# EFFECT OF PLASMA ROTATION ON THE RESONANCE MAGNETIC PERTURBATIONS AT THE EDGE OF TOKAMAK PLASMAS

***I.M. Pankratov[1,2], I.V. Pavlenko[2], O.A. Pomazan[2] and A.Ya. Omelchenko[1]***

***[1]Institute of Plasma Physics NSC "Kharkov Institute of Physics and Technology", Kharkov, Ukraine;***
***[2]V.N. Karazin Kharkov National University, Kharkov, Ukraine***
***E-mail: pankratov@kipt.kharkov.ua***

In the frame of one-fluid MHD the pressure perturbation resonant excitation by external low frequency helical magnetic perturbations near the plasma edge is investigated. The plasma rotation plays a key role in this phenomenon. The plasma response has been taken into account. These pressure perturbations may affect stability of the ballooning and peeling modes.
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## INTRODUCTION

Control of Edge Localized Modes (ELMs) is a critical issue of the present day large tokamaks and future ITER operation [1, 2].

Experiments at DIII-D have shown that ELMs can be suppressed by small external low frequency helical magnetic perturbations [3, 4].

In Ref. [5] the influence of an external helical field on the equilibrium of ideal plasma was investigated in the frame of MHD theory. A perfect shielding of the external resonant field was assumed.

Early in the frame of one-fluid MHD a possibility of the pressure perturbation resonant excitation (due to the plasma rotation) by external helical magnetic perturbations near the plasma edge has been shown [6], when the plasma response has being taken into account (a perfect shielding is not assumed).

In the present paper, the influence of these pressure perturbations on external helical magnetic field near the plasma edge is investigated. Considered plasma parameters are closed to DIII-D experiments [3, 4]. Poloidal and toroidal plasma rotations are taken into account. The plasma response takes into account.

Note, that the toroidal rotation effects on ELM behavior were observed in experiments [4, 7].

## 1. BASIC EQUATIONS

We start from the one-fluid MHD equations

$$\rho\frac{d\mathbf{V}}{dt} = -\nabla p - \nabla\cdot\boldsymbol{\pi}_i + \frac{1}{c}[\mathbf{J}\times\mathbf{B}],\ \ \frac{dp}{dt} + \gamma_0 p\, div\mathbf{V} = 0, \quad (1)$$

$$rot\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},\ \ rot\mathbf{B} = \frac{4\pi}{c}\mathbf{J},\ \ div\mathbf{B} = 0, \quad (2)$$

$$div\mathbf{J} = 0, \quad (3)$$

$$\mathbf{J} = \sigma\left(\mathbf{E} + \frac{1}{c}[\mathbf{V}\times\mathbf{B}]\right), \quad (4)$$

where $\rho$ is the plasma mass densities, $p$ is the plasma pressure, $\mathbf{J}$ is the current density, $\sigma$ is the conductivity and $\boldsymbol{\pi}_i$ is the ion gyroviscosity tensor, respectively.

We consider a current carrying toroidal plasma with nested equilibrium circular magnetic surfaces ( $\rho_0$ is the radius of the magnetic surfaces, $\omega_0$ is the poloidal angle in the cross-section $\varsigma = const$, $\varsigma$ is the toroidal angle). Each magnetic surface is shifted with respect to the magnetic axis ( $\xi$ is the shift, $R$ is the radius of the magnetic axis). The equilibrium toroidal contravariant component of the magnetic field, $B_0^{\varsigma} = \Phi'/\left(2\pi\sqrt{g}\right)$, is large with respect to the poloidal one, $B_0^{\theta} = \chi'/\left(2\pi\sqrt{g}\right)$, $q(a) = \Phi'/\chi'$, $\Phi'$ and $\chi'$ are the radial derivatives of toroidal and poloidal fluxes, respectively. The known expressions for metric tensor are used [8].

