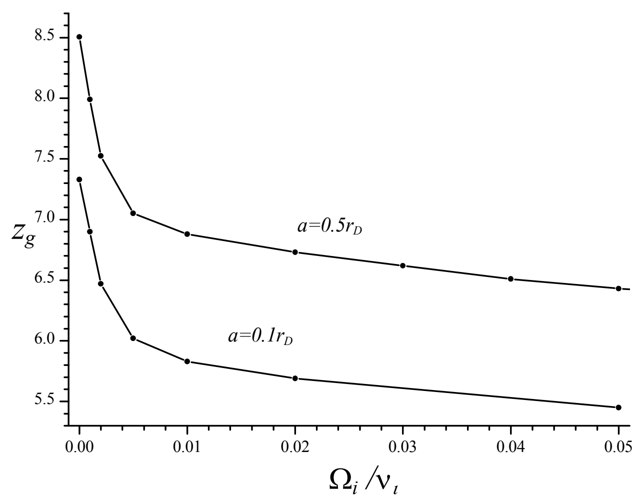

Fig. 7. Dimensionless grain charge $z_g = -eq / aT_e$ vs. Ω_i / ν_i for $a = 0.5 r_D, 0.1 r_D$

domain $\Omega_i / \nu_i :\ 0.01$. It is explained by the decrease of plasma particles fluxes, see Fig. 8. As expected, the smaller grain collects the smaller current. Also, the computations have shown, that potential distribution perpendicularly the magnetic field is similar to Fig. 6 (potential isolines are close to circular).

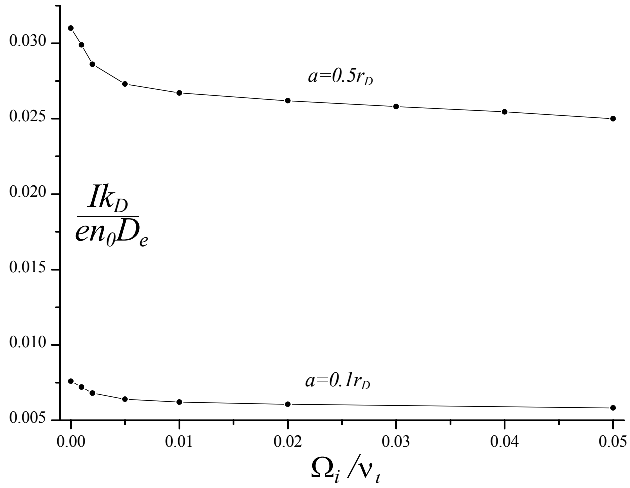

Fig. 8. Dimensionless flux $Ik_D / en_0 D_e$ vs. Ω_i / ν_i for $a = 0.5r_D, 0.1r_D$

Charging currents of electrons and ions to a spherical dust grain in a uniform magnetized collisionless dusty plasma have been examined in [23]. It was found that the external magnetic field reduces the charging currents.

The expression of Fourier image of the potential in weakly ionized magnetized plasma was obtained in [8]

$$\phi_k=\frac{4\pi}{k^2+k_D^2}\left(q-\sum_\alpha \frac{I_\alpha}{k_\perp^2 D_{\alpha\perp}+k_z^2 D_{\alpha\parallel}}\right). \qquad (8)$$

The inverse transformation yields

$$\phi(r)=\frac{1}{(2\pi)^3}\int dk e^{ikr}\phi(k)=\frac{q}{r}e^{-k_D r}-\sum_\alpha \phi_{2\alpha}, \qquad (9)$$

where

$$\phi_{2\alpha}=I_\alpha\int_0^\infty dk_\perp \frac{k_\perp J_0(k_\perp r_\perp)}{k_\perp^2(D_{\alpha\parallel}-D_{\alpha\perp})+k_D^2 D_{\alpha\parallel}} \times\left(\frac{e^{-k_\perp|z|\sqrt{D_{\alpha\perp}/D_{\alpha\parallel}}}}{k_\perp\sqrt{D_{\alpha\perp}/D_{\alpha\parallel}}}-\frac{e^{-|z|\sqrt{k_\perp^2+k_D^2}}}{\sqrt{k_\perp^2+k_D^2}}\right). \qquad (10)$$