On each magnetic equilibrium surface (see, e.g. [8]) we introduce a straight magnetic field line coordinate system ( $a, \theta, \varsigma$ ) $\rho_0 = a$, $\omega_0 = \theta + \lambda(a)\sin\theta$,

$$\lambda(a) = -\xi'(a) - a/R, \quad (5)$$

$$\xi'(a) = \frac{1}{aR}\left(\frac{\chi'(a)}{2\pi R}\right)^{-2}\int_0^a\left[16\pi p_0(b) + \left(\frac{\chi'(b)}{2\pi R}\right)^2\right]bdb\cdot \quad (6)$$

Assuming periodicity in both $\theta$ and $\zeta$, we take the perturbations in the form

$$X(a,\theta,\zeta,t) = \sum_{m,n} X_{mn}(a)\exp[i(m\theta - n\zeta - \omega t)], \quad (7)$$

where $\omega$ is the frequency of the external perturbation**.**

Assuming that magnetic perturbation $B^{\varsigma} \approx 0$, for perturbations with $m >> 1,\ nq >> 1$ from Eqs. (1) - (4) in a linear approximation in *1/R* the next equations were found (derivatives with respect to radius are denoted by the prime) [6]:

$$F_m(a)\left[i(a^2B_m^{\theta})' + mB_m^a\right] + \frac{4\pi SqR}{B_{0\zeta}^2(a)}p_0'(B_{m-1}^a + B_{m+1}^a) + \frac{4\pi iqR}{B_{0\zeta}^2(a)}p_0'a(B_{m-1}^{\theta} - B_{m+1}^{\theta}) + \frac{2aR}{c}\frac{B_m^a}{B_{0\zeta}(a)}(J'/a)' -$$
$$-\frac{8\pi im}{B_{0\zeta}(a)}\frac{a}{R}\left(\mu^2 - 1 + \frac{ap_0'}{B_{0\omega_0}^2(a)} - \frac{R}{a}S\xi'\right)p_m - \frac{4\pi i}{B_{0\zeta}(a)}(ap_{m-1}' - ap_{m+1}') + \frac{4\pi i}{B_{0\zeta}(a)}\left[(m-1)p_{m-1} + (m+1)p_{m+1}\right] = 0, \quad (8)$$

$$p_m = -\frac{i}{\Omega_m^2}\left\{\frac{c_s^2}{R}\frac{B_{\zeta 0}}{B_0}F_m(a)\left(\rho_0 V'_{0\|}V^a_{Em} + p'_0\frac{B^a_m}{B_0}\right) + \omega_m p'_0 V^a_m + \frac{\omega_m \rho_0 c_s^2}{R}\left[\frac{(aV^a_{m-1})'}{m-1} - \frac{(aV^a_{m+1})'}{m+1} - (V^a_{m-1}+V^a_{m+1})\right]\right\}, \quad (9)$$

$$\omega_{im}B^a_m = -F_m(a)\frac{B_{0\varsigma}}{R}V^a_m - \frac{ic^2 m}{4\pi\sigma a^2}\left[i(a^2 B^\theta_m)' + mB^a_m\right]. \quad (10)$$

In Eqs. (8) - (10) $F_m(a) = m\mu(a) - n$, $S = a\frac{q'}{q}$,

$$B_{0\varsigma}(a) = \Phi'/2\pi a, \quad B_{0\omega_0}(a) = \chi'/2\pi R,$$

$$\mu = 1/q, \quad (aB^a_m)' + imaB^\theta_m = 0,$$

$$\Omega_m^2(a) = \omega_{im}\omega_m - \frac{c_s^2}{R^2}F_m^2(a), \quad c_s^2 = \gamma_0\frac{p_0}{\rho_0}, \quad (11)$$

$$\omega_m = \omega - \frac{B_{0\varsigma}}{B_0}\frac{F_m(a)}{R}V_{0\|} + \frac{B_{0\varsigma}}{B_0}\frac{m}{a}c\frac{E_{0a}}{B_0}, \quad (12)$$

$$\omega_{im} = \omega - \frac{B_{0\varsigma}}{B_0}\left[\frac{F_m(a)}{R}V_{0\|} + \frac{m}{a}\left(c\frac{p'_{0i}}{en_0B_0} - c\frac{E_{0a}}{B_0}\right)\right]. \quad (13)$$

In our consideration all poloidal harmonic amplitudes of perturbations have finite values. The number of poloidal harmonics with finite values of amplitudes depends on the antenna spectrum (external perturbation). Equilibrium parameters are denoted by the subscript 0. We took into account the equilibrium poloidal plasma rotation due to the existence of an equilibrium radial electric field $E_{0a}$, the ion diamagnetic drift and the parallel with respect to equilibrium magnetic field plasma rotation with a velocity $V_{0\|}$.