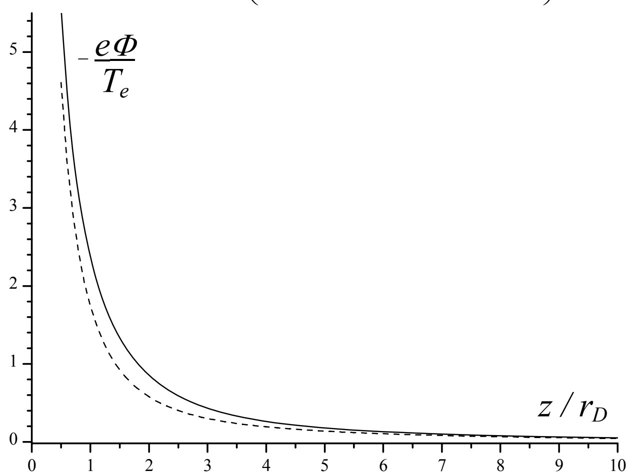

Fig. 9. Dimensionless potential $-e\Phi / T_e$ distribution along the magnetic field, $a = 0.5r_D$, $\Omega_i / \nu_i = 0.05$. Solid line is calculated, dashed line correspond to (9)

Expressions (9), (10) have two parameters, namely grain charge q and current I, which we take from the computation. Fig. 9 shows, that formula (9) gives the correct asymptotic behavior and describe the potential quite well in the rest range of distances. So, (9) is more correct as compare to the simple formula (7).

Isolines of the dimensionless charge density $(n_i - n_e)/n_0$ are presented on Fig. 10.

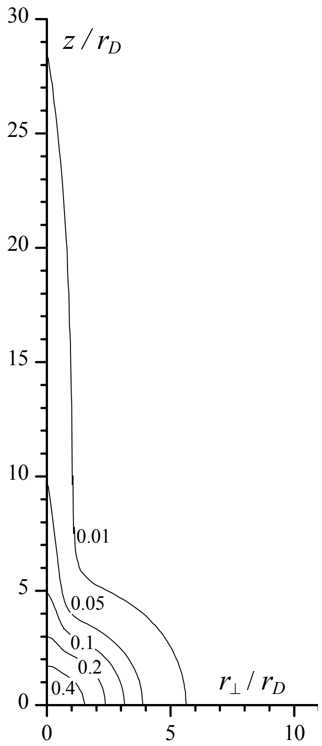

Fig. 10. Isolines of the dimensionless charge density $(n_i - n_e)/n_0$, $\Omega_i / \nu_i = 0.02$, $a = 0.5r_D$

SUMMARY AND CONCLUSIONS

Thus, we presented numerical solution of the problem of highly-charged grain screening in weakly ionized plasma in the presence of external magnetic field. The selfconsistent calculations of the charging currents and grain charge are presented as well. Obtained results for charging currents and grain charge make it possible to use these quantities for analytical estimates of effective potential which turned out to be in rather good agreement with the numerical solutions.

The obtained results show that even weak magnetic field (electrons and ions are unmugnetized) can produce considerable effects on the grain charge and charging currents. In particular, the appearance of external magnetic field leads to sharp decrease of these quantities. At the same time weak magnetic field does not change substantially the spatial symmetry of the potential.

The increase of the magnetic field strength up to the values of electron magnetization leads to the loss of spherical symmetry of the effective potential – in the parallel direction the potential decreases faster than in the perpendicular one. Moreover, the further increase of the external magnetic field can change the sign of the potential at some distance from the grain along the magnetic field line and even generate the non-monotonic spatial dependence. This means that for the grains located on the same magnetic field line weak attraction can be observed. This effect can be explained by the specific selfconsistent spatial charge distribution around the grain.

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ЗАРЯДКА ПЫЛИНКИ В СЛАБОИОНИЗИРОВАННОЙ ПЛАЗМЕ ПРИ НАЛИЧИИ ВНЕШНЕГО МАГНИТНОГО ПОЛЯ

А.И. Момот

Задача экранирования пылинки решается численно для случая слабоионизированной плазмы в присутствии постоянного внешнего магнитного поля. Динамика плазмы описывается в рамках дрейфово-диффузионного подхода. Изучено пространственное распределение экранированного потенциала пылинки и проведено сравнение с аналитическими оценками. Показано, что такой потенциал имеет кулоновскую асимптотику с эффективным зарядом, который зависит от угла между радиус-вектором и направлением магнитного поля.

ЗАРЯДЖАННЯ ПОРОШИНКИ В СЛАБОІОНІЗОВАНІЙ ПЛАЗМІ ЗА НАЯВНІСТЮ ЗОВНІШНЬОГО МАГНІТНОГО ПОЛЯ

А.І. Момот

Задача екранування порошинки розв'язується чисельно для випадку слабоіонізованої плазми за наявністю постійного зовнішнього магнітного поля. Динаміка плазми описується в рамках дрейфово-дифузійного підходу. Вивчено просторовий розподіл екранованого потенціалу порошинки і виконано порівняння з аналітичними оцінками. Показано, що такий потенціал має кулонівську асимптотику з ефективним зарядом, що залежить від кута між радіус-вектором та напрямком магнітного поля.