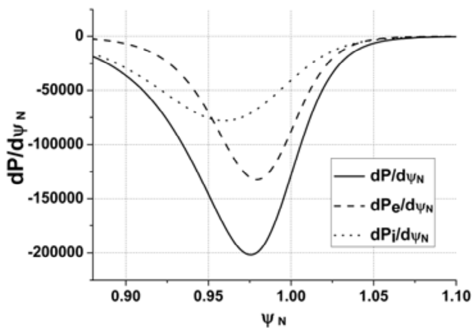


*Fig.1. Equilibrium pressure gradients (in Pa)*

For simplicity we consider case $c_s$ = 0 and $\omega = 0$. Near the plasma edge the inequality $S\xi' >> 1$ ($S$ ~ 4) takes place. From Eqs. (8) - (10) we get in this case

$$p_m = -\frac{ip'_0 V^a_m}{\omega_{im}} = \frac{ip'_0 R}{F_m(a)}\frac{1}{B_{0\varsigma}}\left[B^a_m + \frac{i}{\omega_{im}}\frac{c^2 m}{4\pi\sigma(a)a^2}\left[i(a^2B^\theta_m)' + mB^a_m\right]\right], \quad (14)$$

$$\frac{1}{a_N}\frac{d}{da_N}\left(a_N\frac{d}{da_N}(a_NB^a_m)\right) - \frac{m^2}{a_N^2}(a_NB^a_m) - \frac{m}{a_N^2}Q_m(a_NB^a_m) = 0, \quad (15)$$

where

$$Q_m(a_N) = \frac{K_m(a_N)A(a_N)\left(mK_m(a_N)F_m^2(a_N) + i\frac{B_{0\varsigma}}{B_0}A(a_N)\frac{c}{4\pi\sigma(a_N)a}\right)}{\left(mK_m(a_N)F_m^2(a_N)\right)^2 + \left(A(a_N)\frac{c}{4\pi\sigma(a_N)a}\right)^2}, \quad (16)$$

$$K_m(a_N) = F_m(a_N)\frac{a}{R}\frac{V_{0\|}}{mc} + \frac{1}{B_0}\left(\frac{1}{p_{0i}}\frac{dp_{0i}}{da_N}\frac{T_i(a_N)}{ea_{pl}} - E_{0a}(a_N)\right), \quad (17)$$

$$A(a_N) = \frac{8\pi}{B_{0\varsigma}^2}a_N\frac{dp_0}{da_N}m^2(\mu^2 - 1 - \frac{R}{a}S\xi'), \quad a_N = a/a_{\rm pl}. \quad (18)$$

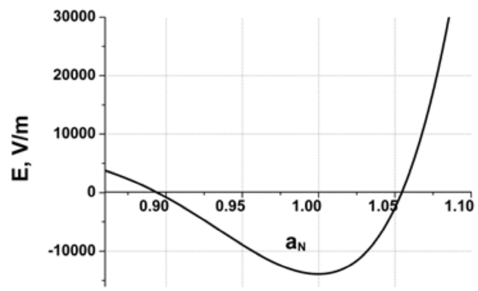


*Fig. 2. Equilibrium radial electric field*

## 2. DISCUSSIONS

Poloidal modes $m$ = 9…14 and toroidal mode $n$ = 3 are considered. The profile $q(\psi_N)$ near plasma edge close to the DIII-D experiments is used ([3, 4]). From Eqs. (14), (15) the pressure perturbation is presented in the next form:

$$p_m(a_N) = a_N\frac{dp_0}{da_N}\frac{R}{a}\frac{mK_m(a_N)F_m(a_N)\left(im\frac{B_{0\varsigma}}{B_0}K_m(a_N)F_m^2(a_N) - A(a_N)\frac{c}{4\pi\sigma(a_N)a}\right)}{\left(mK_m(a_N)F_m^2(a_N)\right)^2 + \left(A(a_N)\frac{c}{4\pi\sigma(a_N)a}\right)^2}\frac{B^a_m}{B_0}. \quad (19)$$

The pressure perturbation resonant excitation by external low frequency helical magnetic perturbations near the plasma edge is possible when $F_m(a_N) \approx 0$ or $K_m(a_N) \approx 0$ (Eq. (19)). The case $K_m(a_N) \approx 0$ occurs during the plasma rotation only. It may affect the excitation of ballooning and peeling modes because of a plasma pressure change. In Figs. 1, 2 the behaviors of the equilibrium pressure gradients and equilibrium radial electric field $E_{0a}$ are shown for typical DIII-D experimental conditions ([3, 4]) as functions of the normalized poloidal flux $\psi_N$ .

In Fig. 3 and Fig. 4 the radial profiles of $ReQ_m$ and $ImQ_m$ are shown, respectively ($m = 11$). Here $B_{0\varsigma} > 0$ . If $V_{0\|} = 0$ the strong change in profile of $Q_m(a_N)$ is visible near $a_N \approx 0.948$ where $F_{11}(a_N) = 0$ only. And effect of the resonance $K_m(a_N) \approx 0$ at $a_N \approx 0.958$ is small. When $V_{0\|} \neq 0$ the effect of the resonance $K_m(a_N) \approx 0$ at $a_N \approx 0.958$ is strong and depends on direction of rotation. Note that the position of this resonance does not depend on *m* practically. But position of $F_m(a_N) \approx 0$ resonance depends on *m* strongly.

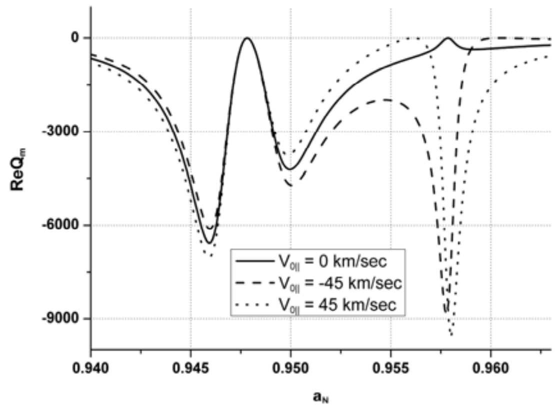


*Fig. 3. Radial profiles of $ReQ_m$*

Note that

$$\left(rot\mathbf{B}\right)^{\varsigma} \approx -\frac{i}{aR}\left[i(a^2B_m^{\theta})' + mB_m^a\right] = \\ = \frac{i}{maR}\left[\frac{d}{da}\left(a\frac{d}{da}(aB_m^a)\right) - \frac{m^2}{a}(aB_m^a)\right]. \quad (20)$$

Hence, parameter $Q_m$ is characteristic of the plasma current response on penetration of external perturbation (see Eq. (15)).

## CONCLUSIONS

The strong influence of toroidal plasma rotation on pressure perturbation resonant excitation by external low frequency helical magnetic perturbations near the plasma edge is shown. The plasma rotation and plasma response play a key role in this phenomenon.

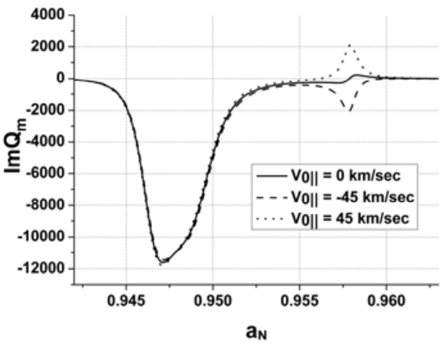


*Fig. 4. Radial profiles of Im $Q_m$*

Obtained results may be used to control of the plasma stability for experiments in tokamaks JET, DIII-D, TEXTOR and future ITER operation.

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### ВЛИЯНИЕ ВРАЩЕНИЯ ПЛАЗМЫ НА РЕЗОНАНСНЫЕ МАГНИТНЫЕ ВОЗМУЩЕНИЯ ВБЛИЗИ КРАЯ ПЛАЗМЫ ТОКАМАКА

***И.М. Панкратов, И.В. Павленко, О.А. Помазан, А.Я. Омельченко***

В рамках одножидкостной МГД исследовано резонансное возбуждение возмущений давления у края плазмы внешними низкочастотными винтовыми возмущениями магнитного поля. Вращение плазмы играет ключевую роль в этом явлении. Учтен отклик плазмы. Эти возмущения давления могут влиять на устойчивость баллонных и пилинг-мод.

### ВПЛИВ ОБЕРТАННЯ ПЛАЗМИ НА РЕЗОНАНСНІ МАГНІТНІ ЗБУРЕННЯ ПОБЛИЗУ КРАЮ ПЛАЗМИ ТОКАМАКА

***І.М. Панкратов, І.В. Павленко, О.О. Помазан, О.Я. Омельченко***

У рамках однорідинної МГД досліджено резонансне збудження збурень тиску біля краю плазми зовнішніми низькочастотними гвинтовими збуреннями магнітного поля. Обертання плазми відіграє ключову роль у цьому явищі. Враховано відгук плазми. Ці збурення тиску можуть впливати на стійкість балонних та пілінг-мод.